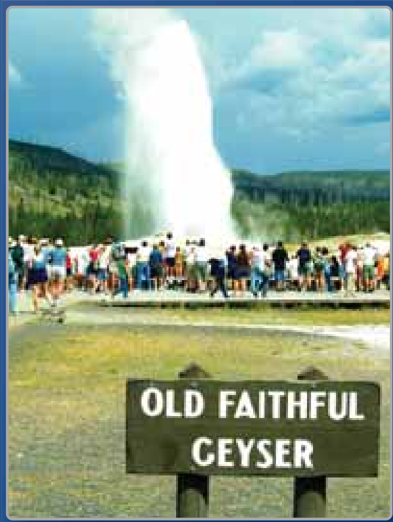
4 Writing Linear Functions

Karaoke Machine *(p. 220)*



Old Faithful Geyser *(p. 204)*

Helicopter Rescue *(p. 190)*

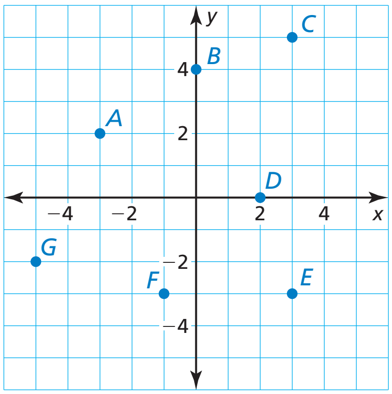
School Spirit *(p. 184)*

Renewable Energy *(p. 178)*

Maintaining Mathematical Proficiency

Using a Coordinate Plane

Example 1 **What ordered pair corresponds to point *A*?**



Point A is 3 units to the left of the origin and 2 units up. So, the x-coordinate is -3 and the y-coordinate is 2.

▶ The ordered pair $(-3, 2)$ corresponds to point A.

Use the graph to answer the question.

1. What ordered pair corresponds to point G?

2. What ordered pair corresponds to point D?

3. Which point is located in Quadrant I?

4. Which point is located in Quadrant IV?

Rewriting Equations

Example 2 **Solve the equation $3x - 2y = 8$ for y.**

$$3x - 2y = 8 \quad \text{Write the equation.}$$

$$3x - 2y - 3x = 8 - 3x \quad \text{Subtract } 3x \text{ from each side.}$$

$$-2y = 8 - 3x \quad \text{Simplify.}$$

$$\frac{-2y}{-2} = \frac{8 - 3x}{-2} \quad \text{Divide each side by } -2.$$

$$y = -4 + \frac{3}{2}x \quad \text{Simplify.}$$

Solve the equation for *y*.

5. $x - y = 5$

6. $6x + 3y = -1$

7. $0 = 2y - 8x + 10$

8. $-x + 4y - 28 = 0$

9. $2y + 1 - x = 7x$

10. $y - 4 = 3x + 5y$

11. **ABSTRACT REASONING** Both coordinates of the point (x, y) are multiplied by a negative number. How does this change the location of the point? Be sure to consider points originally located in all four quadrants.

Mathematical Practices

Mathematically proficient students try simpler forms of the original problem.

Problem-Solving Strategies

Core Concept

Solve a Simpler Problem

When solving a real-life problem, if the numbers in the problem seem complicated, then try solving a simpler form of the problem. After you have solved the simpler problem, look for a general strategy. Then apply that strategy to the original problem.

EXAMPLE 1 Using a Problem-Solving Strategy

In the deli section of a grocery store, a half pound of sliced roast beef costs \$3.19. You buy 1.81 pounds. How much do you pay?

SOLUTION

Step 1 Solve a simpler problem.

Suppose the roast beef costs \$3 per half pound, and you buy 2 pounds.

$$\text{Total cost} = \frac{\$3}{1/2 \text{ lb}} \cdot 2 \text{ lb}$$ Use unit analysis to write a verbal model.

$$= \frac{\$6}{1 \cancel{\text{lb}}} \cdot 2 \cancel{\text{lb}}$$ Rewrite \$3 per $\frac{1}{2}$ pound as \$6 per pound.

$$= \$12$$ Simplify.

► In the simpler problem, you pay \$12.

Step 2 Apply the strategy to the original problem.

$$\text{Total cost} = \frac{\$3.19}{1/2 \text{ lb}} \cdot 1.81 \text{ lb}$$ Use unit analysis to write a verbal model.

$$= \frac{\$6.38}{1 \cancel{\text{lb}}} \cdot 1.81 \cancel{\text{lb}}$$ Rewrite \$3.19 per $\frac{1}{2}$ pound as \$6.38 per pound.

$$= \$11.55$$ Simplify.

► In the original problem, you pay \$11.55.

Your answer is reasonable because you bought about 2 pounds.

Monitoring Progress

1. You work $37\frac{1}{2}$ hours and earn \$352.50. What is your hourly wage?
2. You drive 1244.5 miles and use 47.5 gallons of gasoline. What is your car's gas mileage (in miles per gallon)?
3. You drive 236 miles in 4.6 hours. At the same rate, how long will it take you to drive 450 miles?

4.1 Writing Equations in Slope-Intercept Form

Essential Question

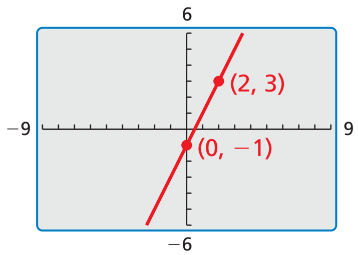
Given the graph of a linear function, how can you write an equation of the line?

