5 Solving Systems of Linear Equations

SEE the Big Idea

Fishing *(p. 279)*

Pets *(p. 266)*

Delivery Vans *(p. 250)*

Roofing Contractor *(p. 238)*

Drama Club *(p. 244)*

Maintaining Mathematical Proficiency

Graphing Linear Functions

Example 1 Graph $3 + y = \frac{1}{2}x$.

Step 1 Rewrite the equation in slope-intercept form.
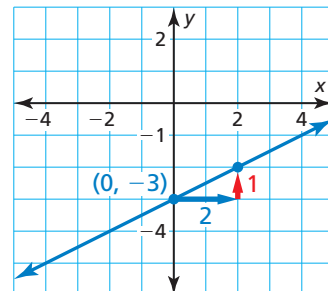$$y = \frac{1}{2}x - 3$$

Step 2 Find the slope and the y-intercept.
$$m = \frac{1}{2} \text{ and } b = -3$$

Step 3 The y-intercept is -3. So, plot $(0, -3)$.

Step 4 Use the slope to find another point on the line.
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$$

Plot the point that is 2 units right and 1 unit up from $(0, -3)$. Draw a line through the two points.

Graph the equation.

1. $y + 4 = x$ **2.** $6x - y = -1$ **3.** $4x + 5y = 20$ **4.** $-2y + 12 = -3x$

Solving and Graphing Linear Inequalities

Example 2 Solve $2x - 17 \le 8x - 5$. Graph the solution.

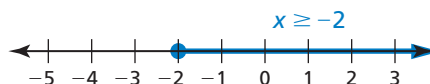|	
$2x - 17 \le \quad 8x - 5$	Write the inequality.
$\underline{+ 5 \qquad\qquad + 5}$	Add 5 to each side.
$2x - 12 \le \quad 8x$	Simplify.
$\underline{- 2x \qquad\quad - 2x}$	Subtract $2x$ from each side.
$-12 \le \quad 6x$	Simplify.
$\dfrac{-12}{6} \le \dfrac{6x}{6}$	Divide each side by 6.
$-2 \le x$	Simplify.

▶ The solution is $x \ge -2$.

Solve the inequality. Graph the solution.

5. $m + 4 > 9$ **6.** $24 \le -6t$ **7.** $2a - 5 \le 13$

8. $-5z + 1 < -14$ **9.** $4k - 16 < k + 2$ **10.** $7w + 12 \ge 2w - 3$

11. ABSTRACT REASONING The graphs of the linear functions g and h have different slopes. The value of both functions at $x = a$ is b. When g and h are graphed in the same coordinate plane, what happens at the point (a, b)?

Mathematical Practices

Mathematically proficient students use technological tools to explore concepts.

Using a Graphing Calculator

Core Concept

Finding the Point of Intersection

You can use a graphing calculator to find the point of intersection, if it exists, of the graphs of two linear equations.

1. Enter the equations into a graphing calculator.

2. Graph the equations in an appropriate viewing window, so that the point of intersection is visible.

3. Use the *intersect* feature of the graphing calculator to find the point of intersection.

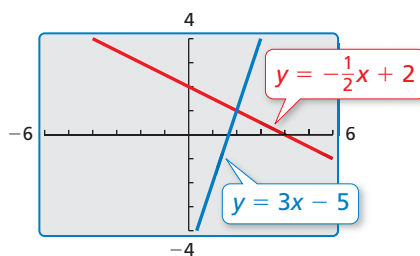**EXAMPLE 1** **Using a Graphing Calculator**

Use a graphing calculator to find the point of intersection, if it exists, of the graphs of the two linear equations.
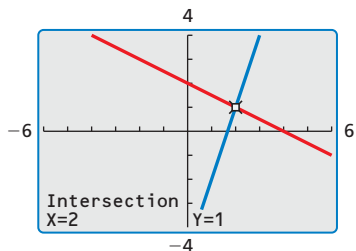
$$y = -\tfrac{1}{2}x + 2 \qquad \text{Equation 1}$$

$$y = 3x - 5 \qquad \text{Equation 2}$$

SOLUTION

The slopes of the lines are not the same, so you know that the lines intersect. Enter the equations into a graphing calculator. Then graph the equations in an appropriate viewing window.

Use the *intersect* feature to find the point of intersection of the lines.

▶ The point of intersection is (2, 1).

Monitoring Progress

Use a graphing calculator to find the point of intersection of the graphs of the two linear equations.

1. $y = -2x - 3$
   $y = \tfrac{1}{2}x - 3$

2. $y = -x + 1$
   $y = x - 2$

3. $3x - 2y = 2$
   $2x - y = 2$

5.1 Solving Systems of Linear Equations by Graphing

Essential Question How can you solve a system of linear equations?

Writing a System of Linear Equations

Work with a partner. Your family opens a bed-and-breakfast. They spend $600 preparing a bedroom to rent. The cost to your family for food and utilities is $15 per night. They charge $75 per night to rent the bedroom.

a. Write an equation that represents the costs.

$$\underset{\text{(in dollars)}}{\text{Cost, } C} = \underset{\text{night}}{\$15 \text{ per}} \cdot \underset{\text{nights, } x}{\text{Number of}} + \$600$$

b. Write an equation that represents the revenue (income).

$$\underset{\text{(in dollars)}}{\text{Revenue, } R} = \underset{\text{night}}{\$75 \text{ per}} \cdot \underset{\text{nights, } x}{\text{Number of}}$$

c. A set of two (or more) linear equations is called a **system of linear equations**. Write the system of linear equations for this problem.

MODELING WITH MATHEMATICS

To be proficient in math, you need to identify important quantities in real-life problems and map their relationships using tools such as diagrams, tables, and graphs.

Using a Table or Graph to Solve a System

Work with a partner. Use the cost and revenue equations from Exploration 1 to determine how many nights your family needs to rent the bedroom before recovering the cost of preparing the bedroom. This is the *break-even point*.

a. Copy and complete the table.

x (nights)	0	1	2	3	4	5	6	7	8	9	10	11
C (dollars)												
R (dollars)												

b. How many nights does your family need to rent the bedroom before breaking even?

c. In the same coordinate plane, graph the cost equation and the revenue equation from Exploration 1.

d. Find the point of intersection of the two graphs. What does this point represent? How does this compare to the break-even point in part (b)? Explain.

Communicate Your Answer

3. How can you solve a system of linear equations? How can you check your solution?

4. Solve each system by using a table or sketching a graph. Explain why you chose each method. Use a graphing calculator to check each solution.

   a. $y = -4.3x - 1.3$ **b.** $y = x$ **c.** $y = -x - 1$

       $y = 1.7x + 4.7$ $y = -3x + 8$ $y = 3x + 5$

What You Will Learn

▶ Check solutions of systems of linear equations.

▶ Solve systems of linear equations by graphing.

▶ Use systems of linear equations to solve real-life problems.

Systems of Linear Equations

A **system of linear equations** is a set of two or more linear equations in the same variables. An example is shown below.

$$x + y = 7 \qquad \text{Equation 1}$$

$$2x - 3y = -11 \qquad \text{Equation 2}$$

A **solution of a system of linear equations** in two variables is an ordered pair that is a solution of each equation in the system.

EXAMPLE 1   Checking Solutions

Tell whether the ordered pair is a solution of the system of linear equations.

a. $(2, 5)$; $\begin{array}{l} x + y = 7 \\ 2x - 3y = -11 \end{array}$ Equation 1   Equation 2

b. $(-2, 0)$; $\begin{array}{l} y = -2x - 4 \\ y = x + 4 \end{array}$ Equation 1   Equation 2

SOLUTION

a. Substitute 2 for x and 5 for y in each equation.

Equation 1

$$x + y = 7$$
$$2 + 5 \stackrel{?}{=} 7$$
$$7 = 7 \checkmark$$

Equation 2

$$2x - 3y = -11$$
$$2(2) - 3(5) \stackrel{?}{=} -11$$
$$-11 = -11 \checkmark$$

▶ Because the ordered pair $(2, 5)$ is a solution of each equation, it is a solution of the linear system.

b. Substitute -2 for x and 0 for y in each equation.

Equation 1

$$y = -2x - 4$$
$$0 \stackrel{?}{=} -2(-2) - 4$$
$$0 = 0 \checkmark$$

Equation 2

$$y = x + 4$$
$$0 \stackrel{?}{=} -2 + 4$$
$$0 \neq 2 \; ✗$$

▶ The ordered pair $(-2, 0)$ is a solution of the first equation, but it is not a solution of the second equation. So, $(-2, 0)$ is *not* a solution of the linear system.

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Tell whether the ordered pair is a solution of the system of linear equations.

1. $(1, -2)$; $\begin{array}{l} 2x + y = 0 \\ -x + 2y = 5 \end{array}$

2. $(1, 4)$; $\begin{array}{l} y = 3x + 1 \\ y = -x + 5 \end{array}$

Solving Systems of Linear Equations by Graphing

The solution of a system of linear equations is the point of intersection of the graphs of the equations.

Core Concept

Solving a System of Linear Equations by Graphing

Step 1 Graph each equation in the same coordinate plane.

Step 2 Estimate the point of intersection.

Step 3 Check the point from Step 2 by substituting for x and y in each equation of the original system.

REMEMBER

Note that the linear equations are in slope-intercept form. You can use the method presented in Section 3.5 to graph the equations.

EXAMPLE 2 Solving a System of Linear Equations by Graphing

Solve the system of linear equations by graphing.

$y = -2x + 5$ Equation 1

$y = 4x - 1$ Equation 2

SOLUTION

Step 1 Graph each equation.

Step 2 Estimate the point of intersection.
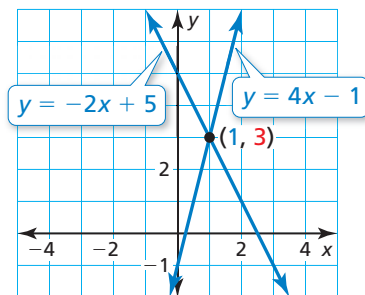The graphs appear to intersect at $(1, 3)$.

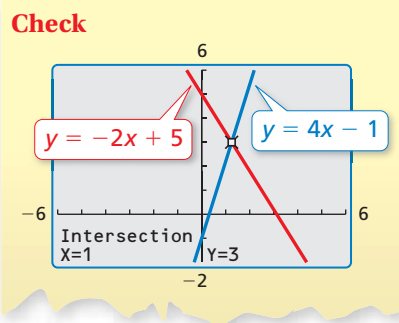**Step 3** Check your point from Step 2.

Equation 1	Equation 2
$y = -2x + 5$	$y = 4x - 1$
$3 \overset{?}{=} -2(1) + 5$	$3 \overset{?}{=} 4(1) - 1$
$3 = 3$ ✓	$3 = 3$ ✓

▶ The solution is $(1, 3)$.

Check

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Solve the system of linear equations by graphing.

3. $y = x - 2$
$y = -x + 4$

4. $y = \frac{1}{2}x + 3$
$y = -\frac{3}{2}x - 5$

5. $2x + y = 5$
$3x - 2y = 4$

Solving Real-Life Problems

EXAMPLE 3 **Modeling with Mathematics**

A roofing contractor buys 30 bundles of shingles and 4 rolls of roofing paper for $1040. In a second purchase (at the same prices), the contractor buys 8 bundles of shingles for $256. Find the price per bundle of shingles and the price per roll of roofing paper.

SOLUTION

1. **Understand the Problem** You know the total price of each purchase and how many of each item were purchased. You are asked to find the price of each item.

2. **Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.

3. **Solve the Problem**

Words $30 \cdot \boxed{\text{Price per bundle}} + 4 \cdot \boxed{\text{Price per roll}} = 1040$

$8 \cdot \boxed{\text{Price per bundle}} + 0 \cdot \boxed{\text{Price per roll}} = 256$

Variables Let x be the price (in dollars) per bundle and let y be the price (in dollars) per roll.

System
$30x + 4y = 1040$ **Equation 1**
$8x = 256$ **Equation 2**

Step 1 Graph each equation. Note that only the first quadrant is shown because x and y must be positive.

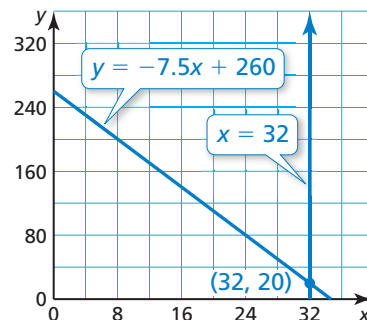**Step 2** Estimate the point of intersection. The graphs appear to intersect at (32, 20).

Step 3 Check your point from Step 2.

$y = -7.5x + 260$

$x = 32$

(32, 20)

Equation 1	Equation 2
$30x + 4y = 1040$	$8x = 256$
$30(32) + 4(20) \overset{?}{=} 1040$	$8(32) \overset{?}{=} 256$
$1040 = 1040$ ✓	$256 = 256$ ✓

▶ The solution is (32, 20). So, the price per bundle of shingles is $32, and the price per roll of roofing paper is $20.

4. **Look Back** You can use estimation to check that your solution is reasonable. A bundle of shingles costs about $30. So, 30 bundles of shingles and 4 rolls of roofing paper (at $20 per roll) cost about $30(30) + 4(20) = 980, and 8 bundles of shingles costs about $8(30) = 240. These prices are close to the given values, so the solution seems reasonable.

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6. You have a total of 18 math and science exercises for homework. You have six more math exercises than science exercises. How many exercises do you have in each subject?

Vocabulary and Core Concept Check

1. **VOCABULARY** Do the equations $5y - 2x = 18$ and $6x = -4y - 10$ form a system of linear equations? Explain.

2. **DIFFERENT WORDS, SAME QUESTION** Consider the system of linear equations $-4x + 2y = 4$ and $4x - y = -6$. Which is different? Find "both" answers.

> Solve the system of linear equations.

> Solve each equation for y.

> Find the point of intersection of the graphs of the equations.

> Find an ordered pair that is a solution of each equation in the system.

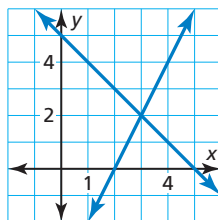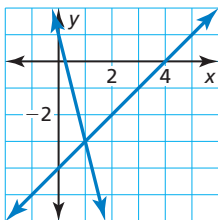## Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, tell whether the ordered pair is a solution of the system of linear equations. *(See Example 1.)*
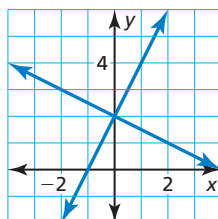
3. $(2, 6)$; $\begin{array}{l} x + y = 8 \\ 3x - y = 0 \end{array}$

4. $(8, 2)$; $\begin{array}{l} x - y = 6 \\ 2x - 10y = 4 \end{array}$

5. $(-1, 3)$; $\begin{array}{l} y = -7x - 4 \\ y = 8x + 5 \end{array}$

6. $(-4, -2)$; $\begin{array}{l} y = 2x + 6 \\ y = -3x - 14 \end{array}$

7. $(-2, 1)$; $\begin{array}{l} 6x + 5y = -7 \\ 2x - 4y = -8 \end{array}$

8. $(5, -6)$; $\begin{array}{l} 6x + 3y = 12 \\ 4x + y = 14 \end{array}$

In Exercises 9–12, use the graph to solve the system of linear equations. Check your solution.

9. $x - y = 4$
   $4x + y = 1$

10. $x + y = 5$
    $y - 2x = -4$

11. $6y + 3x = 18$
    $-x + 4y = 24$

12. $2x - y = -2$
    $2x + 4y = 8$
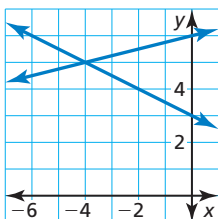
In Exercises 13–20, solve the system of linear equations by graphing. *(See Example 2.)*

13. $y = -x + 7$
    $y = x + 1$

14. $y = -x + 4$
    $y = 2x - 8$

15. $y = \frac{1}{3}x + 2$
    $y = \frac{2}{3}x + 5$

16. $y = \frac{3}{4}x - 4$
    $y = -\frac{1}{2}x + 11$

17. $9x + 3y = -3$
    $2x - y = -4$

18. $4x - 4y = 20$
    $y = -5$

19. $x - 4y = -4$
    $-3x - 4y = 12$

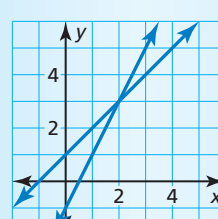20. $3y + 4x = 3$
    $x + 3y = -6$

ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in solving the system of linear equations.

21.
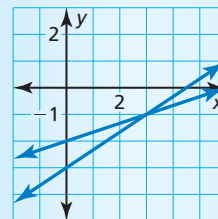
The solution of the linear system $x - 3y = 6$ and $2x - 3y = 3$ is $(3, -1)$.

22.

The solution of the linear system $y = 2x - 1$ and $y = x + 1$ is $x = 2$.

USING TOOLS In Exercises 23–26, use a graphing calculator to solve the system of linear equations.

23. $0.2x + 0.4y = 4$
$-0.6x + 0.6y = -3$

24. $-1.6x - 3.2y = -24$
$2.6x + 2.6y = 26$

25. $-7x + 6y = 0$
$0.5x + y = 2$

26. $4x - y = 1.5$
$2x + y = 1.5$

27. **MODELING WITH MATHEMATICS** You have 40 minutes to exercise at the gym, and you want to burn 300 calories total using both machines. How much time should you spend on each machine? *(See Example 3.)*

Elliptical Trainer   **Stationary Bike**

8 calories per minute   6 calories per minute

28. **MODELING WITH MATHEMATICS** You sell small and large candles at a craft fair. You collect $144 selling a total of 28 candles. How many of each type of candle did you sell?

$6 each $4 each

29. **MATHEMATICAL CONNECTIONS** Write a linear equation that represents the area and a linear equation that represents the perimeter of the rectangle. Solve the system of linear equations by graphing. Interpret your solution.