EXPLORATION 1 Writing Equations in Slope-Intercept Form

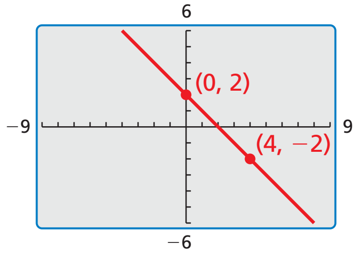
Work with a partner.

- Find the slope and y-intercept of each line.
- Write an equation of each line in slope-intercept form.
- Use a graphing calculator to verify your equation.

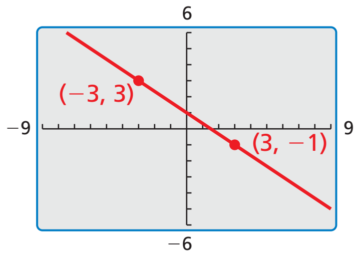
a.



b.



c.



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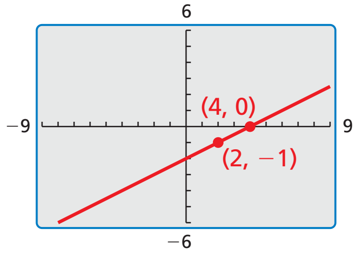
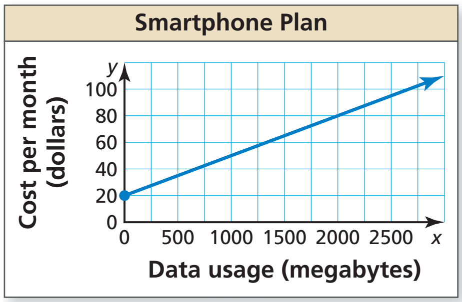


INTERPRETING MATHEMATICAL RESULTS

To be proficient in math, you need to routinely interpret your results in the context of the situation. The reason for studying mathematics is to enable you to model and solve real-life problems.

EXPLORATION 2 Mathematical Modeling

Work with a partner. The graph shows the cost of a smartphone plan.



a. What is the y-intercept of the line? Interpret the y-intercept in the context of the problem.

b. Approximate the slope of the line. Interpret the slope in the context of the problem.

c. Write an equation that represents the cost as a function of data usage.

Communicate Your Answer

3. Given the graph of a linear function, how can you write an equation of the line?

4. Give an example of a graph of a linear function that is different from those above. Then use the graph to write an equation of the line.

4.1 Lesson

Core Vocabulary

linear model, *p. 178*

Previous
slope-intercept form
function
rate

What You Will Learn

- Write equations in slope-intercept form.
- Use linear equations to solve real-life problems.

Writing Equations in Slope-Intercept Form

EXAMPLE 1 Using Slopes and y-Intercepts to Write Equations

Write an equation of each line with the given slope and y-intercept.

a. slope $= -3$; y-intercept $= \frac{1}{2}$

b. slope $= 0$; y-intercept $= -2$

SOLUTION

a. $y = mx + b$ — Write the slope-intercept form.

$y = -3x + \frac{1}{2}$ — Substitute -3 for m and $\frac{1}{2}$ for b.

▶ An equation is $y = -3x + \frac{1}{2}$.

b. $y = mx + b$ — Write the slope-intercept form.

$y = 0x + (-2)$ — Substitute 0 for m and -2 for b.

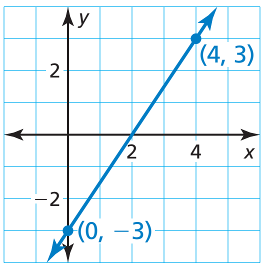
$y = -2$ — Simplify.

▶ An equation is $y = -2$.

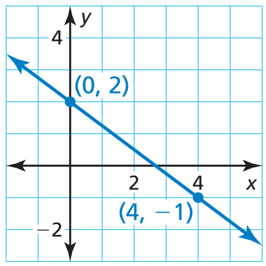
EXAMPLE 2 Using Graphs to Write Equations

Write an equation of each line in slope-intercept form.

a.



b.



SOLUTION

a. Find the slope and y-intercept.

Let $(x_1, y_1) = (0, -3)$ and $(x_2, y_2) = (4, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-3)}{4 - 0} = \frac{6}{4}, \text{ or } \frac{3}{2}$$

Because the line crosses the y-axis at $(0, -3)$, the y-intercept is -3.

▶ So, the equation is $y = \frac{3}{2}x - 3$.

b. Find the slope and y-intercept.

Let $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (4, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{4 - 0} = \frac{-3}{4}, \text{ or } -\frac{3}{4}$$

Because the line crosses the y-axis at $(0, 2)$, the y-intercept is 2.

▶ So, the equation is $y = -\frac{3}{4}x + 2$.

STUDY TIP
You can use any two points on a line to find the slope.

STUDY TIP
After writing an equation, check that the given points are solutions of the equation.

EXAMPLE 3 Using Points to Write Equations

Write an equation of each line that passes through the given points.

a. $(-3, 5), (0, -1)$

b. $(0, -5), (8, -5)$

SOLUTION

a. Find the slope and y-intercept.

$$m = \frac{-1 - 5}{0 - (-3)} = -2$$

Because the line crosses the y-axis at $(0, -1)$, the y-intercept is -1.

▶ So, an equation is $y = -2x - 1$.

b. Find the slope and y-intercept.

$$m = \frac{-5 - (-5)}{8 - 0} = 0$$

Because the line crosses the y-axis at $(0, -5)$, the y-intercept is -5.

▶ So, an equation is $y = -5$.

REMEMBER

If f is a function and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x.

EXAMPLE 4 Writing a Linear Function

Write a linear function f with the values $f(0) = 10$ and $f(6) = 34$.

SOLUTION

Step 1 Write $f(0) = 10$ as $(0, 10)$ and $f(6) = 34$ as $(6, 34)$.

Step 2 Find the slope of the line that passes through $(0, 10)$ and $(6, 34)$.

$$m = \frac{34 - 10}{6 - 0} = \frac{24}{6}, \text{ or } 4$$

Step 3 Write an equation of the line. Because the line crosses the y-axis at $(0, 10)$, the y-intercept is 10.

$y = mx + b$ — Write the slope-intercept form.

$y = 4x + 10$ — Substitute 4 for m and 10 for b.

▶ A function is $f(x) = 4x + 10$.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

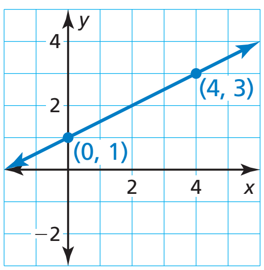
Write an equation of the line with the given slope and y-intercept.

1. slope = 7; y-intercept = 2

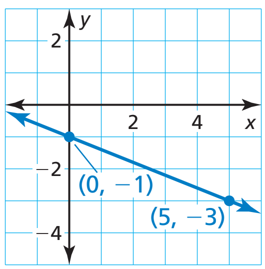
2. slope = $\frac{1}{3}$; y-intercept = -1

Write an equation of the line in slope-intercept form.

3.



4.



5. Write an equation of the line that passes through $(0, -2)$ and $(4, 10)$.

6. Write a linear function g with the values $g(0) = 9$ and $g(8) = 7$.

Solving Real-Life Problems

A **linear model** is a linear function that models a real-life situation. When a quantity y changes at a constant rate with respect to a quantity x, you can use the equation $y = mx + b$ to model the relationship. The value of m is the constant rate of change, and the value of b is the initial, or starting, value of y.

EXAMPLE 5 Modeling with Mathematics

Excluding hydropower, U.S. power plants used renewable energy sources to generate 105 million megawatt hours of electricity in 2007. By 2012, the amount of electricity generated had increased to 219 million megawatt hours. Write a linear model that represents the number of megawatt hours generated by non-hydropower renewable energy sources as a function of the number of years since 2007. Use the model to predict the number of megawatt hours that will be generated in 2017.

SOLUTION

1. **Understand the Problem** You know the amounts of electricity generated in two distinct years. You are asked to write a linear model that represents the amount of electricity generated each year since 2007 and then predict a future amount.
2. **Make a Plan** Break the problem into parts and solve each part. Then combine the results to help you solve the original problem.

   Part 1 Define the variables. Find the initial value and the rate of change.

   Part 2 Write a linear model and predict the amount in 2017.
3. **Solve the Problem**

   Part 1 Let x represent the time (in years) since 2007 and let y represent the number of megawatt hours (in millions). Because time x is defined in years since 2007, 2007 corresponds to $x = 0$ and 2012 corresponds to $x = 5$. Let $(x_1, y_1) = (0, 105)$ and $(x_2, y_2) = (5, 219)$. The initial value is the y-intercept b, which is 105. The rate of change is the slope m.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{219 - 105}{5 - 0} = \frac{114}{5} = 22.8$$

   Part 2

Megawatt hours (millions)	=	Initial value	+	Rate of change	•	Years since 2007
y	=	105	+	22.8	•	x

$y = 105 + 22.8x$ Write the equation.

2017 corresponds to $x = 10$. → $y = 105 + 22.8(10)$ Substitute 10 for x.

$y = 333$ Simplify.

▶ The linear model is $y = 22.8x + 105$. The model predicts non-hydropower renewable energy sources will generate 333 million megawatt hours in 2017.

4. **Look Back** To check that your model is correct, verify that (0, 105) and (5, 219) are solutions of the equation.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

7. The corresponding data for electricity generated by hydropower are 248 million megawatt hours in 2007 and 277 million megawatt hours in 2012. Write a linear model that represents the number of megawatt hours generated by hydropower as a function of the number of years since 2007.

4.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. COMPLETE THE SENTENCE A linear function that models a real-life situation is called a ___________.