$(3x - 3)$ cm

6 cm
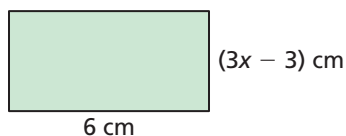
30. **THOUGHT PROVOKING** Your friend's bank account balance (in dollars) is represented by the equation $y = 25x + 250$, where x is the number of months. Graph this equation. After 6 months, you want to have the same account balance as your friend. Write a linear equation that represents your account balance. Interpret the slope and y-intercept of the line that represents your account balance.

31. **COMPARING METHODS** Consider the equation $x + 2 = 3x - 4$.

  a. Solve the equation using algebra.

  b. Solve the system of linear equations $y = x + 2$ and $y = 3x - 4$ by graphing.

  c. How is the linear system and the solution in part (b) related to the original equation and the solution in part (a)?

32. **HOW DO YOU SEE IT?** A teacher is purchasing binders for students. The graph shows the total costs of ordering x binders from three different companies.

Buying Binders
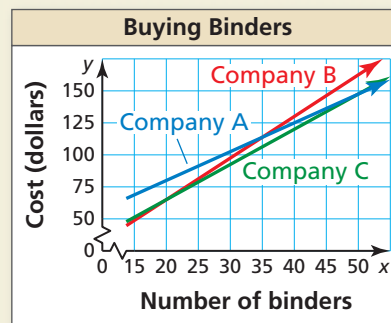
  a. For what numbers of binders are the costs the same at two different companies? Explain.

  b. How do your answers in part (a) relate to systems of linear equations?

33. **MAKING AN ARGUMENT** You and a friend are going hiking but start at different locations. You start at the trailhead and walk 5 miles per hour. Your friend starts 3 miles from the trailhead and walks 3 miles per hour.

  a. Write and graph a system of linear equations that represents this situation.

  b. Your friend says that after an hour of hiking you will both be at the same location on the trail. Is your friend correct? Use the graph from part (a) to explain your answer.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Solve the literal equation for y. (*Section 1.5*)

34. $10x + 5y = 5x + 20$

35. $9x + 18 = 6y - 3x$

36. $\frac{3}{4}x + \frac{1}{4}y = 5$

5.2 Solving Systems of Linear Equations by Substitution

Essential Question How can you use substitution to solve a system of linear equations?

Using Substitution to Solve Systems

Work with a partner. Solve each system of linear equations using two methods.

Method 1   Solve for x first.

Solve for x in one of the equations. Substitute the expression for x into the other equation to find y. Then substitute the value of y into one of the original equations to find x.

Method 2   Solve for y first.

Solve for y in one of the equations. Substitute the expression for y into the other equation to find x. Then substitute the value of x into one of the original equations to find y.

Is the solution the same using both methods? Explain which method you would prefer to use for each system.

a. $x + y = -7$ **b.** $x - 6y = -11$ **c.** $4x + y = -1$
 $-5x + y = 5$ $3x + 2y = 7$ $3x - 5y = -18$

Writing and Solving a System of Equations

Work with a partner.

a. Write a random ordered pair with integer coordinates. One way to do this is to use a graphing calculator. The ordered pair generated at the right is $(-2, -3)$.

ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely with others.

b. Write a system of linear equations that has your ordered pair as its solution.

c. Exchange systems with your partner and use one of the methods from Exploration 1 to solve the system. Explain your choice of method.

> Choose two random integers between -5 and 5.

```
randInt(-5,5,2)
             {-2 -3}
```

Communicate Your Answer

3. How can you use substitution to solve a system of linear equations?

4. Use one of the methods from Exploration 1 to solve each system of linear equations. Explain your choice of method. Check your solutions.

 **a.** $x + 2y = -7$ **b.** $x - 2y = -6$ **c.** $-3x + 2y = -10$
 $2x - y = -9$ $2x + y = -2$ $-2x + y = -6$

 **d.** $3x + 2y = 13$ **e.** $3x - 2y = 9$ **f.** $3x - y = -6$
 $x - 3y = -3$ $-x - 3y = 8$ $4x + 5y = 11$

What You Will Learn

▶ Solve systems of linear equations by substitution.

▶ Use systems of linear equations to solve real-life problems.

Core Vocabulary

Previous
system of linear equations
solution of a system of
linear equations

Solving Linear Systems by Substitution

Another way to solve a system of linear equations is to use substitution.

🔄 Core Concept

Solving a System of Linear Equations by Substitution

Step 1 Solve one of the equations for one of the variables.

Step 2 Substitute the expression from Step 1 into the other equation and solve for the other variable.

Step 3 Substitute the value from Step 2 into one of the original equations and solve.

EXAMPLE 1 **Solving a System of Linear Equations by Substitution**

Solve the system of linear equations by substitution.

$$y = -2x - 9 \qquad \text{Equation 1}$$

$$6x - 5y = -19 \qquad \text{Equation 2}$$

SOLUTION

Step 1 Equation 1 is already solved for y.

Step 2 Substitute $-2x - 9$ for y in Equation 2 and solve for x.

$6x - 5y = -19$	Equation 2
$6x - 5(-2x - 9) = -19$	Substitute $-2x - 9$ for y.
$6x + 10x + 45 = -19$	Distributive Property
$16x + 45 = -19$	Combine like terms.
$16x = -64$	Subtract 45 from each side.
$x = -4$	Divide each side by 16.

Step 3 Substitute -4 for x in Equation 1 and solve for y.

$y = -2x - 9$	Equation 1
$= -2(-4) - 9$	Substitute -4 for x.
$= 8 - 9$	Multiply.
$= -1$	Subtract.

▶ The solution is $(-4, -1)$.

Check

Equation 1

$$y = -2x - 9$$

$$-1 \overset{?}{=} -2(-4) - 9$$

$$-1 = -1 \checkmark$$

Equation 2

$$6x - 5y = -19$$

$$6(-4) - 5(-1) \overset{?}{=} -19$$

$$-19 = -19 \checkmark$$

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Solve the system of linear equations by substitution. Check your solution.

1. $y = 3x + 14$
  $y = -4x$

2. $3x + 2y = 0$
  $y = \frac{1}{2}x - 1$

3. $x = 6y - 7$
  $4x + y = -3$

EXAMPLE 2 **Solving a System of Linear Equations by Substitution**

Solve the system of linear equations by substitution.

$$-x + y = 3 \qquad \text{Equation 1}$$
$$3x + y = -1 \qquad \text{Equation 2}$$

SOLUTION

Step 1 Solve for *y* in Equation 1.

$$y = x + 3 \qquad \text{Revised Equation 1}$$

Step 2 Substitute $x + 3$ for *y* in Equation 2 and solve for *x*.

$3x + y = -1$	Equation 2
$3x + (x + 3) = -1$	Substitute $x + 3$ for *y*.
$4x + 3 = -1$	Combine like terms.
$4x = -4$	Subtract 3 from each side.
$x = -1$	Divide each side by 4.

Step 3 Substitute -1 for *x* in Equation 1 and solve for *y*.

$-x + y = 3$	Equation 1
$-(-1) + y = 3$	Substitute -1 for *x*.
$y = 2$	Subtract 1 from each side.

▶ The solution is $(-1, 2)$.

Algebraic Check

Equation 1
$$-x + y = 3$$
$$-(-1) + 2 \overset{?}{=} 3$$
$$3 = 3 ✓$$

Equation 2
$$3x + y = -1$$
$$3(-1) + 2 \overset{?}{=} -1$$
$$-1 = -1 ✓$$

Graphical Check
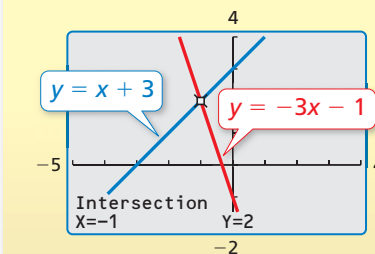
Monitoring Progress   Help in English and Spanish at *BigIdeasMath.com*

Solve the system of linear equations by substitution. Check your solution.

4. $x + y = -2$
$-3x + y = 6$

5. $-x + y = -4$
$4x - y = 10$

6. $2x - y = -5$
$3x - y = 1$

7. $x - 2y = 7$
$3x - 2y = 3$

Solving Real-Life Problems

EXAMPLE 3   Modeling with Mathematics

A drama club earns $1040 from a production. A total of 64 adult tickets and 132 student tickets are sold. An adult ticket costs twice as much as a student ticket. Write a system of linear equations that represents this situation. What is the price of each type of ticket?

SOLUTION

1. **Understand the Problem** You know the amount earned, the total numbers of adult and student tickets sold, and the relationship between the price of an adult ticket and the price of a student ticket. You are asked to write a system of linear equations that represents the situation and find the price of each type of ticket.

2. **Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.

3. **Solve the Problem**

   Words $64 \cdot$ [Adult ticket price] $+ 132 \cdot$ [Student ticket price] $= 1040$

   [Adult ticket price] $= 2 \cdot$ [Student ticket price]

   Variables   Let x be the price (in dollars) of an adult ticket and let y be the price (in dollars) of a student ticket.

   System $64x + 132y = 1040$ **Equation 1**

   $x = 2y$ **Equation 2**

   Step 1 Equation 2 is already solved for x.

   Step 2 Substitute $2y$ for x in Equation 1 and solve for y.

$$64x + 132y = 1040 \qquad \text{Equation 1}$$
$$64(2y) + 132y = 1040 \qquad \text{Substitute } 2y \text{ for } x.$$
$$260y = 1040 \qquad \text{Simplify.}$$
$$y = 4 \qquad \text{Simplify.}$$

   Step 3 Substitute 4 for y in Equation 2 and solve for x.

$$x = 2y \qquad \text{Equation 2}$$
$$x = 2(4) \qquad \text{Substitute 4 for } y.$$
$$x = 8 \qquad \text{Simplify.}$$

▶   The solution is (8, 4). So, an adult ticket costs $8 and a student ticket costs $4.

4. **Look Back** To check that your solution is correct, substitute the values of x and y into both of the original equations and simplify.

$$64(8) + 132(4) = 1040 \qquad\qquad 8 = 2(4)$$
$$1040 = 1040 ✓ \qquad\qquad 8 = 8 ✓$$

STUDY TIP

You can use either of the original equations to solve for x. However, using Equation 2 requires fewer calculations.

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8. There are a total of 64 students in a drama club and a yearbook club. The drama club has 10 more students than the yearbook club. Write a system of linear equations that represents this situation. How many students are in each club?

Vocabulary and Core Concept Check

1. **WRITING** Describe how to solve a system of linear equations by substitution.

2. **NUMBER SENSE** When solving a system of linear equations by substitution, how do you decide which variable to solve for in Step 1?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, tell which equation you would choose to solve for one of the variables. Explain.

3. $x + 4y = 30$
   $x - 2y = 0$

4. $3x - y = 0$
   $2x + y = -10$

5. $5x + 3y = 11$
   $5x - y = 5$

6. $3x - 2y = 19$
   $x + y = 8$

7. $x - y = -3$
   $4x + 3y = -5$

8. $3x + 5y = 25$
   $x - 2y = -6$

In Exercises 9–16, solve the sytem of linear equations by substitution. Check your solution. *(See Examples 1 and 2.)*

9. $x = 17 - 4y$
   $y = x - 2$

10. $6x - 9 = y$
    $y = -3x$

11. $x = 16 - 4y$
    $3x + 4y = 8$

12. $-5x + 3y = 51$
    $y = 10x - 8$

13. $2x = 12$
    $x - 5y = -29$

14. $2x - y = 23$
    $x - 9 = -1$

15. $5x + 2y = 9$
    $x + y = -3$

16. $11x - 7y = -14$
    $x - 2y = -4$

17. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $8x + 2y = -12$ and $5x - y = 4$.

Step 1 $5x - y = 4$
           $-y = -5x + 4$
           $y = 5x - 4$

Step 2 $5x - (5x - 4) = 4$
           $5x - 5x + 4 = 4$
                   $4 = 4$

18. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $4x + 2y = 6$ and $3x + y = 9$.

Step 1 $3x + y = 9$
           $y = 9 - 3x$

Step 2 $4x + 2(9 - 3x) = 6$
           $4x + 18 - 6x = 6$
                   $-2x = -12$
                     $x = 6$

Step 3 $3x + y = 9$
           $3x + 6 = 9$
           $3x = 3$
           $x = 1$

19. **MODELING WITH MATHEMATICS** A farmer plants corn and wheat on a 180-acre farm. The farmer wants to plant three times as many acres of corn as wheat. Write a system of linear equations that represents this situation. How many acres of each crop should the farmer plant? *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** A company that offers tubing trips down a river rents tubes for a person to use and "cooler" tubes to carry food and water. A group spends $270 to rent a total of 15 tubes. Write a system of linear equations that represents this situation. How many of each type of tube does the group rent?

RIVER
Tubing
1 Person Tube $20
Cooler Tube $12.50

In Exercises 21–24, write a system of linear equations that has the ordered pair as its solution.

21. $(3, 5)$　　　　**22.** $(-2, 8)$

23. $(-4, -12)$　　　**24.** $(15, -25)$

25. **PROBLEM SOLVING** A math test is worth 100 points and has 38 problems. Each problem is worth either 5 points or 2 points. How many problems of each point value are on the test?

26. **PROBLEM SOLVING** An investor owns shares of Stock A and Stock B. The investor owns a total of 200 shares with a total value of $4000. How many shares of each stock does the investor own?

Stock	Price
A	$9.50
B	$27.00

MATHEMATICAL CONNECTIONS **In Exercises 27 and 28, (a) write an equation that represents the sum of the angle measures of the triangle and (b) use your equation and the equation shown to find the values of x and y.**

27.

$x°$

$x + 2 = 3y$

$y°$

28.

$(y − 18)°$

$y°$

$3x − 5y = −22$

$x°$
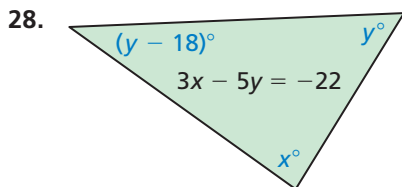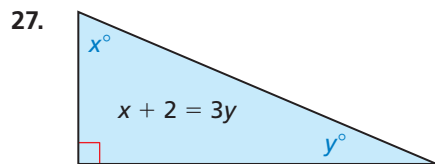
29. **REASONING** Find the values of a and b so that the solution of the linear system is $(-9, 1)$.

$$ax + by = -31 \quad \text{Equation 1}$$
$$ax - by = -41 \quad \text{Equation 2}$$

30. **MAKING AN ARGUMENT** Your friend says that given a linear system with an equation of a horizontal line and an equation of a vertical line, you cannot solve the system by substitution. Is your friend correct? Explain.

31. **OPEN-ENDED** Write a system of linear equations in which $(3, -5)$ is a solution of Equation 1 but not a solution of Equation 2, and $(-1, 7)$ is a solution of the system.

32. **HOW DO YOU SEE IT?** The graphs of two linear equations are shown.

$y = x + 1$

$y = 6 - \frac{1}{4}x$
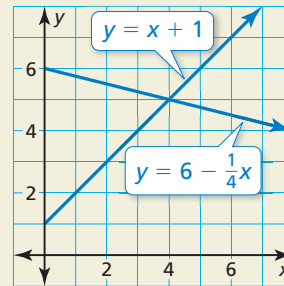
a. At what point do the lines appear to intersect?

b. Could you solve a system of linear equations by substitution to check your answer in part (a)? Explain.

33. **REPEATED REASONING** A radio station plays a total of 272 pop, rock, and hip-hop songs during a day. The number of pop songs is 3 times the number of rock songs. The number of hip-hop songs is 32 more than the number of rock songs. How many of each type of song does the radio station play?

34. **THOUGHT PROVOKING** You have $2.65 in coins. Write a system of equations that represents this situation. Use variables to represent the number of each type of coin.

35. **NUMBER SENSE** The sum of the digits of a two-digit number is 11. When the digits are reversed, the number increases by 27. Find the original number.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Find the sum or difference. (*Skills Review Handbook*)

36. $(x − 4) + (2x − 7)$　　　**37.** $(5y − 12) + (−5y − 1)$

38. $(t − 8) − (t + 15)$　　　**39.** $(6d + 2) − (3d − 3)$

40. $4(m + 2) + 3(6m − 4)$　　**41.** $2(5v + 6) − 6(−9v + 2)$

5.3 Solving Systems of Linear Equations by Elimination

Essential Question How can you use elimination to solve a system of linear equations?

Writing and Solving a System of Equations

Work with a partner. You purchase a drink and a sandwich for $4.50. Your friend purchases a drink and five sandwiches for $16.50. You want to determine the price of a drink and the price of a sandwich.

a. Let x represent the price (in dollars) of one drink. Let y represent the price (in dollars) of one sandwich. Write a system of equations for the situation. Use the following verbal model.

$$\boxed{\begin{array}{c}\text{Number}\\\text{of drinks}\end{array}} \cdot \boxed{\begin{array}{c}\text{Price}\\\text{per drink}\end{array}} + \boxed{\begin{array}{c}\text{Number of}\\\text{sandwiches}\end{array}} \cdot \boxed{\begin{array}{c}\text{Price per}\\\text{sandwich}\end{array}} = \boxed{\begin{array}{c}\text{Total}\\\text{price}\end{array}}$$

Label one of the equations Equation 1 and the other equation Equation 2.

b. Subtract Equation 1 from Equation 2. Explain how you can use the result to solve the system of equations. Then find and interpret the solution.

CHANGING COURSE

To be proficient in math, you need to monitor and evaluate your progress and change course using a different solution method, if necessary.

Using Elimination to Solve Systems

Work with a partner. Solve each system of linear equations using two methods.

Method 1 Subtract. Subtract Equation 2 from Equation 1. Then use the result to solve the system.

Method 2 Add. Add the two equations. Then use the result to solve the system.