2. WRITING Explain how you can use slope-intercept form to write an equation of a line given its slope and y-intercept.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write an equation of the line with the given slope and y-intercept. *(See Example 1.)*

3. slope: 2
y-intercept: 9

4. slope: 0
y-intercept: 5

5. slope: -3
y-intercept: 0

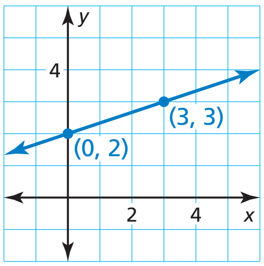
6. slope: -7
y-intercept: 1

7. slope: $\frac{2}{3}$
y-intercept: -8

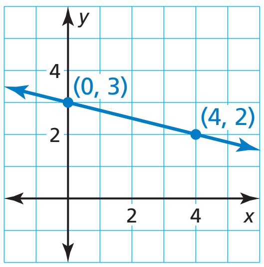
8. slope: $-\frac{3}{4}$
y-intercept: -6

In Exercises 9–12, write an equation of the line in slope-intercept form. *(See Example 2.)*

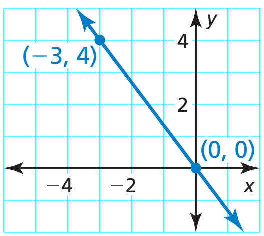
9.



10.



11.



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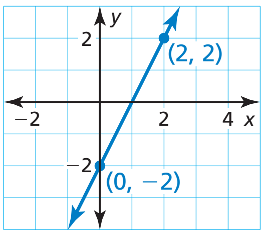


In Exercises 13–18, write an equation of the line that passes through the given points. *(See Example 3.)*

13. (3, 1), (0, 10)

14. (2, 7), (0, -5)

15. (2, -4), (0, -4)

16. (-6, 0), (0, -24)

17. (0, 5), (-1.5, 1)

18. (0, 3), (-5, 2.5)

In Exercises 19–24, write a linear function f with the given values. *(See Example 4.)*

19. $f(0) = 2, f(2) = 4$

20. $f(0) = 7, f(3) = 1$

21. $f(4) = -3, f(0) = -2$

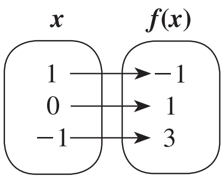
22. $f(5) = -1, f(0) = -5$

23. $f(-2) = 6, f(0) = -4$

24. $f(0) = 3, f(-6) = 3$

In Exercises 25 and 26, write a linear function f with the given values.

25.



26.

x	$f(x)$
-4	-2
-2	-1
0	0

27. ERROR ANALYSIS Describe and correct the error in writing an equation of the line with a slope of 2 and a y-intercept of 7.

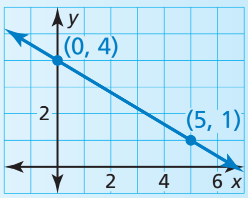
✗ $y = 7x + 2$

28. ERROR ANALYSIS Describe and correct the error in writing an equation of the line shown.

✗

$$\text{slope} = \frac{1-4}{0-5} = \frac{-3}{-5} = \frac{3}{5}$$

$$y = \frac{3}{5}x + 4$$



29. MODELING WITH MATHEMATICS In 1960, the world record for the men's mile was 3.91 minutes. In 1980, the record time was 3.81 minutes. *(See Example 5.)*

a. Write a linear model that represents the world record (in minutes) for the men's mile as a function of the number of years since 1960.

b. Use the model to estimate the record time in 2000 and predict the record time in 2020.

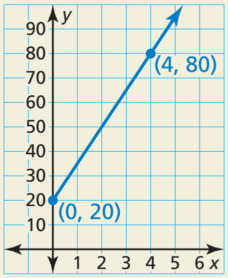
30. MODELING WITH MATHEMATICS A recording studio charges musicians an initial fee of \$50 to record an album. Studio time costs an additional \$75 per hour.

a. Write a linear model that represents the total cost of recording an album as a function of studio time (in hours).

b. Is it less expensive to purchase 12 hours of recording time at the studio or a \$750 music software program that you can use to record on your own computer? Explain.

31. WRITING A line passes through the points $(0, -2)$ and $(0, 5)$. Is it possible to write an equation of the line in slope-intercept form? Justify your answer.

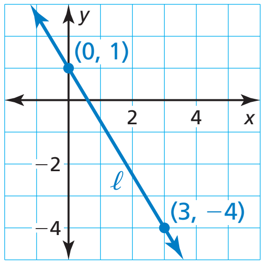
32. THOUGHT PROVOKING Describe a real-life situation involving a linear function whose graph passes through the points.



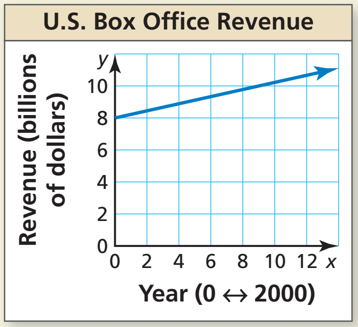
33. REASONING Recall that the standard form of a linear equation is $Ax + By = C$. Rewrite this equation in slope-intercept form. Use your answer to find the slope and y-intercept of the graph of the equation $-6x + 5y = 9$.

34. MAKING AN ARGUMENT Your friend claims that given $f(0)$ and any other value of a linear function f, you can write an equation in slope-intercept form that represents the function. Your cousin disagrees, claiming that the two points could lie on a vertical line. Who is correct? Explain.

35. ANALYZING A GRAPH Line ℓ is a reflection in the x-axis of line k. Write an equation that represents line k.



36. HOW DO YOU SEE IT? The graph shows the approximate U.S. box office revenues (in billions of dollars) from 2000 to 2012, where $x = 0$ represents the year 2000.



a. Estimate the slope and y-intercept of the graph.

b. Interpret your answers in part (a) in the context of the problem.

c. How can you use your answers in part (a) to predict the U.S. box office revenue in 2018?

37. ABSTRACT REASONING Show that the equation of the line that passes through the points $(0, b)$ and $(1, b + m)$ is $y = mx + b$. Explain how you can be sure that the point $(-1, b - m)$ also lies on the line.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. *(Section 1.3)*

38. $3(x - 15) = x + 11$

39. $-4y - 10 = 4(y - 3)$

40. $2(3d + 3) = 7 + 6d$

41. $-5(4 - 3n) = 10(n - 2)$

Use intercepts to graph the linear equation. *(Section 3.4)*

42. $-4x + 2y = 16$

43. $3x + 5y = -15$

44. $x - 6y = 24$

45. $-7x - 2y = -21$

4.2 Writing Equations in Point-Slope Form

Essential Question

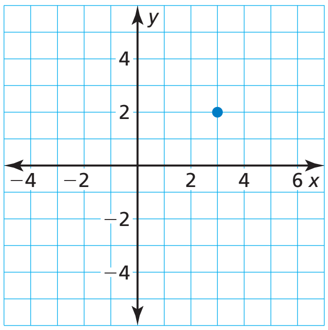
How can you write an equation of a line when you are given the slope and a point on the line?

EXPLORATION 1 Writing Equations of Lines

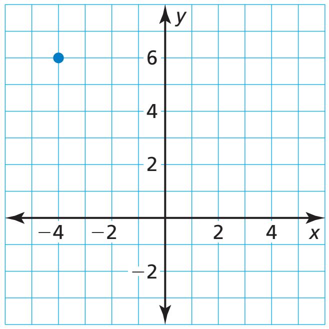
Work with a partner.

- Sketch the line that has the given slope and passes through the given point.
- Find the y-intercept of the line.
- Write an equation of the line.

a. $m = \frac{1}{2}$



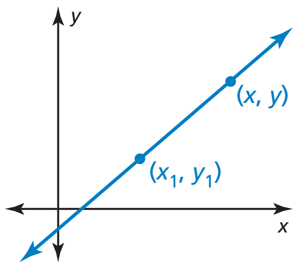
b. $m = -2$



EXPLORATION 2 Writing a Formula

Work with a partner.

The point (x_1, y_1) is a given point on a nonvertical line. The point (x, y) is any other point on the line. Write an equation that represents the slope m of the line. Then rewrite this equation by multiplying each side by the difference of the x-coordinates to obtain the **point-slope form** of a linear equation.



USING A GRAPHING CALCULATOR

To be proficient in math, you need to understand the feasibility, appropriateness, and limitations of the technological tools at your disposal. For instance, in real-life situations such as the one given in Exploration 3, it may not be feasible to use a square viewing window on a graphing calculator.

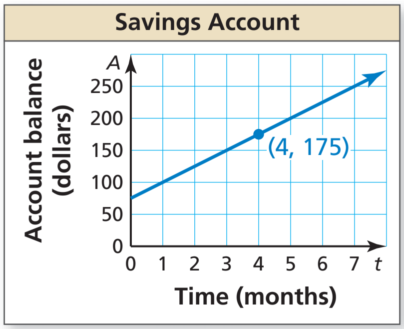
EXPLORATION 3 Writing an Equation

Work with a partner.

For four months, you have saved \$25 per month. You now have \$175 in your savings account.

a. Use your result from Exploration 2 to write an equation that represents the balance A after t months.

b. Use a graphing calculator to verify your equation.



Communicate Your Answer

4. How can you write an equation of a line when you are given the slope and a point on the line?

5. Give an example of how to write an equation of a line when you are given the slope and a point on the line. Your example should be different from those above.

4.2 Lesson

Core Vocabulary

point-slope form, *p. 182*

Previous

slope-intercept form
function
linear model
rate

What You Will Learn

- Write an equation of a line given its slope and a point on the line.
- Write an equation of a line given two points on the line.
- Use linear equations to solve real-life problems.

Writing Equations of Lines in Point-Slope Form

Given a point on a line and the slope of the line, you can write an equation of the line. Consider the line that passes through (2, 3) and has a slope of $\frac{1}{2}$. Let (x, y) be another point on the line where $x \neq 2$. You can write an equation relating x and y using the slope formula with $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (x, y)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Write the slope formula.

$$\frac{1}{2} = \frac{y - 3}{x - 2}$$ Substitute values.

$$\frac{1}{2}(x - 2) = y - 3$$ Multiply each side by $(x - 2)$.

The equation in *point-slope form* is $y - 3 = \frac{1}{2}(x - 2)$.

Core Concept

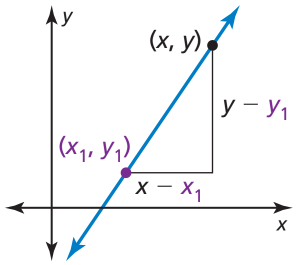
Point-Slope Form

Words A linear equation written in the form $y - y_1 = m(x - x_1)$ is in **point-slope form**. The line passes through the point (x_1, y_1), and the slope of the line is m.

Algebra $y - y_1 = m(x - x_1)$

slope (m)

passes through (x_1, y_1)



EXAMPLE 1 Using a Slope and a Point to Write an Equation

Write an equation in point-slope form of the line that passes through the point (−8, 3) and has a slope of $\frac{1}{4}$.