Is the solution the same using both methods? Which method do you prefer?

a. $3x - y = 6$
$3x + y = 0$

b. $2x + y = 6$
$2x - y = 2$

c. $x - 2y = -7$
$x + 2y = 5$

Using Elimination to Solve a System

Work with a partner.

$2x + y = 7$ Equation 1
$x + 5y = 17$ Equation 2

a. Can you eliminate a variable by adding or subtracting the equations as they are? If not, what do you need to do to one or both equations so that you can?

b. Solve the system individually. Then exchange solutions with your partner and compare and check the solutions.

Communicate Your Answer

4. How can you use elimination to solve a system of linear equations?

5. When can you add or subtract the equations in a system to solve the system? When do you have to multiply first? Justify your answers with examples.

6. In Exploration 3, why can you multiply an equation in the system by a constant and not change the solution of the system? Explain your reasoning.

What You Will Learn

▶ Solve systems of linear equations by elimination.

▶ Use systems of linear equations to solve real-life problems.

Solving Linear Systems by Elimination

🌀 Core Concept

Solving a System of Linear Equations by Elimination

Step 1 Multiply, if necessary, one or both equations by a constant so at least one pair of like terms has the same or opposite coefficients.

Step 2 Add or subtract the equations to eliminate one of the variables.

Step 3 Solve the resulting equation.

Step 4 Substitute the value from Step 3 into one of the original equations and solve for the other variable.

You can use elimination to solve a system of equations because replacing one equation in the system with the sum of that equation and a multiple of the other produces a system that has the same solution. Here is why.

System 1

$$a = b \qquad \text{Equation 1}$$
$$c = d \qquad \text{Equation 2}$$

System 2

$$a + kc = b + kd \qquad \text{Equation 3}$$
$$c = d \qquad \text{Equation 2}$$

Consider System 1. In this system, a and c are algebraic expressions, and b and d are constants. Begin by multiplying each side of Equation 2 by a constant k. By the Multiplication Property of Equality, $kc = kd$. You can rewrite Equation 1 as Equation 3 by adding kc on the left and kd on the right. You can rewrite Equation 3 as Equation 1 by subtracting kc on the left and kd on the right. Because you can rewrite either system as the other, System 1 and System 2 have the same solution.

EXAMPLE 1 **Solving a System of Linear Equations by Elimination**

Solve the system of linear equations by elimination.

$$3x + 2y = 4 \qquad \text{Equation 1}$$
$$3x - 2y = -4 \qquad \text{Equation 2}$$

SOLUTION

Step 1 Because the coefficients of the y-terms are opposites, you do not need to multiply either equation by a constant.

Step 2 Add the equations.

$$3x + 2y = 4 \qquad \text{Equation 1}$$
$$\underline{3x - 2y = -4} \qquad \text{Equation 2}$$
$$6x \qquad = 0 \qquad \text{Add the equations.}$$

Step 3 Solve for x.

$$6x = 0 \qquad \text{Resulting equation from Step 2}$$
$$x = 0 \qquad \text{Divide each side by 6.}$$

Step 4 Substitute 0 for x in one of the original equations and solve for y.

$$3x + 2y = 4 \qquad \text{Equation 1}$$
$$3(0) + 2y = 4 \qquad \text{Substitute 0 for } x.$$
$$y = 2 \qquad \text{Solve for } y.$$

▶ The solution is $(0, 2)$.

Check

Equation 1

$$3x + 2y = 4$$
$$3(0) + 2(2) \overset{?}{=} 4$$
$$4 = 4 ✓$$

Equation 2

$$3x - 2y = -4$$
$$3(0) - 2(2) \overset{?}{=} -4$$
$$-4 = -4 ✓$$

EXAMPLE 2 **Solving a System of Linear Equations by Elimination**

Solve the system of linear equations by elimination.

$$-10x + 3y = 1 \qquad \text{Equation 1}$$
$$-5x - 6y = 23 \qquad \text{Equation 2}$$

ANOTHER WAY

To use subtraction to eliminate one of the variables, multiply Equation 2 by 2 and then subtract the equations.

$$-10x + 3y = 1$$
$$\underline{-(-10x - 12y = 46)}$$
$$15y = -45$$

SOLUTION

Step 1 Multiply Equation 2 by -2 so that the coefficients of the x-terms are opposites.

$$-10x + 3y = 1 \qquad\qquad\qquad -10x + 3y = 1 \qquad \text{Equation 1}$$
$$-5x - 6y = 23 \quad \boxed{\text{Multiply by }-2.} \quad 10x + 12y = -46 \quad \text{Revised Equation 2}$$

Step 2 Add the equations.

$$-10x + 3y = 1 \qquad\qquad \text{Equation 1}$$
$$\underline{\quad 10x + 12y = -46} \qquad \text{Revised Equation 2}$$
$$15y = -45 \qquad\qquad \text{Add the equations.}$$

Step 3 Solve for y.

$$15y = -45 \qquad\qquad \text{Resulting equation from Step 2}$$
$$y = -3 \qquad\qquad \text{Divide each side by 15.}$$

Step 4 Substitute -3 for y in one of the original equations and solve for x.

$$-5x - 6y = 23 \qquad\qquad \text{Equation 2}$$
$$-5x - 6(-3) = 23 \qquad\qquad \text{Substitute } -3 \text{ for } y.$$
$$-5x + 18 = 23 \qquad\qquad \text{Multiply.}$$
$$-5x = 5 \qquad\qquad \text{Subtract 18 from each side.}$$
$$x = -1 \qquad\qquad \text{Divide each side by } -5.$$

▶ The solution is $(-1, -3)$.
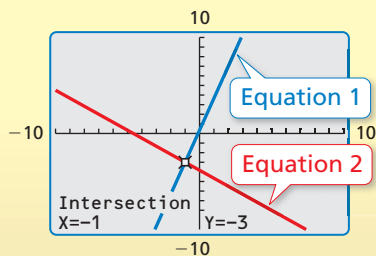
Check

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Solve the system of linear equations by elimination. Check your solution.

1. $3x + 2y = 7$
   $-3x + 4y = 5$

2. $x - 3y = 24$
   $3x + y = 12$

3. $x + 4y = 22$
   $4x + y = 13$

Concept Summary

Methods for Solving Systems of Linear Equations

Method	When to Use
Graphing *(Lesson 5.1)*	To estimate solutions
Substitution *(Lesson 5.2)*	When one of the variables in one of the equations has a coefficient of 1 or -1
Elimination *(Lesson 5.3)*	When at least one pair of like terms has the same or opposite coefficients
Elimination (Multiply First) *(Lesson 5.3)*	When one of the variables cannot be eliminated by adding or subtracting the equations

Solving Real-Life Problems
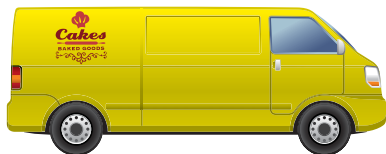
EXAMPLE 3 **Modeling with Mathematics**

A business with two locations buys seven large delivery vans and five small delivery vans. Location A receives five large vans and two small vans for a total cost of $235,000. Location B receives two large vans and three small vans for a total cost of $160,000. What is the cost of each type of van?

SOLUTION

1. **Understand the Problem** You know how many of each type of van each location receives. You also know the total cost of the vans for each location. You are asked to find the cost of each type of van.

2. **Make a Plan** Use a verbal model to write a system of linear equations that represents the problem. Then solve the system of linear equations.

3. **Solve the Problem**

   Words $5 \cdot \boxed{\text{Cost of large van}} + 2 \cdot \boxed{\text{Cost of small van}} = 235{,}000$

   $2 \cdot \boxed{\text{Cost of large van}} + 3 \cdot \boxed{\text{Cost of small van}} = 160{,}000$

   Variables Let x be the cost (in dollars) of a large van and let y be the cost (in dollars) of a small van.

   System $5x + 2y = 235{,}000$ Equation 1
   $2x + 3y = 160{,}000$ Equation 2

   Step 1 Multiply Equation 1 by -3. Multiply Equation 2 by 2.

   $5x + 2y = 235{,}000$ | **Multiply by −3.** | $-15x - 6y = -705{,}000$ Revised Equation 1
   $2x + 3y = 160{,}000$ | **Multiply by 2.** | $4x + 6y = 320{,}000$ Revised Equation 2

   Step 2 Add the equations.

   $$
   \begin{array}{ll}
   -15x - 6y = -705{,}000 & \text{Revised Equation 1} \\
   \underline{4x + 6y = 320{,}000} & \text{Revised Equation 2} \\
   -11x = -385{,}000 & \text{Add the equations.}
   \end{array}
   $$

   Step 3 Solving the equation $-11x = -385{,}000$ gives $x = 35{,}000$.

   Step 4 Substitute 35,000 for x in one of the original equations and solve for y.

   $$
   \begin{array}{ll}
   5x + 2y = 235{,}000 & \text{Equation 1} \\
   5(35{,}000) + 2y = 235{,}000 & \text{Substitute 35,000 for } x. \\
   y = 30{,}000 & \text{Solve for } y.
   \end{array}
   $$

   ▶ The solution is (35,000, 30,000). So, a large van costs $35,000 and a small van costs $30,000.

4. **Look Back** Check to make sure your solution makes sense with the given information. For Location A, the total cost is $5(35{,}000) + 2(30{,}000) = \$235{,}000$. For Location B, the total cost is $2(35{,}000) + 3(30{,}000) = \$160{,}000$. So, the solution makes sense.

STUDY TIP

In Example 3, both equations are multiplied by a constant so that the coefficients of the y-terms are opposites.

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   4. Solve the system in Example 3 by eliminating x.

Vocabulary and Core Concept Check

1. **OPEN-ENDED** Give an example of a system of linear equations that can be solved by first adding the equations to eliminate one variable.

2. **WRITING** Explain how to solve the system of linear equations by elimination.

   $2x - 3y = -4$ Equation 1
   $-5x + 9y = 7$ Equation 2

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, solve the system of linear equations by elimination. Check your solution. *(See Example 1.)*

3. $x + 2y = 13$
    $-x + y = 5$

4. $9x + y = 2$
    $-4x - y = -17$

5. $5x + 6y = 50$
    $x - 6y = -26$

6. $-x + y = 4$
    $x + 3y = 4$

7. $-3x - 5y = -7$
    $-4x + 5y = 14$

8. $4x - 9y = -21$
    $-4x - 3y = 9$

9. $-y - 10 = 6x$
    $5x + y = -10$

10. $3x - 30 = y$
    $7y - 6 = 3x$

In Exercises 11–18, solve the system of linear equations by elimination. Check your solution. *(See Examples 2 and 3.)*

11. $x + y = 2$
    $2x + 7y = 9$

12. $8x - 5y = 11$
    $4x - 3y = 5$

13. $11x - 20y = 28$
    $3x + 4y = 36$

14. $10x - 9y = 46$
    $-2x + 3y = 10$

15. $4x - 3y = 8$
    $5x - 2y = -11$

16. $-2x - 5y = 9$
    $3x + 11y = 4$

17. $9x + 2y = 39$
    $6x + 13y = -9$

18. $12x - 7y = -2$
    $8x + 11y = 30$

19. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $5x - 7y = 16$ and $x + 7y = 8$.

$$5x - 7y = 16$$
$$\underline{x + 7y = 8}$$
$$4x \qquad = 24$$
$$x = 6$$

20. **ERROR ANALYSIS** Describe and correct the error in solving for one of the variables in the linear system $4x + 3y = 8$ and $x - 2y = -13$.

$4x + 3y = 8$
$x - 2y = -13$ Multiply by −4.

$4x + 3y = 8$
$\underline{-4x + 8y = -13}$
$11y = -5$
$$y = \frac{-5}{11}$$

21. **MODELING WITH MATHEMATICS** A service center charges a fee of x dollars for an oil change plus y dollars per quart of oil used. A sample of its sales record is shown. Write a system of linear equations that represents this situation. Find the fee and cost per quart of oil.

	A	B	C	
1	Customer	Oil Tank Size (quarts)	Total Cost	
2	A	5	$22.45	
3	B	7	$25.45	
4				

22. **MODELING WITH MATHEMATICS** A music website charges x dollars for individual songs and y dollars for entire albums. Person A pays $25.92 to download 6 individual songs and 2 albums. Person B pays $33.93 to download 4 individual songs and 3 albums. Write a system of linear equations that represents this situation. How much does the website charge to download a song? an entire album?

In Exercises 23–26, solve the system of linear equations using any method. Explain why you chose the method.

23. $3x + 2y = 4$
$2y = 8 - 5x$

24. $-6y + 2 = -4x$
$y - 2 = x$

25. $y - x = 2$
$y = -\frac{1}{4}x + 7$

26. $3x + y = \frac{1}{3}$
$2x - 3y = \frac{8}{3}$

27. **WRITING** For what values of a can you solve the linear system $ax + 3y = 2$ and $4x + 5y = 6$ by elimination without multiplying first? Explain.

28. **HOW DO YOU SEE IT?** The circle graph shows the results of a survey in which 50 students were asked about their favorite meal.

Favorite Meal
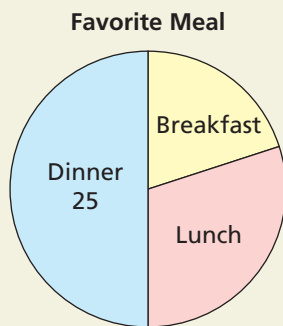
a. Estimate the numbers of students who chose breakfast and lunch.

b. The number of students who chose lunch was 5 more than the number of students who chose breakfast. Write a system of linear equations that represents the numbers of students who chose breakfast and lunch.

c. Explain how you can solve the linear system in part (b) to check your answers in part (a).

29. **MAKING AN ARGUMENT** Your friend says that any system of equations that can be solved by elimination can be solved by substitution in an equal or fewer number of steps. Is your friend correct? Explain.

30. **THOUGHT PROVOKING** Write a system of linear equations that can be added to eliminate a variable *or* subtracted to eliminate a variable.

31. **MATHEMATICAL CONNECTIONS** A rectangle has a perimeter of 18 inches. A new rectangle is formed by doubling the width w and tripling the length ℓ, as shown. The new rectangle has a perimeter P of 46 inches.
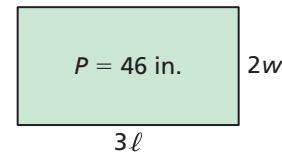
a. Write and solve a system of linear equations to find the length and width of the original rectangle.

b. Find the length and width of the new rectangle.

32. **CRITICAL THINKING** Refer to the discussion of System 1 and System 2 on page 248. Without solving, explain why the two systems shown have the same solution.

System 1		System 2	
$3x - 2y = 8$	Equation 1	$5x = 20$	Equation 3
$x + y = 6$	Equation 2	$x + y = 6$	Equation 2

33. **PROBLEM SOLVING** You are making 6 quarts of fruit punch for a party. You have bottles of 100% fruit juice and 20% fruit juice. How many quarts of each type of juice should you mix to make 6 quarts of 80% fruit juice?

34. **PROBLEM SOLVING** A motorboat takes 40 minutes to travel 20 miles downstream. The return trip takes 60 minutes. What is the speed of the current?

35. **CRITICAL THINKING** Solve for x, y, and z in the system of equations. Explain your steps.

$x + 7y + 3z = 29$	Equation 1
$3z + x - 2y = -7$	Equation 2
$5y = 10 - 2x$	Equation 3

Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Solve the equation. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*. *(Section 1.3)*

36. $5d - 8 = 1 + 5d$

37. $9 + 4t = 12 - 4t$

38. $3n + 2 = 2(n - 3)$

39. $-3(4 - 2v) = 6v - 12$

Write an equation of the line that passes through the given point and is parallel to the given line. *(Section 4.3)*

40. $(4, 1)$; $y = -2x + 7$

41. $(0, 6)$; $y = 5x - 3$

42. $(-5, -2)$; $y = \frac{2}{3}x + 1$

Solving Special Systems of Linear Equations

Essential Question   Can a system of linear equations have no solution or infinitely many solutions?

EXPLORATION 1   **Using a Table to Solve a System**

Work with a partner. You invest $450 for equipment to make skateboards. The materials for each skateboard cost $20. You sell each skateboard for $20.

a. Write the cost and revenue equations. Then copy and complete the table for your cost C and your revenue R.

x (skateboards)	0	1	2	3	4	5	6	7	8	9	10
C (dollars)											
R (dollars)											

b. When will your company break even? What is wrong?

EXPLORATION 2   **Writing and Analyzing a System**

> **MODELING WITH MATHEMATICS**
>
> To be proficient in math, you need to interpret mathematical results in real-life contexts.

Work with a partner. A necklace and matching bracelet have two types of beads. The necklace has 40 small beads and 6 large beads and weighs 10 grams. The bracelet has 20 small beads and 3 large beads and weighs 5 grams. The threads holding the beads have no significant weight.

a. Write a system of linear equations that represents the situation. Let x be the weight (in grams) of a small bead and let y be the weight (in grams) of a large bead.

b. Graph the system in the coordinate plane shown. What do you notice about the two lines?

c. Can you find the weight of each type of bead? Explain your reasoning.
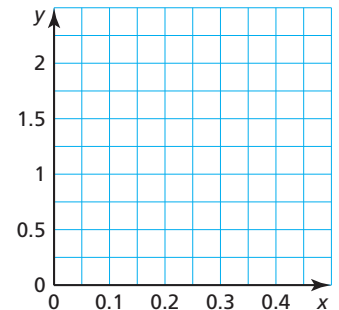
Communicate Your Answer

3. Can a system of linear equations have no solution or infinitely many solutions? Give examples to support your answers.