SOLUTION

$$y - y_1 = m(x - x_1)$$ Write the point-slope form.

$$y - 3 = \frac{1}{4}[x - (-8)]$$ Substitute $\frac{1}{4}$ for m, −8 for x_1, and 3 for y_1.

$$y - 3 = \frac{1}{4}(x + 8)$$ Simplify.

▶ The equation is $y - 3 = \frac{1}{4}(x + 8)$.

Check

$$y - 3 = \frac{1}{4}(x + 8)$$

$$3 - 3 \stackrel{?}{=} \frac{1}{4}(-8 + 8)$$

$$0 = 0 \checkmark$$

Monitoring Progress

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Write an equation in point-slope form of the line that passes through the given point and has the given slope.

1. $(3, -1);\ m = -2$

2. $(4, 0);\ m = -\frac{2}{3}$

Writing Equations of Lines Given Two Points

When you are given two points on a line, you can write an equation of the line using the following steps.

Step 1 Find the slope of the line.

Step 2 Use the slope and one of the points to write an equation of the line in point-slope form.

ANOTHER WAY

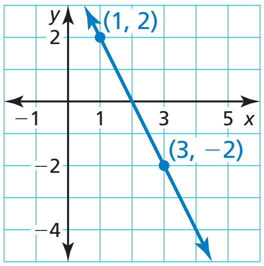
You can use either of the given points to write an equation of the line.

Use $m = -2$ and $(3, -2)$.

$$y - (-2) = -2(x - 3)$$
$$y + 2 = -2x + 6$$
$$y = -2x + 4$$

EXAMPLE 2 Using Two Points to Write an Equation

Write an equation in slope-intercept form of the line shown.



SOLUTION

Step 1 Find the slope of the line.

$$m = \frac{-2 - 2}{3 - 1} = \frac{-4}{2}, \text{ or } -2$$

Step 2 Use the slope $m = -2$ and the point $(1, 2)$ to write an equation of the line.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 2 = -2(x - 1)$	Substitute −2 for m, 1 for x_1, and 2 for y_1.
$y - 2 = -2x + 2$	Distributive Property
$y = -2x + 4$	Write in slope-intercept form.

▶ The equation is $y = -2x + 4$.

EXAMPLE 3 Writing a Linear Function

Write a linear function f with the values $f(4) = -2$ and $f(8) = 4$.

SOLUTION

Note that you can rewrite $f(4) = -2$ as $(4, -2)$ and $f(8) = 4$ as $(8, 4)$.

Step 1 Find the slope of the line that passes through $(4, -2)$ and $(8, 4)$.

$$m = \frac{4 - (-2)}{8 - 4} = \frac{6}{4}, \text{ or } 1.5$$

Step 2 Use the slope $m = 1.5$ and the point $(8, 4)$ to write an equation of the line.

$y - y_1 = m(x - x_1)$	Write the point-slope form.
$y - 4 = 1.5(x - 8)$	Substitute 1.5 for m, 8 for x_1, and 4 for y_1.
$y - 4 = 1.5x - 12$	Distributive Property
$y = 1.5x - 8$	Write in slope-intercept form.

▶ A function is $f(x) = 1.5x - 8$.

Monitoring Progress

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Write an equation in slope-intercept form of the line that passes through the given points.

3. (1, 4), (3, 10)

4. (−4, −1), (8, −4)

5. Write a linear function g with the values $g(2) = 3$ and $g(6) = 5$.

Solving Real-Life Problems

EXAMPLE 4 **Modeling with Mathematics**

The student council is ordering customized foam hands to promote school spirit. The table shows the cost of ordering different numbers of foam hands. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the cost as a function of the number of foam hands.

Number of foam hands	4	6	8	10	12
Cost (dollars)	34	46	58	70	82

SOLUTION

1. **Understand the Problem** You know five data pairs from the table. You are asked whether the data are linear. If so, write a linear model that represents the cost.
2. **Make a Plan** Find the rate of change for consecutive data pairs in the table. If the rate of change is constant, use the point-slope form to write an equation. Rewrite the equation in slope-intercept form so that the cost is a function of the number of foam hands.
3. **Solve the Problem**

Step 1 Find the rate of change for consecutive data pairs in the table.

$$\frac{46-34}{6-4}=6,\ \frac{58-46}{8-6}=6,\ \frac{70-58}{10-8}=6,\ \frac{82-70}{12-10}=6$$

Because the rate of change is constant, the data are linear. So, use the point-slope form to write an equation that represents the data.

Step 2 Use the constant rate of change (slope) $m = 6$ and the data pair (4, 34) to write an equation. Let C be the cost (in dollars) and n be the number of foam hands.

$C - C_1 = m(n - n_1)$ Write the point-slope form.

$C - 34 = 6(n - 4)$ Substitute 6 for m, 4 for n_1, and 34 for C_1.

$C - 34 = 6n - 24$ Distributive Property

$C = 6n + 10$ Write in slope-intercept form.

▶ Because the cost increases at a constant rate, the situation can be modeled by a linear equation. The linear model is $C = 6n + 10$.

4. **Look Back** To check that your model is correct, verify that the other data pairs are solutions of the equation.