4. Does the system of linear equations represented by each graph have *no solution*, *one solution*, or *infinitely many solutions*? Explain.

a. $y = x + 2$ $x + y = 2$

b. $y = x + 2$ $-x + y = 1$

c. $y = x + 2$ $-2x + 2y = 4$
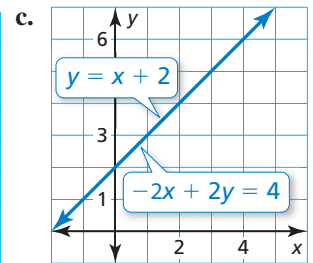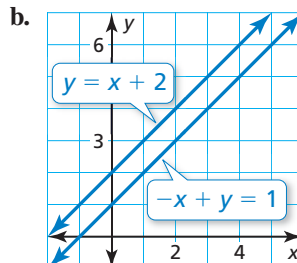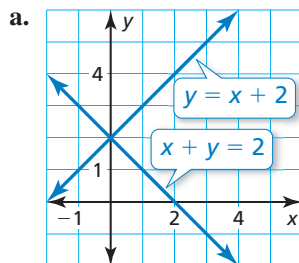
What You Will Learn

▶ Determine the numbers of solutions of linear systems.

▶ Use linear systems to solve real-life problems.

The Numbers of Solutions of Linear Systems

🌀 Core Concept

Solutions of Systems of Linear Equations

A system of linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.
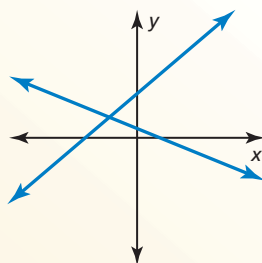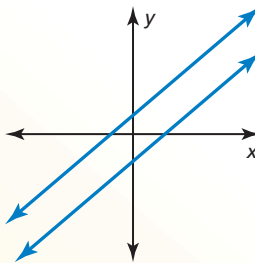
One solution	No solution	Infinitely many solutions
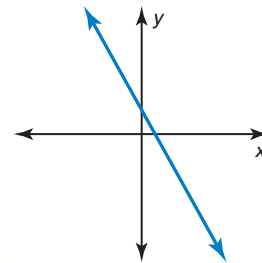
The lines intersect. The lines are parallel. The lines are the same.

ANOTHER WAY

You can solve some linear systems by inspection. In Example 1, notice you can rewrite the system as

$-2x + y = 1$
$-2x + y = -5$.

This system has no solution because $-2x + y$ cannot be equal to both 1 and −5.

EXAMPLE 1 **Solving a System: No Solution**

Solve the system of linear equations.

$y = 2x + 1$ Equation 1
$y = 2x - 5$ Equation 2

SOLUTION

Method 1 Solve by graphing.

Graph each equation.

The lines have the same slope and different y-intercepts. So, the lines are parallel.

Because parallel lines do not intersect, there is no point that is a solution of both equations.

▶ So, the system of linear equations has no solution.

Method 2 Solve by substitution.

Substitute $2x - 5$ for y in Equation 1.

$y = 2x + 1$ Equation 1

$2x - 5 = 2x + 1$ Substitute $2x - 5$ for y.

$-5 = 1$ ✗   Subtract $2x$ from each side.

STUDY TIP

A linear system with no solution is called an *inconsistent system*.

▶ The equation $-5 = 1$ is never true. So, the system of linear equations has no solution.

EXAMPLE 2 **Solving a System: Infinitely Many Solutions**

Solve the system of linear equations.

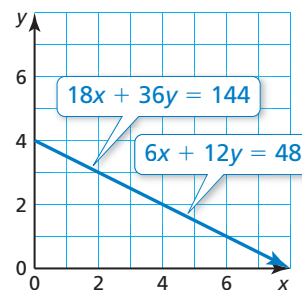$$-2x + y = 3 \qquad \text{Equation 1}$$
$$-4x + 2y = 6 \qquad \text{Equation 2}$$

SOLUTION

Method 1 Solve by graphing.

Graph each equation.
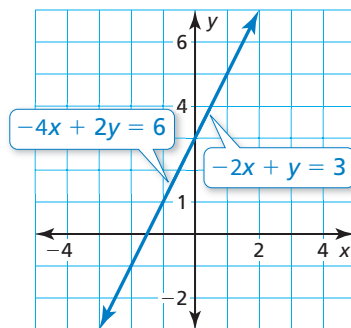
The lines have the same slope and the same y-intercept. So, the lines are the same. Because the lines are the same, all points on the line are solutions of both equations.

▶ So, the system of linear equations has infinitely many solutions.

Method 2 Solve by elimination.

Step 1 Multiply Equation 1 by -2.

$$-2x + y = 3 \qquad \boxed{\text{Multiply by } -2.} \qquad 4x - 2y = -6 \qquad \text{Revised Equation 1}$$
$$-4x + 2y = 6 \qquad\qquad\qquad\qquad -4x + 2y = 6 \qquad \text{Equation 2}$$

Step 2 Add the equations.

$$
\begin{array}{ll}
4x - 2y = -6 & \text{Revised Equation 1} \\
\underline{-4x + 2y = 6} & \text{Equation 2} \\
 0 = 0 & \text{Add the equations.}
\end{array}
$$

STUDY TIP

A linear system with infinitely many solutions is called a *consistent dependent system*.

▶ The equation $0 = 0$ is always true. So, the solutions are all the points on the line $-2x + y = 3$. The system of linear equations has infinitely many solutions.

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Solve the system of linear equations.

1. $x + y = 3$
$\quad 2x + 2y = 6$

2. $y = -x + 3$
$\quad 2x + 2y = 4$

3. $x + y = 3$
$\quad x + 2y = 4$

4. $y = -10x + 2$
$\quad 10x + y = 10$

Solving Real-Life Problems

EXAMPLE 3   Modeling with Mathematics
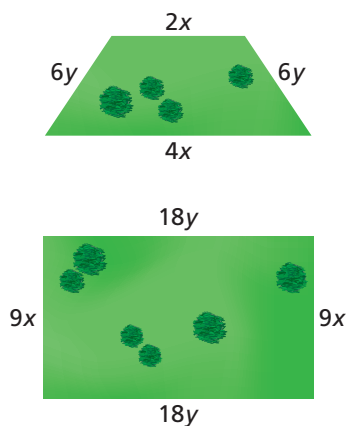
The perimeter of the trapezoidal piece of land is 48 kilometers. The perimeter of the rectangular piece of land is 144 kilometers. Write and solve a system of linear equations to find the values of x and y.

SOLUTION

1. **Understand the Problem** You know the perimeter of each piece of land and the side lengths in terms of x or y. You are asked to write and solve a system of linear equations to find the values of x and y.

2. **Make a Plan** Use the figures and the definition of perimeter to write a system of linear equations that represents the problem. Then solve the system of linear equations.

3. **Solve the Problem**

Perimeter of trapezoid	**Perimeter of rectangle**
$2x + 4x + 6y + 6y = 48$	$9x + 9x + 18y + 18y = 144$
$6x + 12y = 48$ Equation 1	$18x + 36y = 144$ Equation 2

System $6x + 12y = 48$ Equation 1
$18x + 36y = 144$ Equation 2

Method 1 Solve by graphing.

Graph each equation.

The lines have the same slope and the same y-intercept. So, the lines are the same.

In this context, x and y must be positive. Because the lines are the same, all the points on the line in Quadrant I are solutions of both equations.

> So, the system of linear equations has infinitely many solutions.

Method 2 Solve by elimination.

Multiply Equation 1 by -3 and add the equations.

$6x + 12y = 48$ **Multiply by −3.** $-18x - 36y = -144$ Revised Equation 1
$18x + 36y = 144$ $\underline{18x + 36y = 144}$ Equation 2
                                                          $0 = 0$ Add the equations.

> The equation $0 = 0$ is always true. In this context, x and y must be positive. So, the solutions are all the points on the line $6x + 12y = 48$ in Quadrant I. The system of linear equations has infinitely many solutions.

4. **Look Back** Choose a few of the ordered pairs (x, y) that are solutions of Equation 1. You should find that no matter which ordered pairs you choose, they will also be solutions of Equation 2. So, *infinitely many solutions* seems reasonable.

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5. **WHAT IF?** What happens to the solution in Example 3 when the perimeter of the trapezoidal piece of land is 96 kilometers? Explain.

Vocabulary and Core Concept Check

1. **REASONING** Is it possible for a system of linear equations to have exactly two solutions? Explain.

2. **WRITING** Compare the graph of a system of linear equations that has infinitely many solutions and the graph of a system of linear equations that has no solution.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, match the system of linear equations with its graph. Then determine whether the system has *one solution*, *no solution*, **or** *infinitely many solutions*.

3. $-x + y = 1$
$x - y = 1$

4. $2x - 2y = 4$
$-x + y = -2$

5. $2x + y = 4$
$-4x - 2y = -8$

6. $x - y = 0$
$5x - 2y = 6$

7. $-2x + 4y = 1$
$3x - 6y = 9$

8. $5x + 3y = 17$
$x - 3y = -2$

A.

B.

C.

D.

E.

F.
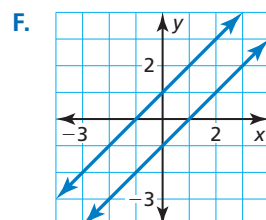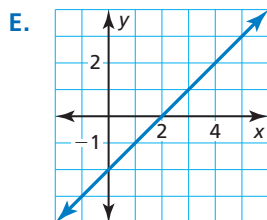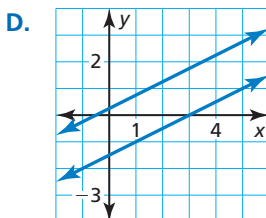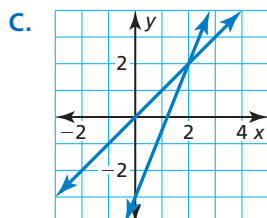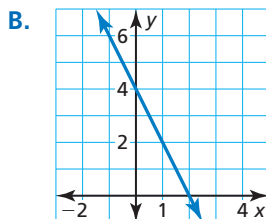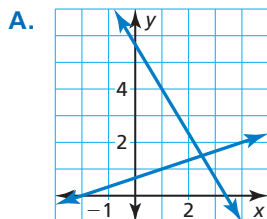
In Exercises 9–16, solve the system of linear equations.
(See Examples 1 and 2.)

9. $y = -2x - 4$
$y = 2x - 4$

10. $y = -6x - 8$
$y = -6x + 8$

11. $3x - y = 6$
$-3x + y = -6$

12. $-x + 2y = 7$
$x - 2y = 7$

13. $4x + 4y = -8$
$-2x - 2y = 4$

14. $15x - 5y = -20$
$-3x + y = 4$

15. $9x - 15y = 24$
$6x - 10y = -16$

16. $3x - 2y = -5$
$4x + 5y = 47$

In Exercises 17–22, use only the slopes and y-intercepts of the graphs of the equations to determine whether the system of linear equations has *one solution*, *no solution*, **or** *infinitely many solutions*. **Explain.**

17. $y = 7x + 13$
$-21x + 3y = 39$

18. $y = -6x - 2$
$12x + 2y = -6$

19. $4x + 3y = 27$
$4x - 3y = -27$

20. $-7x + 7y = 1$
$2x - 2y = -18$

21. $-18x + 6y = 24$
$3x - y = -2$

22. $2x - 2y = 16$
$3x - 6y = 30$

ERROR ANALYSIS In Exercises 23 and 24, describe and correct the error in solving the system of linear equations.

23.

$-4x + y = 4$
$4x + y = 12$

The lines do not intersect. So, the system has no solution.

24.

$y = 3x - 8$
$y = 3x - 12$

The lines have the same slope. So, the system has infinitely many solutions.

25. **MODELING WITH MATHEMATICS** A small bag of trail mix contains 3 cups of dried fruit and 4 cups of almonds. A large bag contains $4\frac{1}{2}$ cups of dried fruit and 6 cups of almonds. Write and solve a system of linear equations to find the price of 1 cup of dried fruit and 1 cup of almonds. *(See Example 3.)*

$9 $6

26. **MODELING WITH MATHEMATICS** In a canoe race, Team A is traveling 6 miles per hour and is 2 miles ahead of Team B. Team B is also traveling 6 miles per hour. The teams continue traveling at their current rates for the remainder of the race. Write a system of linear equations that represents this situation. Will Team B catch up to Team A? Explain.

27. **PROBLEM SOLVING** A train travels from New York City to Washington, D.C., and then back to New York City. The table shows the number of tickets purchased for each leg of the trip. The cost per ticket is the same for each leg of the trip. Is there enough information to determine the cost of one coach ticket? Explain.

Destination	Coach tickets	Business class tickets	Money collected (dollars)
Washington, D.C.	150	80	22,860
New York City	170	100	27,280

28. **THOUGHT PROVOKING** Write a system of three linear equations in two variables so that any two of the equations have exactly one solution, but the entire system of equations has no solution.

29. **REASONING** In a system of linear equations, one equation has a slope of 2 and the other equation has a slope of $-\frac{1}{3}$. How many solutions does the system have? Explain.

30. **HOW DO YOU SEE IT?** The graph shows information about the last leg of a 4 × 200-meter relay for three relay teams. Team A's runner ran about 7.8 meters per second, Team B's runner ran about 7.8 meters per second, and Team C's runner ran about 8.8 meters per second.

Last Leg of 4 × 200-Meter Relay
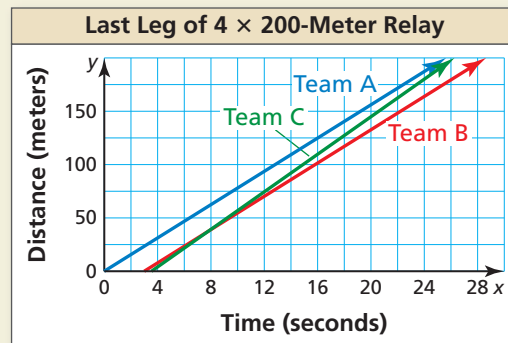
a. Estimate the distance at which Team C's runner passed Team B's runner.

b. If the race was longer, could Team C's runner have passed Team A's runner? Explain.

c. If the race was longer, could Team B's runner have passed Team A's runner? Explain.

31. **ABSTRACT REASONING** Consider the system of linear equations $y = ax + 4$ and $y = bx - 2$, where a and b are real numbers. Determine whether each statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

a. The system has infinitely many solutions.

b. The system has no solution.

c. When $a < b$, the system has one solution.

32. **MAKING AN ARGUMENT** One admission to an ice skating rink costs x dollars, and renting a pair of ice skates costs y dollars. Your friend says she can determine the exact cost of one admission and one skate rental. Is your friend correct? Explain.

D&G ICE RINK			
Check No.	Table No.	Order No. 240796	Server Name
3	Admissions		
2	Skate Rentals		
		Total $ 38.00	

D&G ICE RINK			
Check No.	Table No.	Order No. 240797	Server Name
15	Admissions		
10	Skate Rentals		
		Total $ 190.00	

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solutions. *(Section 1.4)*

33. $|2x + 6| = |x|$

34. $|3x - 45| = |12x|$

35. $|x - 7| = |2x - 8|$

36. $|2x + 1| = |3x - 11|$

Core Vocabulary

system of linear equations, *p. 236*   solution of a system of linear equations, *p. 236*

Core Concepts

Section 5.1

Solving a System of Linear Equations by Graphing, *p. 237*

Section 5.2

Solving a System of Linear Equations by Substitution, *p. 242*

Section 5.3

Solving a System of Linear Equations by Elimination, *p. 248*

Section 5.4

Solutions of Systems of Linear Equations, *p. 254*

Mathematical Practices

1. Describe the given information in Exercise 33 on page 246 and your plan for finding the solution.

2. Describe another real-life situation similar to Exercise 22 on page 251 and the mathematics that you can apply to solve the problem.

3. What question(s) can you ask your friend to help her understand the error in the statement she made in Exercise 32 on page 258?

------------------------------ Study Skills ------------------------------

Analyzing Your Errors

Study Errors

What Happens: You do not study the right material or you do not learn it well enough to remember it on a test without resources such as notes.

How to Avoid This Error: Take a practice test. Work with a study group. Discuss the topics on the test with your teacher. Do not try to learn a whole chapter's worth of material in one night.

Use the graph to solve the system of linear equations. Check your solution.   *(Section 5.1)*

1. $y = -\frac{1}{3}x + 2$

$y = x - 2$
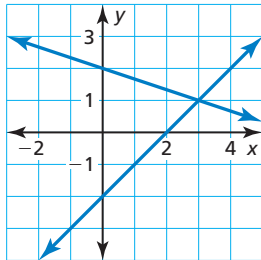
2. $y = \frac{1}{2}x - 1$

$y = 4x + 6$
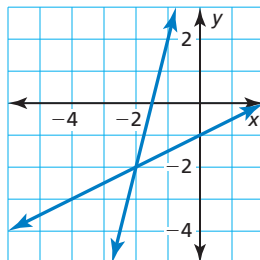
3. $y = 1$

$y = 2x + 1$
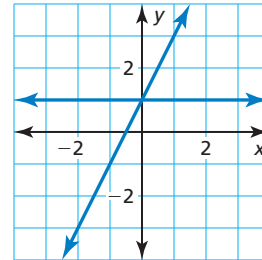
Solve the system of linear equations by substitution. Check your solution. *(Section 5.2)*

4. $y = x - 4$

$-2x + y = 18$

5. $2y + x = -4$

$y - x = -5$

6. $3x - 5y = 13$

$x + 4y = 10$

Solve the system of linear equations by elimination. Check your solution. *(Section 5.3)*

7. $x + y = 4$

$-3x - y = -8$

8. $x + 3y = 1$

$5x + 6y = 14$

9. $2x - 3y = -5$

$5x + 2y = 16$

Solve the system of linear equations. *(Section 5.4)*

10. $x - y = 1$

$x - y = 6$

11. $6x + 2y = 16$

$2x - y = 2$

12. $3x - 3y = -2$

$-6x + 6y = 4$

13. You plant a spruce tree that grows 4 inches per year and a hemlock tree that grows 6 inches per year. The initial heights are shown. *(Section 5.1)*

   a. Write a system of linear equations that represents this situation.

   b. Solve the system by graphing. Interpret your solution.

14 in.