$46 = 6(6) + 10$ ✓ $58 = 6(8) + 10$ ✓

$70 = 6(10) + 10$ ✓ $82 = 6(12) + 10$ ✓

Monitoring Progress

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Number of months	Total cost (dollars)
3	176
6	302
9	428
12	554

6. You pay an installation fee and a monthly fee for Internet service. The table shows the total cost for different numbers of months. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the total cost as a function of the number of months.

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Vocabulary and Core Concept Check

1. **USING STRUCTURE** Without simplifying, identify the slope of the line given by the equation $y - 5 = -2(x + 5)$. Then identify one point on the line.

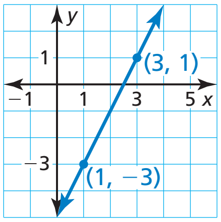
2. **WRITING** Explain how you can use the slope formula to write an equation of the line that passes through $(3, -2)$ and has a slope of 4.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, write an equation in point-slope form of the line that passes through the given point and has the given slope. *(See Example 1.)*

3. $(2, 1)$; $m = 2$
4. $(3, 5)$; $m = -1$
5. $(7, -4)$; $m = -6$
6. $(-8, -2)$; $m = 5$
7. $(9, 0)$; $m = -3$
8. $(0, 2)$; $m = 4$
9. $(-6, 6)$; $m = \frac{3}{2}$
10. $(5, -12)$; $m = -\frac{2}{5}$

In Exercises 11–14, write an equation in slope-intercept form of the line shown. *(See Example 2.)*

11.


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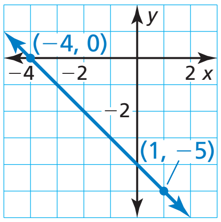

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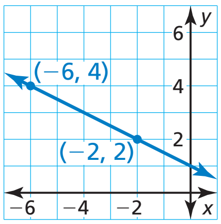
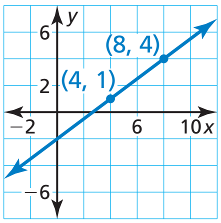


14.


In Exercises 15–20, write an equation in slope-intercept form of the line that passes through the given points.

15. $(7, 2), (2, 12)$
16. $(6, -2), (12, 1)$
17. $(6, -1), (3, -7)$
18. $(-2, 5), (-4, -5)$
19. $(1, -9), (-3, -9)$
20. $(-5, 19), (5, 13)$

In Exercises 21–26, write a linear function *f* with the given values. *(See Example 3.)*

21. $f(2) = -2, f(1) = 1$
22. $f(5) = 7, f(-2) = 0$
23. $f(-4) = 2, f(6) = -3$
24. $f(-10) = 4, f(-2) = 4$
25. $f(-3) = 1, f(13) = 5$
26. $f(-9) = 10, f(-1) = -2$

In Exercises 27–30, tell whether the data in the table can be modeled by a linear equation. Explain. If possible, write a linear equation that represents *y* as a function of *x*. *(See Example 4.)*

27.

x	2	4	6	8	10
y	−1	5	15	29	47

28.

x	−3	−1	1	3	5
y	16	10	4	−2	−8

29.

x	*y*
0	1.2
1	1.4
2	1.6
4	2

30.

x	*y*
1	18
2	15
4	12
8	9

31. **ERROR ANALYSIS** Describe and correct the error in writing a linear function g with the values $g(5) = 4$ and $g(3) = 10$.

✗

$$m = \frac{10 - 4}{3 - 5} = \frac{6}{-2} = -3$$

$$y - y_1 = mx - x_1$$
$$y - 4 = -3x - 5$$
$$y = -3x - 1$$

A function is $g(x) = -3x - 1$.

32. ERROR ANALYSIS Describe and correct the error in writing an equation of the line that passes through the points (1, 2) and (4, 3).

$$m = \frac{3-2}{4-1} = \frac{1}{3} \qquad y - 2 = \frac{1}{3}(x - 4)$$

33. MODELING WITH MATHEMATICS You are designing a sticker to advertise your band. A company charges \$225 for the first 1000 stickers and \$80 for each additional 1000 stickers.

a. Write an equation that represents the total cost (in dollars) of the stickers as a function of the number (in thousands) of stickers ordered.

b. Find the total cost of 9000 stickers.

34. MODELING WITH MATHEMATICS You pay a processing fee and a daily fee to rent a beach house. The table shows the total cost of renting the beach house for different numbers of days.

Days	2	4	6	8
Total cost (dollars)	246	450	654	858

a. Can the situation be modeled by a linear equation? Explain.

b. What is the processing fee? the daily fee?

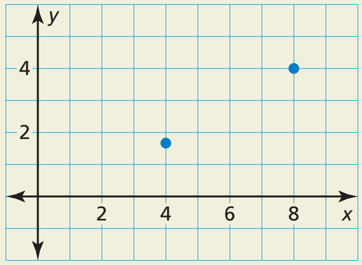
c. You can spend no more than \$1200 on the beach house rental. What is the maximum number of days you can rent the beach house?

35. WRITING Describe two ways to graph the equation $y - 1 = \frac{3}{2}(x - 4)$.

36. THOUGHT PROVOKING The graph of a linear function passes through the point $(12, -5)$ and has a slope of $\frac{2}{5}$. Represent this function in two other ways.

37. REASONING You are writing an equation of the line that passes through two points that are not on the y-axis. Would you use slope-intercept form or point-slope form to write the equation? Explain.