8 in.

spruce tree

hemlock tree
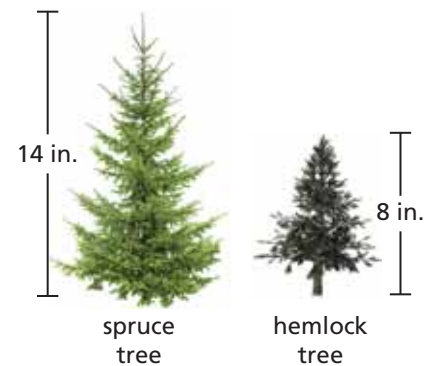
14. It takes you 3 hours to drive to a concert 135 miles away. You drive 55 miles per hour on highways and 40 miles per hour on the rest of the roads. *(Section 5.1, Section 5.2, and Section 5.3)*

   a. How much time do you spend driving at each speed?

   b. How many miles do you drive on highways? the rest of the roads?

15. In a football game, all of the home team's points are from 7-point touchdowns and 3-point field goals. The team scores six times. Write and solve a system of linear equations to find the numbers of touchdowns and field goals that the home team scores. *(Section 5.1, Section 5.2, and Section 5.3)*

HOME   VISITORS

T.O. LEFT   QUARTER   T.O. LEFT

DOWN   YDS. TO GO   BALL ON
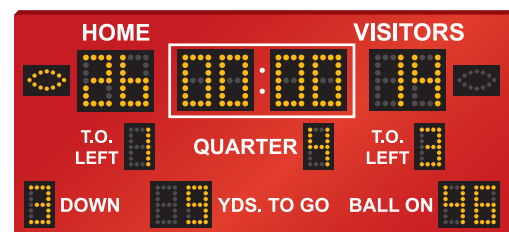
Solving Equations by Graphing

Essential Question How can you use a system of linear equations to solve an equation with variables on both sides?

Previously, you learned how to use algebra to solve equations with variables on both sides. Another way is to use a system of linear equations.

EXPLORATION 1 **Solving an Equation by Graphing**

Work with a partner. Solve $2x - 1 = -\frac{1}{2}x + 4$ by graphing.

a. Use the left side to write a linear equation. Then use the right side to write another linear equation.

b. Graph the two linear equations from part (a). Find the x-value of the point of intersection. Check that the x-value is the solution of

$$2x - 1 = -\frac{1}{2}x + 4.$$

c. Explain why this "graphical method" works.
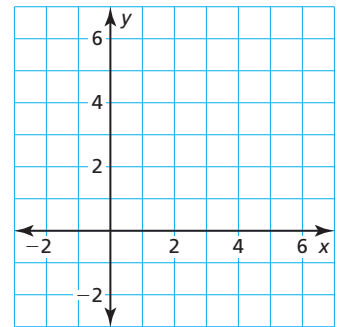
USING TOOLS STRATEGICALLY

To be proficient in math, you need to consider the available tools, which may include pencil and paper or a graphing calculator, when solving a mathematical problem.

EXPLORATION 2 **Solving Equations Algebraically and Graphically**

Work with a partner. Solve each equation using two methods.

　　　Method 1 Use an algebraic method.

　　　Method 2 Use a graphical method.

Is the solution the same using both methods?

a. $\frac{1}{2}x + 4 = -\frac{1}{4}x + 1$ 　　　　　　　**b.** $\frac{2}{3}x + 4 = \frac{1}{3}x + 3$

c. $-\frac{2}{3}x - 1 = \frac{1}{3}x - 4$ 　　　　　　**d.** $\frac{4}{5}x + \frac{7}{5} = 3x - 3$

e. $-x + 2.5 = 2x - 0.5$ 　　　　　　　**f.** $-3x + 1.5 = x + 1.5$

Communicate Your Answer

3. How can you use a system of linear equations to solve an equation with variables on both sides?

4. Compare the algebraic method and the graphical method for solving a linear equation with variables on both sides. Describe the advantages and disadvantages of each method.

What You Will Learn

Solve linear equations by graphing.

▶ Solve absolute value equations by graphing.

▶ Use linear equations to solve real-life problems.

Core Vocabulary

Previous
absolute value equation

Solving Linear Equations by Graphing

You can use a system of linear equations to solve an equation with variables on both sides.

🅖 Core Concept

Solving Linear Equations by Graphing

Step 1 To solve the equation $ax + b = cx + d$, write two linear equations.

$$ax + b = cx + d$$

$y = ax + b$ and $y = cx + d$

Step 2 Graph the system of linear equations. The x-value of the solution of the system of linear equations is the solution of the equation $ax + b = cx + d$.

EXAMPLE 1   **Solving an Equation by Graphing**

Solve $-x + 1 = 2x - 5$ by graphing. Check your solution.

SOLUTION

Step 1 Write a system of linear equations using each side of the original equation.

$$-x + 1 = 2x - 5$$

$y = -x + 1$ $y = 2x - 5$

Step 2 Graph the system.

$y = -x + 1$ **Equation 1**
$y = 2x - 5$ **Equation 2**

Check

$-x + 1 = 2x - 5$

$-(2) + 1 \overset{?}{=} 2(2) - 5$

$-1 = -1$ ✔
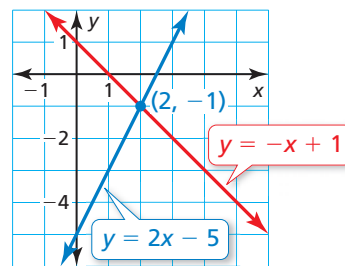
The graphs intersect at $(2, -1)$.

▶ So, the solution of the equation is $x = 2$.

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Solve the equation by graphing. Check your solution.

1. $\frac{1}{2}x - 3 = 2x$

2. $-4 + 9x = -3x + 2$

Solving Absolute Value Equations by Graphing

EXAMPLE 2 Solving an Absolute Value Equation by Graphing

Solve $|x + 1| = |2x - 4|$ by graphing. Check your solutions.

SOLUTION

Recall that an absolute value equation of the form $|ax + b| = |cx + d|$ has two related equations.

$$ax + b = cx + d \qquad \text{Equation 1}$$
$$ax + b = -(cx + d) \qquad \text{Equation 2}$$

So, the related equations of $|x + 1| = |2x - 4|$ are as follows.

$$x + 1 = 2x - 4 \qquad \text{Equation 1}$$
$$x + 1 = -(2x - 4) \qquad \text{Equation 2}$$

Apply the steps for solving an equation by graphing to each of the related equations.

Step 1 Write a system of linear equations for each related equation.

Equation 1

$x + 1 = 2x - 4$
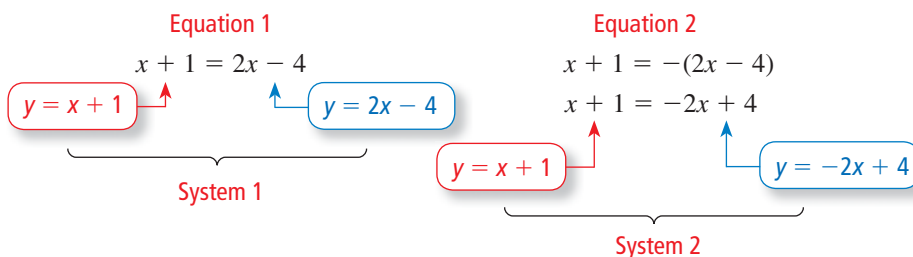
$y = x + 1$ $y = 2x - 4$

System 1

Equation 2

$x + 1 = -(2x - 4)$

$x + 1 = -2x + 4$

$y = x + 1$ $y = -2x + 4$

System 2

Step 2 Graph each system.

System 1

$y = x + 1$

$y = 2x - 4$
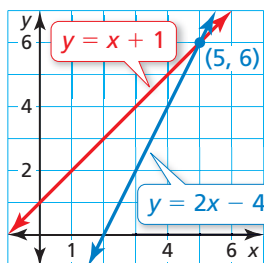
The graphs intersect at (5, 6).

System 2

$y = x + 1$

$y = -2x + 4$
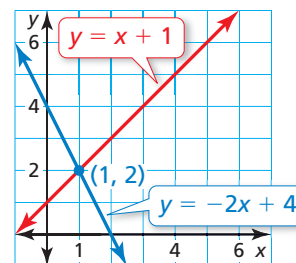
The graphs intersect at (1, 2).

▶ So, the solutions of the equation are $x = 5$ and $x = 1$.

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Solve the equation by graphing. Check your solutions.

3. $|2x + 2| = |x - 2|$

4. $|x - 6| = |-x + 4|$

Solving Real-Life Problems

EXAMPLE 3 Modeling with Mathematics

Your family needs to rent a car for a week while on vacation. Company A charges $3.25 per mile plus a flat fee of $125 per week. Company B charges $3 per mile plus a flat fee of $150 per week. After how many miles of travel are the total costs the same at both companies?

SOLUTION

1. **Understand the Problem** You know the costs of renting a car from two companies. You are asked to determine how many miles of travel will result in the same total costs at both companies.

2. **Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation by graphing.

3. **Solve the Problem**

   Words

Company A				Company B		
Cost per mile	• Miles	+	Flat fee	Cost per mile	• Miles	+	Flat fee

   (= between the two sides)
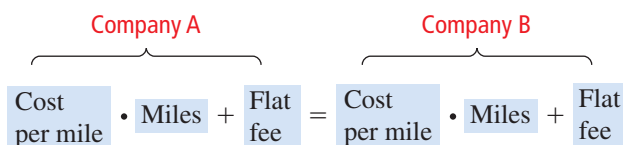
   Variable Let x be the number of miles traveled.

   Equation $3.25x + 125 = 3x + 150$

   Solve the equation by graphing.

   Step 1 Write a system of linear equations using each side of the original equation.

   $$3.25x + 125 = 3x + 150$$

   $y = 3.25x + 125$ $y = 3x + 150$

   Step 2 Use a graphing calculator to graph the system.

   600

   $y = 3x + 150$

   $y = 3.25x + 125$

   Intersection
   X=100   Y=450
   0   150
   0
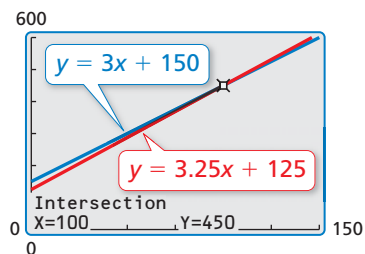
   Because the graphs intersect at (100, 450), the solution of the equation is $x = 100$.

   ▶ So, the total costs are the same after 100 miles.

4. **Look Back** One way to check your solution is to solve the equation algebraically, as shown.

Check

$3.25x + 125 = 3x + 150$
$0.25x + 125 = 150$
$0.25x = 25$
$x = 100$

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5. **WHAT IF?** Company C charges $3.30 per mile plus a flat fee of $115 per week. After how many miles are the total costs the same at Company A and Company C?

Vocabulary and Core Concept Check

1. **REASONING** The graphs of the equations $y = 3x - 20$ and $y = -2x + 10$ intersect at the point $(6, -2)$. Without solving, find the solution of the equation $3x - 20 = -2x + 10$.

2. **WRITING** Explain how to rewrite the absolute value equation $|2x - 4| = |-5x + 1|$ as two systems of linear equations.

Monitoring Progress and Modeling with Mathematics

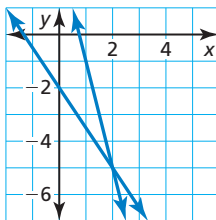In Exercises 3–6, use the graph to solve the equation. Check your solution.

3. $-2x + 3 = x$
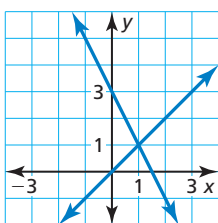
4. $-3 = 4x + 1$
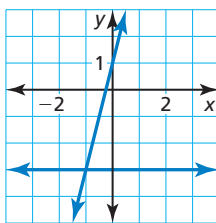
5. $-x - 1 = \frac{1}{3}x + 3$
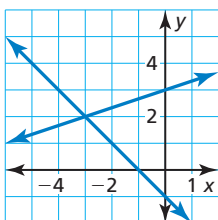
6. $-\frac{3}{2}x - 2 = -4x + 3$

In Exercises 7–14, solve the equation by graphing. Check your solution. *(See Example 1.)*

7. $x + 4 = -x$

8. $4x = x + 3$

9. $x + 5 = -2x - 4$

10. $-2x + 6 = 5x - 1$

11. $\frac{1}{2}x - 2 = 9 - 5x$

12. $-5 + \frac{1}{4}x = 3x + 6$

13. $5x - 7 = 2(x + 1)$

14. $-6(x + 4) = -3x - 6$

In Exercises 15–20, solve the equation by graphing. Determine whether the equation has *one solution*, *no solution*, or *infinitely many solutions*.

15. $3x - 1 = -x + 7$

16. $5x - 4 = 5x + 1$

17. $-4(2 - x) = 4x - 8$

18. $-2x - 3 = 2(x - 2)$

19. $-x - 5 = -\frac{1}{3}(3x + 5)$

20. $\frac{1}{2}(8x + 3) = 4x + \frac{3}{2}$

In Exercises 21 and 22, use the graphs to solve the equation. Check your solutions.

21. $|x - 4| = |3x|$
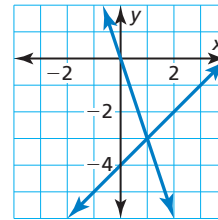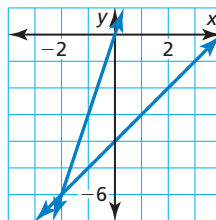
22. $|2x + 4| = |x - 1|$
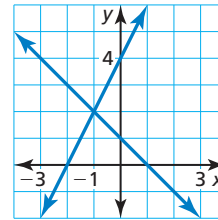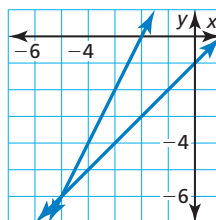
In Exercises 23–30, solve the equation by graphing. Check your solutions. *(See Example 2.)*

23. $|2x| = |x + 3|$

24. $|2x - 6| = |x|$

25. $|-x + 4| = |2x - 2|$

26. $|x + 2| = |-3x + 6|$

27. $|x + 1| = |x - 5|$

28. $|2x + 5| = |-2x + 1|$

29. $|x - 3| = 2|x|$

30. $4|x + 2| = |2x + 7|$

USING TOOLS In Exercises 31 and 32, use a graphing calculator to solve the equation.

31. $0.7x + 0.5 = -0.2x - 1.3$

32. $2.1x + 0.6 = -1.4x + 6.9$

33. **MODELING WITH MATHEMATICS** You need to hire a catering company to serve meals to guests at a wedding reception. Company A charges $500 plus $20 per guest. Company B charges $800 plus $16 per guest. For how many guests are the total costs the same at both companies? *(See Example 3.)*

34. **MODELING WITH MATHEMATICS** Your dog is 16 years old in dog years. Your cat is 28 years old in cat years. For every human year, your dog ages by 7 dog years and your cat ages by 4 cat years. In how many human years will both pets be the same age in their respective types of years?

35. **MODELING WITH MATHEMATICS** You and a friend race across a field to a fence and back. Your friend has a 50-meter head start. The equations shown represent you and your friend's distances d (in meters) from the fence t seconds after the race begins. Find the time at which you catch up to your friend.

You: $d = \left| -5t + 100 \right|$

Your friend: $d = \left| -3\frac{1}{3}t + 50 \right|$

36. **MAKING AN ARGUMENT** The graphs of $y = -x + 4$ and $y = 2x - 8$ intersect at the point $(4, 0)$. So, your friend says the solution of the equation $-x + 4 = 2x - 8$ is $(4, 0)$. Is your friend correct? Explain.

37. **OPEN-ENDED** Find values for m and b so that the solution of the equation $mx + b = -2x - 1$ is $x = -3$.

38. **HOW DO YOU SEE IT?** The graph shows the total revenue and expenses of a company x years after it opens for business.

Revenue and Expenses
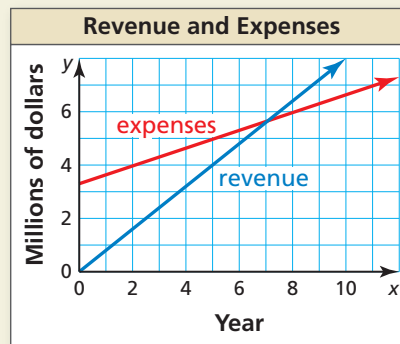
a. Estimate the point of intersection of the graphs.

b. Interpret your answer in part (a).

39. **MATHEMATICAL CONNECTIONS** The value of the perimeter of the triangle (in feet) is equal to the value of the area of the triangle (in square feet). Use a graph to find x.
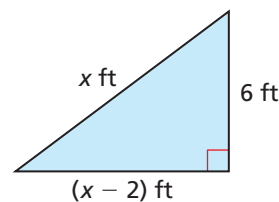
40. **THOUGHT PROVOKING** A car has an initial value of $20,000 and decreases in value at a rate of $1500 per year. Describe a different car that will be worth the same amount as this car in exactly 5 years. Specify the initial value and the rate at which the value decreases.

41. **ABSTRACT REASONING** Use a graph to determine the sign of the solution of the equation $ax + b = cx + d$ in each situation.

a. $0 < b < d$ and $a < c$ b. $d < b < 0$ and $a < c$

Maintaining Mathematical Proficiency   Reviewing what you learned in previous grades and lessons

Graph the inequality.   *(Section 2.1)*

42. $y > 5$ **43.** $x \le -2$ **44.** $n \ge 9$ **45.** $c < -6$

Use the graphs of f and g to describe the transformation from the graph of f to the graph of g.   *(Section 3.6)*

46. $f(x) = x - 5$; $g(x) = f(x + 2)$ **47.** $f(x) = 6x$; $g(x) = -f(x)$

48. $f(x) = -2x + 1$; $g(x) = f(4x)$ **49.** $f(x) = \frac{1}{2}x - 2$; $g(x) = f(x - 1)$

# 5.6 Graphing Linear Inequalities in Two Variables

Essential Question How can you graph a linear inequality in two variables?

A **solution of a linear inequality in two variables** is an ordered pair (x, y) that makes the inequality true. The **graph of a linear inequality** in two variables shows all the solutions of the inequality in a coordinate plane.

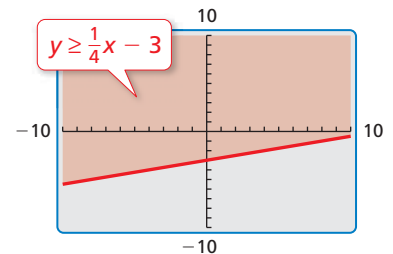### EXPLORATION 1 Writing a Linear Inequality in Two Variables

Work with a partner.

a. Write an equation represented by the dashed line.

b. The solutions of an inequality are represented by the shaded region. In words, describe the solutions of the inequality.

c. Write an inequality represented by the graph. Which inequality symbol did you use? Explain your reasoning.
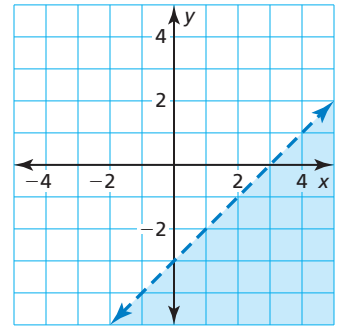
EXPLORATION 2 Using a Graphing Calculator

Work with a partner. Use a graphing calculator to graph $y \geq \frac{1}{4}x - 3$.

a. Enter the equation $y = \frac{1}{4}x - 3$ into your calculator.

b. The inequality has the symbol $\geq$. So, the region to be shaded is above the graph of $y = \frac{1}{4}x - 3$, as shown. Verify this by testing a point in this region, such as $(0, 0)$, to make sure it is a solution of the inequality.

$y \geq \frac{1}{4}x - 3$

Because the inequality symbol is *greater than or equal to*, the line is solid and not dashed. Some graphing calculators always use a solid line when graphing inequalities. In this case, you have to determine whether the line should be solid or dashed, based on the inequality symbol used in the original inequality.

EXPLORATION 3 Graphing Linear Inequalities in Two Variables

Work with a partner. Graph each linear inequality in two variables. Explain your steps. Use a graphing calculator to check your graphs.

a. $y > x + 5$ **b.** $y \leq -\frac{1}{2}x + 1$ **c.** $y \geq -x - 5$

Communicate Your Answer

4. How can you graph a linear inequality in two variables?

5. Give an example of a real-life situation that can be modeled using a linear inequality in two variables.

USING TOOLS STRATEGICALLY

To be proficient in math, you need to use technological tools to explore and deepen your understanding of concepts.

linear inequality in
    two variables, *p. 268*
solution of a linear inequality
    in two variables, *p. 268*
graph of a linear inequality,
    p. 268
half-planes, *p. 268*

Previous
ordered pair

What You Will Learn

▶ Check solutions of linear inequalities.
▶ Graph linear inequalities in two variables.
▶ Use linear inequalities to solve real-life problems.

Linear Inequalities

A **linear inequality in two variables**, *x* and *y*, can be written as

$$ax + by < c \qquad ax + by \le c \qquad ax + by > c \qquad ax + by \ge c$$

where *a*, *b*, and *c* are real numbers. A **solution of a linear inequality in two variables** is an ordered pair (x, y) that makes the inequality true.

EXAMPLE 1   **Checking Solutions**

Tell whether the ordered pair is a solution of the inequality.

a. $2x + y < -3; (-1, 9)$ **b.** $x - 3y \ge 8; (2, -2)$

SOLUTION

a. $2x + y < -3$ Write the inequality.

   $2(-1) + 9 \overset{?}{<} -3$ Substitute −1 for *x* and 9 for *y*.

              $7 \not< -3$ ✗   Simplify. 7 is *not* less than −3.

▶ So, $(-1, 9)$ is *not* a solution of the inequality.

b. $x - 3y \ge 8$ Write the inequality.

   $2 - 3(-2) \overset{?}{\ge} 8$ Substitute 2 for *x* and −2 for *y*.

              $8 \ge 8$ ✓   Simplify. 8 is equal to 8.

▶ So, $(2, -2)$ is a solution of the inequality.

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Tell whether the ordered pair is a solution of the inequality.

1. $x + y > 0; (-2, 2)$ **2.** $4x - y \ge 5; (0, 0)$

3. $5x - 2y \le -1; (-4, -1)$ **4.** $-2x - 3y < 15; (5, -7)$

Graphing Linear Inequalities in Two Variables

The **graph of a linear inequality** in two variables shows all the solutions of the inequality in a coordinate plane.

All solutions of $y < 2x$ lie on one side of the boundary line $y = 2x$.
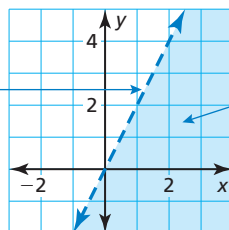
The boundary line divides the coordinate plane into two **half-planes**. The shaded half-plane is the graph of $y < 2x$.

Core Concept

Graphing a Linear Inequality in Two Variables

Step 1 Graph the boundary line for the inequality. Use a dashed line for < or >. Use a solid line for ≤ or ≥.

Step 2 Test a point that is not on the boundary line to determine whether it is a solution of the inequality.

Step 3 When the test point is a solution, shade the half-plane that contains the point. When the test point is *not* a solution, shade the half-plane that does *not* contain the point.

EXAMPLE 2 **Graphing a Linear Inequality in One Variable**

Graph $y \leq 2$ in a coordinate plane.

SOLUTION

Step 1 Graph $y = 2$. Use a solid line because the inequality symbol is ≤.

Step 2 Test $(0, 0)$.

$$y \leq 2 \qquad \text{Write the inequality.}$$

$$0 \leq 2 \quad \checkmark \qquad \text{Substitute.}$$

Step 3 Because $(0, 0)$ is a solution, shade the half-plane that contains $(0, 0)$.

STUDY TIP

It is often convenient to use the origin as a test point. However, you must choose a different test point when the origin is on the boundary line.
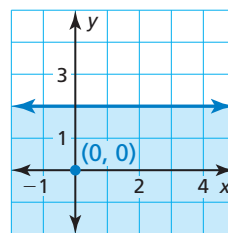
EXAMPLE 3 **Graphing a Linear Inequality in Two Variables**

Graph $-x + 2y > 2$ in a coordinate plane.

SOLUTION

Step 1 Graph $-x + 2y = 2$, or $y = \frac{1}{2}x + 1$. Use a dashed line because the inequality symbol is >.

Step 2 Test $(0, 0)$.

$$-x + 2y > 2 \qquad \text{Write the inequality.}$$

$$-(0) + 2(0) \overset{?}{>} 2 \qquad \text{Substitute.}$$

$$0 \not> 2 \quad \times \qquad \text{Simplify.}$$

Step 3 Because $(0, 0)$ is *not* a solution, shade the half-plane that does *not* contain $(0, 0)$.
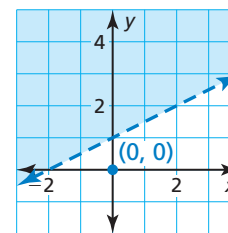
Check
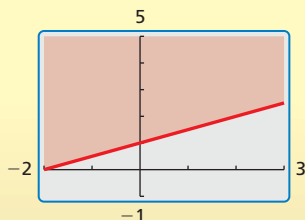
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Graph the inequality in a coordinate plane.

5. $y > -1$

6. $x \leq -4$

7. $x + y \leq -4$

8. $x - 2y < 0$

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**

You can spend at most $10 on grapes and apples for a fruit salad. Grapes cost $2.50 per pound, and apples cost $1 per pound. Write and graph an inequality that represents the amounts of grapes and apples you can buy. Identify and interpret two solutions of the inequality.

SOLUTION

1. **Understand the Problem** You know the most that you can spend and the prices per pound for grapes and apples. You are asked to write and graph an inequality and then identify and interpret two solutions.

2. **Make a Plan** Use a verbal model to write an inequality that represents the problem. Then graph the inequality. Use the graph to identify two solutions. Then interpret the solutions.

3. **Solve the Problem**

Words

Cost per pound of grapes	•	Pounds of grapes	+	Cost per pound of apples	•	Pounds of apples	≤	Amount you can spend

Variables Let x be pounds of grapes and y be pounds of apples.

Inequality $2.50 \cdot x + 1 \cdot y \leq 10$

Step 1 Graph $2.5x + y = 10$, or $y = -2.5x + 10$. Use a solid line because the inequality symbol is $\leq$. Restrict the graph to positive values of x and y because negative values do not make sense in this real-life context.

Step 2 Test $(0, 0)$.

$$2.5x + y \leq 10 \qquad \text{Write the inequality.}$$
$$2.5(0) + 0 \overset{?}{\leq} 10 \qquad \text{Substitute.}$$
$$0 \leq 10 \ \checkmark \qquad \text{Simplify.}$$

Step 3 Because $(0, 0)$ is a solution, shade the half-plane that contains $(0, 0)$.

▶ One possible solution is $(1, 6)$ because it lies in the shaded half-plane. Another possible solution is $(2, 5)$ because it lies on the solid line. So, you can buy 1 pound of grapes and 6 pounds of apples, or 2 pounds of grapes and 5 pounds of apples.

4. **Look Back** Check your solutions by substituting them into the original inequality, as shown.

Fruit Salad

Check

$$2.5x + y \leq 10$$
$$2.5(1) + 6 \overset{?}{\leq} 10$$
$$8.5 \leq 10 \ \checkmark$$

$$2.5x + y \leq 10$$
$$2.5(2) + 5 \overset{?}{\leq} 10$$
$$10 \leq 10 \ \checkmark$$

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9. You can spend at most $12 on red peppers and tomatoes for salsa. Red peppers cost $4 per pound, and tomatoes cost $3 per pound. Write and graph an inequality that represents the amounts of red peppers and tomatoes you can buy. Identify and interpret two solutions of the inequality.

Vocabulary and Core Concept Check

1. **VOCABULARY** How can you tell whether an ordered pair is a solution of a linear inequality?

2. **WRITING** Compare the graph of a linear inequality in two variables with the graph of a linear equation in two variables.
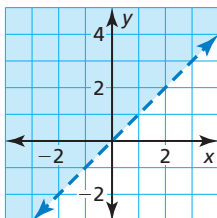
Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, tell whether the ordered pair is a solution of the inequality. *(See Example 1.)*

3. $x + y < 7$; (2, 3)
4. $x - y \le 0$; (5, 2)

5. $x + 3y \ge -2$; (−9, 2)
6. $8x + y > -6$; (−1, 2)

7. $-6x + 4y \le 6$; (−3, −3)

8. $3x - 5y \ge 2$; (−1, −1)
9. $-x - 6y > 12$; (−8, 2)

10. $-4x - 8y < 15$; (−6, 3)

In Exercises 11–16, tell whether the ordered pair is a solution of the inequality whose graph is shown.

11. (0, −1)
12. (−1, 3)

13. (1, 4)
14. (0, 0)

15. (3, 3)
16. (2, 1)

17. **MODELING WITH MATHEMATICS** A carpenter has at most $250 to spend on lumber. The inequality $8x + 12y \le 250$ represents the numbers x of 2-by-8 boards and the numbers y of 4-by-4 boards the carpenter can buy. Can the carpenter buy twelve 2-by-8 boards and fourteen 4-by-4 boards? Explain.

4 in. x 4 in. x 8 ft
$12 each

2 in. x 8 in. x 8 ft
$8 each

18. **MODELING WITH MATHEMATICS** The inequality $3x + 2y \ge 93$ represents the numbers x of multiple-choice questions and the numbers y of matching questions you can answer correctly to receive an A on a test. You answer 20 multiple-choice questions and 18 matching questions correctly. Do you receive an A on the test? Explain.

In Exercises 19–24, graph the inequality in a coordinate plane. *(See Example 2.)*

19. $y \le 5$
20. $y > 6$

21. $x < 2$
22. $x \ge -3$

23. $y > -7$
24. $x < 9$

In Exercises 25–30, graph the inequality in a coordinate plane. *(See Example 3.)*

25. $y > -2x - 4$
26. $y \le 3x - 1$

27. $-4x + y < -7$
28. $3x - y \ge 5$

29. $5x - 2y \le 6$
30. $-x + 4y > -12$

ERROR ANALYSIS In Exercises 31 and 32, describe and correct the error in graphing the inequality.

31. $y < -x + 1$
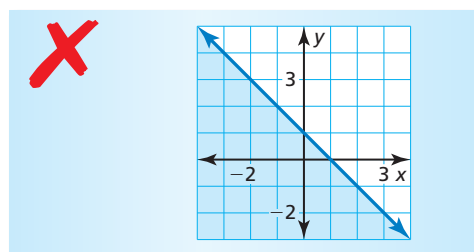
32. $y \le 3x - 2$
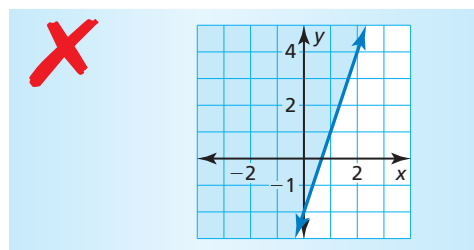
33. MODELING WITH MATHEMATICS You have at most $20 to spend at an arcade. Arcade games cost $0.75 each, and snacks cost $2.25 each. Write and graph an inequality that represents the numbers of games you can play and snacks you can buy. Identify and interpret two solutions of the inequality. *(See Example 4.)*

34. MODELING WITH MATHEMATICS A drama club must sell at least $1500 worth of tickets to cover the expenses of producing a play. Write and graph an inequality that represents how many adult and student tickets the club must sell. Identify and interpret two solutions of the inequality.

School Play
Adults: $10
Students: $6

In Exercises 35–38, write an inequality that represents the graph.

35.

36.

37.

38.
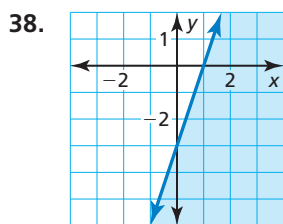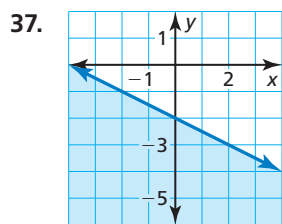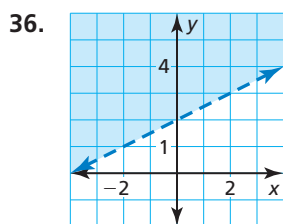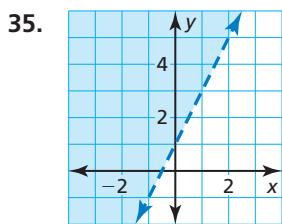
39. PROBLEM SOLVING Large boxes weigh 75 pounds, and small boxes weigh 40 pounds.

Weight limit: 2000 lb

a. Write and graph an inequality that represents the numbers of large and small boxes a 200-pound delivery person can take on the elevator.

b. Explain why some solutions of the inequality might not be practical in real life.

40. HOW DO YOU SEE IT? Match each inequality with its graph.

a. $3x - 2y \le 6$ b. $3x - 2y < 6$

c. $3x - 2y > 6$ d. $3x - 2y \ge 6$

A.

B.

C.

D.
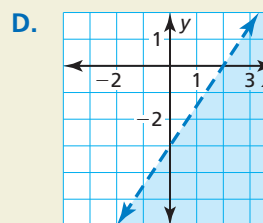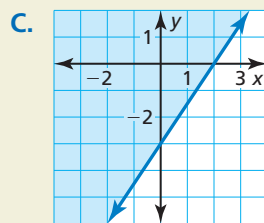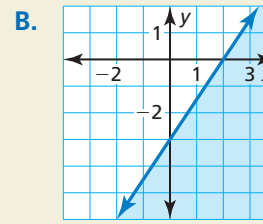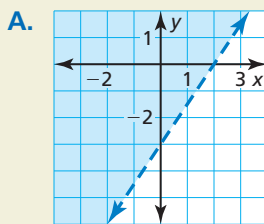
41. REASONING When graphing a linear inequality in two variables, why must you choose a test point that is *not* on the boundary line?

42. THOUGHT PROVOKING Write a linear inequality in two variables that has the following two properties.

• (0, 0), (0, −1), and (0, 1) are not solutions.
• (1, 1), (3, −1), and (−1, 3) are solutions.

43. WRITING Can you always use (0, 0) as a test point when graphing an inequality? Explain.

CRITICAL THINKING In Exercises 44 and 45, write and graph an inequality whose graph is described by the given information.

44. The points (2, 5) and (−3, −5) lie on the boundary line. The points (6, 5) and (−2, −3) are solutions of the inequality.

45. The points (−7, −16) and (1, 8) lie on the boundary line. The points (−7, 0) and (3, 14) are *not* solutions of the inequality.

Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Write the next three terms of the arithmetic sequence. *(Section 4.6)*

46. 0, 8, 16, 24, 32, . . .   **47.** −5, −8, −11, −14, −17, . . .   **48.** $-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$
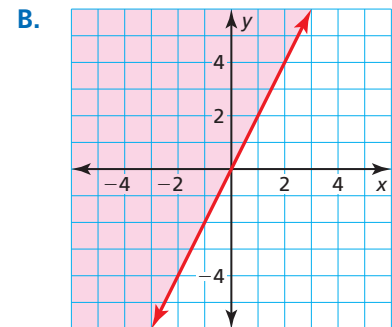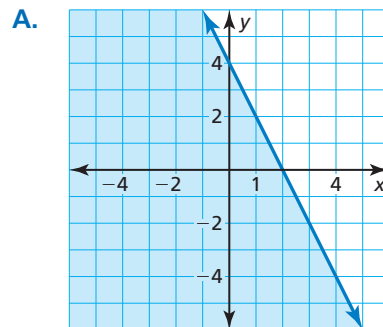
Essential Question   How can you graph a system of linear inequalities?

EXPLORATION 1   **Graphing Linear Inequalities**

Work with a partner. Match each linear inequality with its graph. Explain your reasoning.

$$2x + y \leq 4 \qquad \text{Inequality 1}$$

$$2x - y \leq 0 \qquad \text{Inequality 2}$$

A.

B.

EXPLORATION 2   **Graphing a System of Linear Inequalities**

Work with a partner. Consider the linear inequalities given in Exploration 1.

$$2x + y \leq 4 \qquad \text{Inequality 1}$$

$$2x - y \leq 0 \qquad \text{Inequality 2}$$

a. Use two different colors to graph the inequalities in the same coordinate plane. What is the result?

b. Describe each of the shaded regions of the graph. What does the unshaded region represent?

MAKING SENSE OF PROBLEMS

To be proficient in math, you need to explain to yourself the meaning of a problem.

Communicate Your Answer

3. How can you graph a system of linear inequalities?

4. When graphing a system of linear inequalities, which region represents the solution of the system?

5. Do you think all systems of linear inequalities have a solution? Explain your reasoning.

6. Write a system of linear inequalities represented by the graph.

What You Will Learn

▶ Check solutions of systems of linear inequalities.

▶ Graph systems of linear inequalities.

▶ Write systems of linear inequalities.

▶ Use systems of linear inequalities to solve real-life problems.
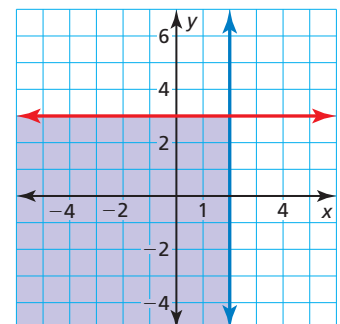
Core Vocabulary

system of linear inequalities,
  p. 274
solution of a system of linear
  inequalities, *p. 274*
graph of a system of linear
  inequalities, *p. 275*

Previous
linear inequality in two
  variables

Systems of Linear Inequalities

A **system of linear inequalities** is a set of two or more linear inequalities in the same variables. An example is shown below.

$y < x + 2$ Inequality 1

$y \geq 2x - 1$ Inequality 2

A **solution of a system of linear inequalities** in two variables is an ordered pair that is a solution of each inequality in the system.

EXAMPLE 1   Checking Solutions

Tell whether each ordered pair is a solution of the system of linear inequalities.

$y < 2x$ Inequality 1

$y \geq x + 1$ Inequality 2

a. $(3, 5)$ **b.** $(-2, 0)$

SOLUTION

a. Substitute 3 for x and 5 for y in each inequality.

Inequality 1	Inequality 2
$y < 2x$	$y \geq x + 1$
$5 \overset{?}{<} 2(3)$	$5 \overset{?}{\geq} 3 + 1$
$5 < 6$ ✓	$5 \geq 4$ ✓

▶ Because the ordered pair $(3, 5)$ is a solution of each inequality, it is a solution of the system.

b. Substitute -2 for x and 0 for y in each inequality.

Inequality 1	Inequality 2
$y < 2x$	$y \geq x + 1$
$0 \overset{?}{<} 2(-2)$	$0 \overset{?}{\geq} -2 + 1$
$0 \not< -4$ ✗	$0 \geq -1$ ✓

▶ Because $(-2, 0)$ is not a solution of each inequality, it is *not* a solution of the system.

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Tell whether the ordered pair is a solution of the system of linear inequalities.

1. $(-1, 5)$; $\begin{array}{l} y < 5 \\ y > x - 4 \end{array}$ **2.** $(1, 4)$; $\begin{array}{l} y \geq 3x + 1 \\ y > x - 1 \end{array}$

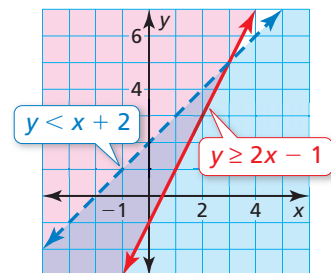# Graphing Systems of Linear Inequalities

The **graph of a system of linear inequalities** is the graph of all the solutions of the system.

Core Concept

Graphing a System of Linear Inequalities

Step 1 Graph each inequality in the same coordinate plane.

Step 2 Find the intersection of the half-planes that are solutions of the inequalities. This intersection is the graph of the system.

$y < x + 2$

$y \geq 2x - 1$

EXAMPLE 2 **Graphing a System of Linear Inequalities**

Graph the system of linear inequalities.

$y \leq 3$ Inequality 1

$y > x + 2$ Inequality 2

SOLUTION

Step 1 Graph each inequality.

Step 2 Find the intersection of the half-planes. One solution is $(-3, 1)$.

The solution is the purple-shaded region.

$(-3, 1)$
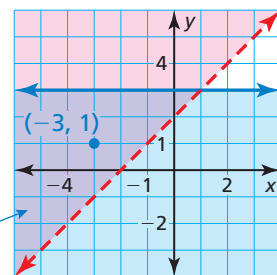
EXAMPLE 3 **Graphing a System of Linear Inequalities: No Solution**

Graph the system of linear inequalities.

$2x + y < -1$ Inequality 1

$2x + y > 3$ Inequality 2

SOLUTION

Step 1 Graph each inequality.

Step 2 Find the intersection of the half-planes. Notice that the lines are parallel, and the half-planes do not intersect.

▶ So, the system has no solution.
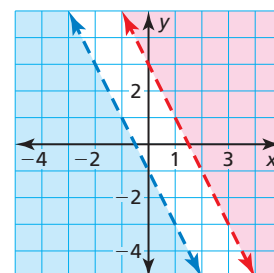
Monitoring Progress   🔊   Help in English and Spanish at *BigIdeasMath.com*

Graph the system of linear inequalities.

3. $y \geq -x + 4$

$x + y \leq 0$

4. $y > 2x - 3$

$y \geq \frac{1}{2}x + 1$

5. $-2x + y < 4$

$2x + y > 4$

Writing Systems of Linear Inequalities

EXAMPLE 4   **Writing a System of Linear Inequalities**

Write a system of linear inequalities represented
by the graph.
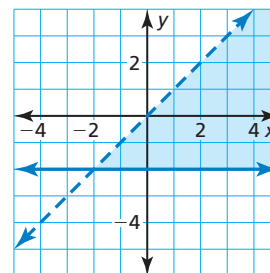
SOLUTION

Inequality 1 The horizontal boundary line passes
through $(0, -2)$. So, an equation of the
line is $y = -2$. The shaded region is
above the *solid* boundary line, so the
inequality is $y \geq -2$.

Inequality 2 The slope of the other boundary line is 1, and the y-intercept
is 0. So, an equation of the line is $y = x$. The shaded region
is *below* the *dashed* boundary line, so the inequality is $y < x$.

▶ The system of linear inequalities represented by the graph is

$y \geq -2$ Inequality 1

$y < x.$ Inequality 2

EXAMPLE 5   **Writing a System of Linear Inequalities**

Write a system of linear inequalities represented
by the graph.
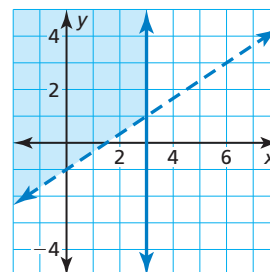
SOLUTION

Inequality 1 The vertical boundary line passes
through $(3, 0)$. So, an equation of the
line is $x = 3$. The shaded region is to
the *left* of the *solid* boundary line,
so the inequality is $x \leq 3$.

Inequality 2 The slope of the other boundary line is $\frac{2}{3}$, and the y-intercept is -1.
So, an equation of the line is $y = \frac{2}{3}x - 1$. The shaded region is *above*
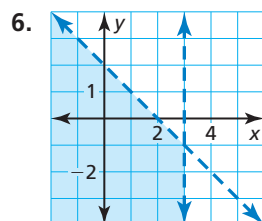the *dashed* boundary line, so the inequality is $y > \frac{2}{3}x - 1$.

▶ The system of linear inequalities represented by the graph is

$x \leq 3$ Inequality 1

$y > \frac{2}{3}x - 1.$ Inequality 2

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Write a system of linear inequalities represented by the graph.

6.

7.
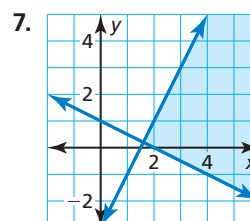
Solving Real-Life Problems

EXAMPLE 6 **Modeling with Mathematics**

You have at most 8 hours to spend at the mall and at the beach. You want to spend at least 2 hours at the mall and more than 4 hours at the beach. Write and graph a system that represents the situation. How much time can you spend at each location?
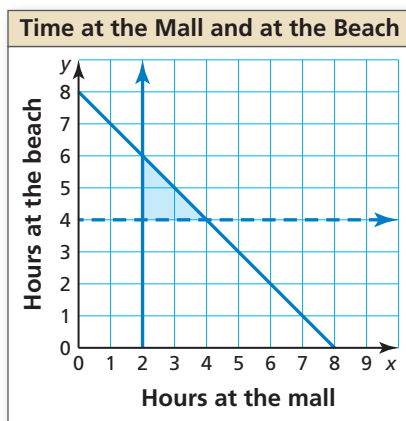
SOLUTION

1. **Understand the Problem** You know the total amount of time you can spend at the mall and at the beach. You also know how much time you want to spend at each location. You are asked to write and graph a system that represents the situation and determine how much time you can spend at each location.

2. **Make a Plan** Use the given information to write a system of linear inequalities. Then graph the system and identify an ordered pair in the solution region.

3. **Solve the Problem** Let x be the number of hours at the mall and let y be the number of hours at the beach.

$$x + y \leq 8 \qquad \text{at most 8 hours at the mall and at the beach}$$
$$x \geq 2 \qquad \text{at least 2 hours at the mall}$$
$$y > 4 \qquad \text{more than 4 hours at the beach}$$

Graph the system.

Check

$$x + y \leq 8$$
$$2.5 + 5 \overset{?}{\leq} 8$$
$$7.5 \leq 8 \ \checkmark$$

$$x \geq 2$$
$$2.5 \geq 2 \ \checkmark$$

$$y > 4$$
$$5 > 4 \ \checkmark$$

Time at the Mall and at the Beach

One ordered pair in the solution region is (2.5, 5).

▶ So, you can spend 2.5 hours at the mall and 5 hours at the beach.

4. **Look Back** Check your solution by substituting it into the inequalities in the system, as shown.

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8. Name another solution of Example 6.

9. **WHAT IF?** You want to spend at least 3 hours at the mall. How does this change the system? Is (2.5, 5) still a solution? Explain.

Vocabulary and Core Concept Check

1. **VOCABULARY** How can you verify that an ordered pair is a solution of a system of linear inequalities?

2. **WHICH ONE DOESN'T BELONG?** Use the graph shown. Which of the ordered pairs does *not* belong with the other three? Explain your reasoning.
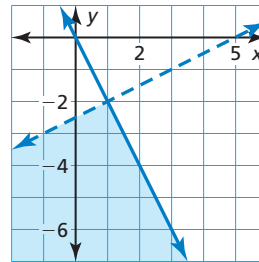
$(1, -2)$ $(0, -4)$ $(-1, -6)$ $(2, -4)$
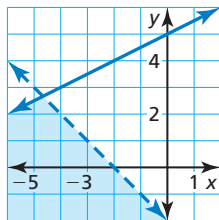
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, tell whether the ordered pair is a solution of the system of linear inequalities.

3. $(-4, 3)$

4. $(-3, -1)$

5. $(-2, 5)$

6. $(1, 1)$

In Exercises 7–10, tell whether the ordered pair is a solution of the system of linear inequalities. *(See Example 1.)*

7. $(-5, 2)$; $\begin{array}{l} y < 4 \\ y > x + 3 \end{array}$

8. $(1, -1)$; $\begin{array}{l} y > -2 \\ y > x - 5 \end{array}$

9. $(0, 0)$; $\begin{array}{l} y \le x + 7 \\ y \ge 2x + 3 \end{array}$

10. $(4, -3)$; $\begin{array}{l} y \le -x + 1 \\ y \le 5x - 2 \end{array}$

In Exercises 11–20, graph the system of linear inequalities. *(See Examples 2 and 3.)*

11. $y > -3$
    $y \ge 5x$

12. $y < -1$
    $x > 4$

13. $y < -2$
    $y > 2$

14. $y < x - 1$
    $y \ge x + 1$

15. $y \ge -5$
    $y - 1 < 3x$

16. $x + y > 4$
    $y \ge \frac{3}{2}x - 9$

17. $x + y > 1$
    $-x - y < -3$

18. $2x + y \le 5$
    $y + 2 \ge -2x$

19. $x < 4$
    $y > 1$
    $y \ge -x + 1$

20. $x + y \le 10$
    $x - y \ge 2$
    $y > 2$

In Exercises 21–26, write a system of linear inequalities represented by the graph. *(See Examples 4 and 5.)*

21.

22.

23.

24.

25.

26.
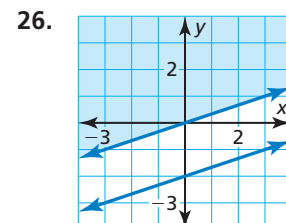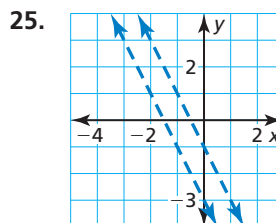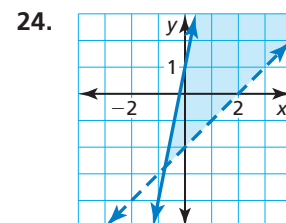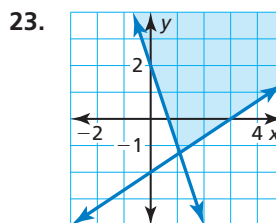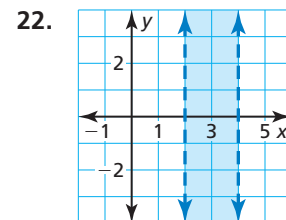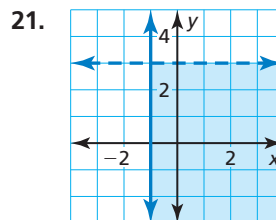
ERROR ANALYSIS In Exercises 27 and 28, describe and correct the error in graphing the system of linear inequalities.

27.

✗ $y \le x - 1$
   $y \ge x + 3$

28.

✗ $y \le 3x + 4$
   $y > \frac{1}{2}x + 2$
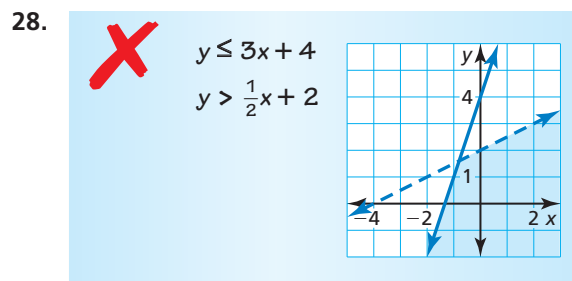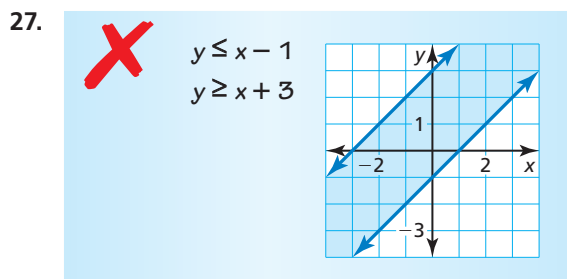
29. **MODELING WITH MATHEMATICS** You can spend at most $21 on fruit. Blueberries cost $4 per pound, and strawberries cost $3 per pound. You need at least 3 pounds of fruit to make muffins. *(See Example 6.)*

   a. Write and graph a system of linear inequalities that represents the situation.

   b. Identify and interpret a solution of the system.

   c. Use the graph to determine whether you can buy 4 pounds of blueberries and 1 pound of strawberries.

30. **MODELING WITH MATHEMATICS** You earn $10 per hour working as a manager at a grocery store. You are required to work at the grocery store at least 8 hours per week. You also teach music lessons for $15 per hour. You need to earn at least $120 per week, but you do not want to work more than 20 hours per week.

   a. Write and graph a system of linear inequalities that represents the situation.

   b. Identify and interpret a solution of the system.

   c. Use the graph to determine whether you can work 8 hours at the grocery store and teach 1 hour of music lessons.

31. **MODELING WITH MATHEMATICS** You are fishing for surfperch and rockfish, which are species of bottomfish. Gaming laws allow you to catch no more than 15 surfperch per day, no more than 10 rockfish per day, and no more than 20 total bottomfish per day.

   a. Write and graph a system of linear inequalities that represents the situation.

   b. Use the graph to determine whether you can catch 11 surfperch and 9 rockfish in 1 day.

surfperch

rockfish

32. **REASONING** Describe the intersection of the half-planes of the system shown.

$$x - y \le 4$$
$$x - y \ge 4$$

33. **MATHEMATICAL CONNECTIONS** The following points are the vertices of a shaded rectangle.

$$(-1, 1), (6, 1), (6, -3), (-1, -3)$$

   a. Write a system of linear inequalities represented by the shaded rectangle.

   b. Find the area of the rectangle.

34. **MATHEMATICAL CONNECTIONS** The following points are the vertices of a shaded triangle.

$$(2, 5), (6, -3), (-2, -3)$$

   a. Write a system of linear inequalities represented by the shaded triangle.

   b. Find the area of the triangle.

35. **PROBLEM SOLVING** You plan to spend less than half of your monthly $2000 paycheck on housing and savings. You want to spend at least 10% of your paycheck on savings and at most 30% of it on housing. How much money can you spend on savings and housing?

36. **PROBLEM SOLVING** On a road trip with a friend, you drive about 70 miles per hour, and your friend drives about 60 miles per hour. The plan is to drive less than 15 hours and at least 600 miles each day. Your friend will drive more hours than you. How many hours can you and your friend each drive in 1 day?

37. WRITING How are solving systems of linear inequalities and solving systems of linear equations similar? How are they different?

38. HOW DO YOU SEE IT? The graphs of two linear equations are shown.
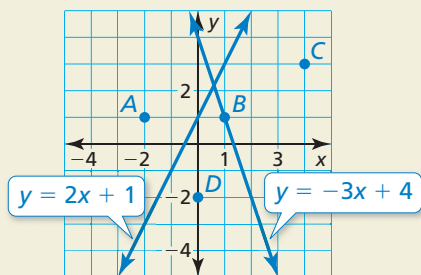
Replace the equal signs with inequality symbols to create a system of linear inequalities that has point C as a solution, but not points A, B, and D. Explain your reasoning.

$$y \;\boxed{\phantom{<}}\; -3x + 4$$
$$y \;\boxed{\phantom{<}}\; 2x + 1$$

39. USING STRUCTURE Write a system of linear inequalities that is equivalent to $|y| < x$, where $x > 0$. Graph the system.

40. MAKING AN ARGUMENT Your friend says that a system of linear inequalities in which the boundary lines are parallel must have no solution. Is your friend correct? Explain.

41. CRITICAL THINKING Is it possible for the solution set of a system of linear inequalities to be all real numbers? Explain your reasoning.

OPEN-ENDED In Exercises 42–44, write a system of linear inequalities with the given characteristic.

42. All solutions are in Quadrant I.

43. All solutions have one positive coordinate and one negative coordinate.

44. There are no solutions.

45. OPEN-ENDED One inequality in a system is $-4x + 2y > 6$. Write another inequality so the system has (a) no solution and (b) infinitely many solutions.

46. THOUGHT PROVOKING You receive a gift certificate for a clothing store and plan to use it to buy T-shirts and sweatshirts. Describe a situation in which you can buy 9 T-shirts and 1 sweatshirt, but you cannot buy 3 T-shirts and 8 sweatshirts. Write and graph a system of linear inequalities that represents the situation.

47. CRITICAL THINKING Write a system of linear inequalities that has exactly one solution.

48. MODELING WITH MATHEMATICS You make necklaces and key chains to sell at a craft fair. The table shows the amounts of time and money it takes to make a necklace and a key chain, and the amounts of time and money you have available for making them.

	Necklace	Key chain	Available
Time to make (hours)	0.5	0.25	20
Cost to make (dollars)	2	3	120

a. Write and graph a system of four linear inequalities that represents the number x of necklaces and the number y of key chains that you can make.

b. Find the vertices (corner points) of the graph of the system.

c. You sell each necklace for $10 and each key chain for $8. The revenue R is given by the equation $R = 10x + 8y$. Find the revenue corresponding to each ordered pair in part (b). Which vertex results in the maximum revenue?

Maintaining Mathematical Proficiency

Write the product using exponents. *(Skills Review Handbook)*

49. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

50. $(-13) \cdot (-13) \cdot (-13)$

51. $x \cdot x \cdot x \cdot x \cdot x$

Write an equation of the line with the given slope and y-intercept. *(Section 4.1)*

52. slope: 1
y-intercept: -6

53. slope: -3
y-intercept: 5

54. slope: $-\frac{1}{4}$
y-intercept: -1

55. slope: $\frac{4}{3}$
y-intercept: 0

Core Vocabulary

Core Concepts

Section 5.5

Section 5.6

Section 5.7

Mathematical Practices

1. Why do the equations in Exercise 35 on page 266 contain absolute value expressions?

2. Why is it important to be precise when answering part (a) of Exercise 39 on page 272?

3. Describe the overall step-by-step process you used to solve Exercise 35 on page 279.

Performance Task

Prize Patrol

You have been selected to drive a prize patrol cart and place prizes on the competing teams' predetermined paths. You know the teams' routes and you can only make one pass. Where will you place the prizes so that each team will have a chance to find a prize on their route?

To explore the answers to these questions and more, go to *BigIdeasMath.com*.

5.1 Solving Systems of Linear Equations by Graphing *(pp. 235–240)*

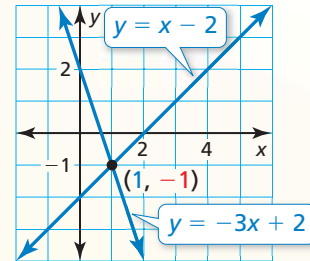**Solve the system by graphing.**
$$y = x - 2 \qquad \text{Equation 1}$$
$$y = -3x + 2 \qquad \text{Equation 2}$$

Step 1 Graph each equation.

Step 2 Estimate the point of intersection.
The graphs appear to intersect at $(1, -1)$.

Step 3 Check your point from Step 2.

Equation 1	Equation 2
$y = x - 2$	$y = -3x + 2$
$-1 \overset{?}{=} 1 - 2$	$-1 \overset{?}{=} -3(1) + 2$
$-1 = -1$ ✓	$-1 = -1$ ✓

▶ The solution is $(1, -1)$.

Solve the system of linear equations by graphing.

1. $y = -3x + 1$
$y = x - 7$

2. $y = -4x + 3$
$4x - 2y = 6$

3. $5x + 5y = 15$
$2x - 2y = 10$

5.2 Solving Systems of Linear Equations by Substitution *(pp. 241–246)*

Solve the system by substitution.
$$-2x + y = -8 \qquad \text{Equation 1}$$
$$7x + y = 10 \qquad \text{Equation 2}$$

Step 1 Solve for y in Equation 1.
$$y = 2x - 8 \qquad \text{Revised Equation 1}$$

Step 2 Substitute $2x - 8$ for y in Equation 2 and solve for x.

$7x + y = 10$	Equation 2
$7x + (2x - 8) = 10$	Substitute $2x - 8$ for y.
$9x - 8 = 10$	Combine like terms.
$9x = 18$	Add 8 to each side.
$x = 2$	Divide each side by 9.

Step 3 Substituting 2 for x in Equation 1 and solving for y gives $y = -4$.

▶ The solution is $(2, -4)$.

Solve the system of linear equations by substitution. Check your solution.

4. $3x + y = -9$
$y = 5x + 7$

5. $x + 4y = 6$
$x - y = 1$

6. $2x + 3y = 4$
$y + 3x = 6$

7. You spend $20 total on tubes of paint and disposable brushes for an art project. Tubes of paint cost $4.00 each and paintbrushes cost $0.50 each. You purchase twice as many brushes as tubes of paint. How many brushes and tubes of paint do you purchase?

Solving Systems of Linear Equations by Elimination *(pp. 247–252)*

Solve the system by elimination. $4x + 6y = -8$ Equation 1

$x - 2y = -2$ Equation 2

Step 1 Multiply Equation 2 by 3 so that the coefficients of the *y*-terms are opposites.

$4x + 6y = -8$ $4x + 6y = -8$ Equation 1

$x - 2y = -2$ **Multiply by 3.** $3x - 6y = -6$ Revised Equation 2

Step 2 Add the equations.

$4x + 6y = -8$ Equation 1

$\underline{3x - 6y = -6}$ Revised Equation 2

$7x \quad = -14$ Add the equations.

Check

Equation 1

$4x + 6y = -8$

$4(-2) + 6(0) \overset{?}{=} -8$

$-8 = -8$ ✓

Step 3 Solve for *x*.

$7x = -14$ Resulting equation from Step 2

$x = -2$ Divide each side by 7.

Step 4 Substitute -2 for *x* in one of the original equations and solve for *y*.

$4x + 6y = -8$ Equation 1

$4(-2) + 6y = -8$ Substitute -2 for *x*.

$-8 + 6y = -8$ Multiply.

$y = 0$ Solve for *y*.

Equation 2

$x - 2y = -2$

$(-2) - 2(0) \overset{?}{=} -2$

$-2 = -2$ ✓

▶ The solution is $(-2, 0)$.

Solve the system of linear equations by elimination. Check your solution.

8. $9x - 2y = 34$
$5x + 2y = -6$

9. $x + 6y = 28$
$2x - 3y = -19$

10. $8x - 7y = -3$
$6x - 5y = -1$

Solving Special Systems of Linear Equations *(pp. 253–258)*

Solve the system. $4x + 2y = -14$ Equation 1

$y = -2x - 6$ Equation 2

Solve by substitution. Substitute $-2x - 6$ for *y* in Equation 1.

$4x + 2y = -14$ Equation 1

$4x + 2(-2x - 6) = -14$ Substitute $-2x - 6$ for *y*.

$4x - 4x - 12 = -14$ Distributive Property

$-12 = -14$ ✗   Combine like terms.

▶ The equation $-12 = -14$ is never true. So, the system has no solution.

Solve the system of linear equations.

11. $x = y + 2$
$-3x + 3y = 6$

12. $3x - 6y = -9$
$-5x + 10y = 10$

13. $-4x + 4y = 32$
$3x + 24 = 3y$

Solve $3x - 1 = -2x + 4$ by graphing. Check your solution.

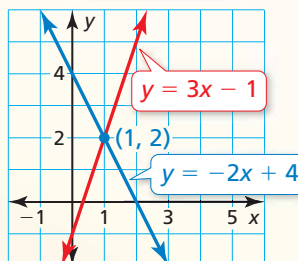**Step 1** Write a system of linear equations using each side of the original equation.

$$\boxed{y = 3x - 1} \rightarrow 3x - 1 = -2x + 4 \leftarrow \boxed{y = -2x + 4}$$

Step 2 Graph the system.

$y = 3x - 1$ Equation 1

$y = -2x + 4$ Equation 2

The graphs intersect at $(1, 2)$.

▶ So, the solution of the equation is $x = 1$.

Check

$$3x - 1 = -2x + 4$$
$$3(1) - 1 \overset{?}{=} -2(1) + 4$$
$$2 = 2 \checkmark$$

Solve the equation by graphing. Check your solution(s).

14. $\frac{1}{3}x + 5 = -2x - 2$ **15.** $|x + 1| = |-x - 9|$ **16.** $|2x - 8| = |x + 5|$
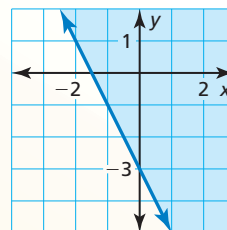
Graph $4x + 2y \geq -6$ in a coordinate plane.

Step 1 Graph $4x + 2y = -6$, or $y = -2x - 3$. Use a solid line because the inequality symbol is $\geq$.

Step 2 Test $(0, 0)$.

$$4x + 2y \geq -6 \qquad \text{Write the inequality.}$$
$$4(0) + 2(0) \overset{?}{\geq} -6 \qquad \text{Substitute.}$$
$$0 \geq -6 \checkmark \qquad \text{Simplify.}$$

Step 3 Because $(0, 0)$ is a solution, shade the half-plane that contains $(0, 0)$.

Graph the inequality in a coordinate plane.
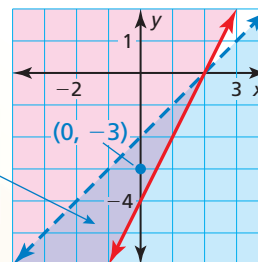
17. $y > -4$ **18.** $-9x + 3y \geq 3$ **19.** $5x + 10y < 40$

Graph the system. $y < x - 2$ Inequality 1

                     $y \geq 2x - 4$ Inequality 2

Step 1 Graph each inequality.

Step 2 Find the intersection of the half-planes. One solution is $(0, -3)$.

The solution is the purple-shaded region.

Graph the system of linear inequalities.

20. $y \leq x - 3$ **21.** $y > -2x + 3$ **22.** $x + 3y > 6$

     $y \geq x + 1$ $y \geq \frac{1}{4}x - 1$ $2x + y < 7$

5 Chapter Test

Solve the system of linear equations using any method. Explain why you chose the method.

1. $8x + 3y = -9$
$-8x + y = 29$

2. $\frac{1}{2}x + y = -6$
$y = \frac{3}{5}x + 5$

3. $y = 4x + 4$
$-8x + 2y = 8$

4. $x = y - 11$
$x - 3y = 1$

5. $6x - 4y = 9$
$9x - 6y = 15$

6. $y = 5x - 7$
$-4x + y = -1$

7. Write a system of linear inequalities so the points $(1, 2)$ and $(4, -3)$ are solutions of the system, but the point $(-2, 8)$ is not a solution of the system.

8. How is solving the equation $|2x + 1| = |x - 7|$ by graphing similar to solving the equation $4x + 3 = -2x + 9$ by graphing? How is it different?

Graph the system of linear inequalities.

9. $y > \frac{1}{2}x + 4$
$2y \le x + 4$

10. $x + y < 1$
$5x + y > 4$

11. $y \ge -\frac{2}{3}x + 1$
$-3x + y > -2$

12. You pay \$45.50 for 10 gallons of gasoline and 2 quarts of oil at a gas station. Your friend pays \$22.75 for 5 gallons of the same gasoline and 1 quart of the same oil.

   a. Is there enough information to determine the cost of 1 gallon of gasoline and 1 quart of oil? Explain.

   b. The receipt shown is for buying the same gasoline and same oil. Is there now enough information to determine the cost of 1 gallon of gasoline and 1 quart of oil? Explain.

   c. Determine the cost of 1 gallon of gasoline and 1 quart of oil.

```
        WELCOME

DATE 11/12/13        16:25
PUMP # 03
PRODUCT:           REGUNL
GALLONS:             8.00
2 QUARTS OIL
TOTAL:            $38.40

      THANK YOU
   HAVE A NICE DAY
```

13. Describe the advantages and disadvantages of solving a system of linear equations by graphing.

14. You have at most \$60 to spend on trophies and medals to give as prizes for a contest.

   a. Write and graph an inequality that represents the numbers of trophies and medals you can buy. Identify and interpret a solution of the inequality.

   b. You want to purchase at least 6 items. Write and graph a system that represents the situation. How many of each item can you buy?
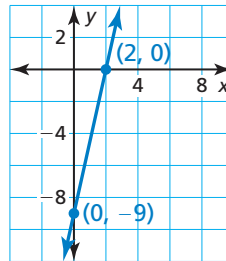
Medals
\$3 each

Trophies
\$12 each

15. Compare the slopes and y-intercepts of the graphs of the equations in the linear system $8x + 4y = 12$ and $3y = -6x - 15$ to determine whether the system has one solution, no solution, or infinitely many solutions. Explain.

1. The graph of which equation is shown?

   (A) $9x - 2y = -18$

   (B) $-9x - 2y = 18$

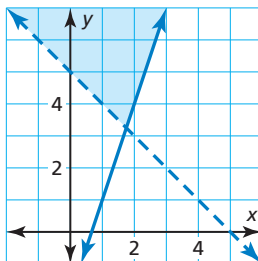   (C) $9x + 2y = 18$

   (D) $-9x + 2y = -18$

2. A van rental company rents out 6-, 8-, 12-, and 16-passenger vans. The function $C(x) = 100 + 5x$ represents the cost C (in dollars) of renting an x-passenger van for a day. Choose the numbers that are in the range of the function.

   | 130 | 140 | 150 | 160 | 170 | 180 | 190 | 200 |

3. Fill in the system of linear inequalities with $<$, $\leq$, $>$, or $\geq$ so that the graph represents the system.

   $y \;\boxed{}\; 3x - 2$

   $y \;\boxed{}\; -x + 5$

4. Your friend claims to be able to fill in each box with a constant so that when you set each side of the equation equal to y and graph the resulting equations, the lines will intersect exactly once. Do you support your friend's claim? Explain.

   $$4x + \boxed{} = 4x + \boxed{}$$

5. Select the phrases you should use when describing the transformations from the graph of f to the graph of g.

reflection in the x-axis	reflection in the y-axis
horizontal translation	vertical translation
horizontal stretch	vertical stretch
horizontal shrink	vertical shrink
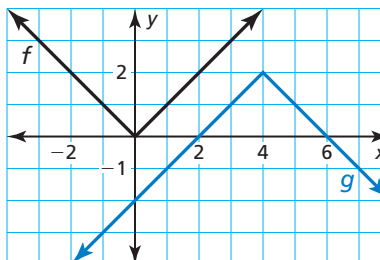
6. Which two equations form a system of linear equations that has no solution?

$y = 3x + 2$ $y = \frac{1}{3}x + 2$ $y = 2x + 3$ $y = 3x + \frac{1}{2}$

7. Fill in a value for a so that each statement is true for the equation $ax - 8 = 4 - x$.

  a. When $a =$ ____ , the solution is $x = -2$.

  b. When $a =$ ____ , the solution is $x = 12$.

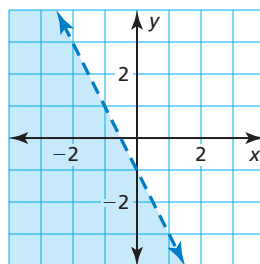 **c.** When $a =$ ____ , the solution is $x = 3$.

8. Which ordered pair is a solution of the linear inequality whose graph is shown?

  Ⓐ $(1, 1)$

  Ⓑ $(-1, 1)$

  Ⓒ $(-1, -1)$

  Ⓓ $(1, -1)$

9. Which of the systems of linear equations are equivalent?

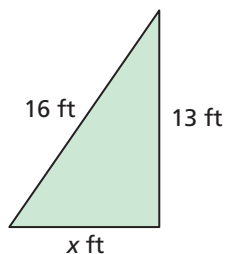| $4x - 5y = 3$ | $4x - 5y = 3$ | $4x - 5y = 3$ | $12x - 15y = 9$ |
| $2x + 15y = -1$ | $-4x - 30y = 2$ | $4x + 30y = -1$ | $2x + 15y = -1$ |

10. The value of x is more than 9. Which of the inequalities correctly describe the triangle? The perimeter (in feet) is represented by P, and the area (in square feet) is represented by A.

16 ft   13 ft

x ft

$P < 29$ $A > 117$ $P > 38$ $A > 58.5$ $A > 104$