38. HOW DO YOU SEE IT? The graph shows two points that lie on the graph of a linear function.

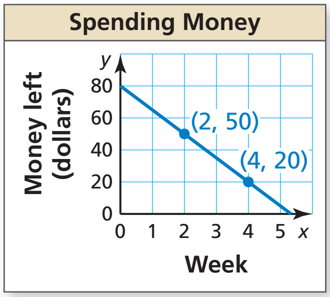


a. Does the y-intercept of the graph of the linear function appear to be positive or negative? Explain.

b. Estimate the coordinates of the two points. How can you use your estimates to confirm your answer in part (a)?

39. CONNECTION TO TRANSFORMATIONS Compare the graph of $y = 2x$ to the graph of $y - 1 = 2(x + 3)$. Make a conjecture about the graphs of $y = mx$ and $y - k = m(x - h)$.

40. COMPARING FUNCTIONS Three siblings each receive money for a holiday and then spend it at a constant weekly rate. The graph describes Sibling A's spending, the table describes Sibling B's spending, and the equation $y = -22.5x + 90$ describes Sibling C's spending. The variable y represents the amount of money left after x weeks.



Week, x	Money left, y
1	\$100
2	\$75
3	\$50
4	\$25

a. Which sibling received the most money? the least money?

b. Which sibling spends money at the fastest rate? the slowest rate?

c. Which sibling runs out of money first? last?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the reciprocal of the number. *(Skills Review Handbook)*

41. 5

42. -8

43. $-\frac{2}{7}$

44. $\frac{3}{2}$

4.3 Writing Equations of Parallel and Perpendicular Lines

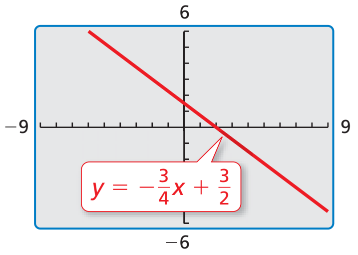
Essential Question

How can you recognize lines that are parallel or perpendicular?

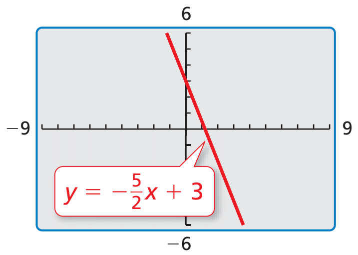
EXPLORATION 1 Recognizing Parallel Lines

Work with a partner. Write each linear equation in slope-intercept form. Then use a graphing calculator to graph the three equations in the same square viewing window. (The graph of the first equation is shown.) Which two lines appear parallel? How can you tell?

a. $3x + 4y = 6$
$3x + 4y = 12$
$4x + 3y = 12$



b. $5x + 2y = 6$
$2x + y = 3$
$2.5x + y = 5$



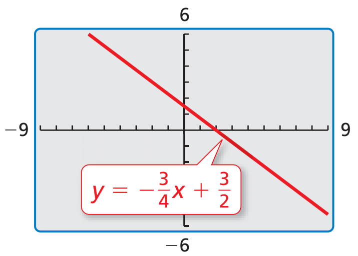
USING TOOLS STRATEGICALLY

To be proficient in math, you need to use a graphing calculator and other available technological tools, as appropriate, to help you explore relationships and deepen your understanding of concepts.

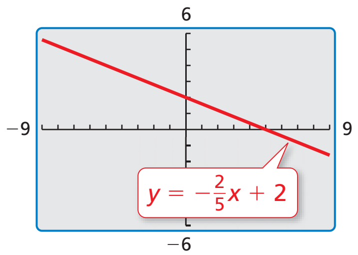
EXPLORATION 2 Recognizing Perpendicular Lines

Work with a partner. Write each linear equation in slope-intercept form. Then use a graphing calculator to graph the three equations in the same square viewing window. (The graph of the first equation is shown.) Which two lines appear perpendicular? How can you tell?

a. $3x + 4y = 6$
$3x - 4y = 12$
$4x - 3y = 12$



b. $2x + 5y = 10$
$-2x + y = 3$
$2.5x - y = 5$



Communicate Your Answer

3. How can you recognize lines that are parallel or perpendicular?

4. Compare the slopes of the lines in Exploration 1. How can you use slope to determine whether two lines are parallel? Explain your reasoning.

5. Compare the slopes of the lines in Exploration 2. How can you use slope to determine whether two lines are perpendicular? Explain your reasoning.

4.3 Lesson

What You Will Learn

- Identify and write equations of parallel lines.
- Identify and write equations of perpendicular lines.
- Use parallel and perpendicular lines in real-life problems.

Core Vocabulary

parallel lines, *p. 188*
perpendicular lines, *p. 189*

Previous
reciprocal

Identifying and Writing Equations of Parallel Lines

Core Concept

Parallel Lines and Slopes

Two lines in the same plane that never intersect are **parallel lines**. Two distinct nonvertical lines are parallel if and only if they have the same slope.

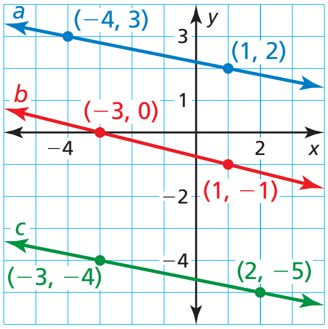
All vertical lines are parallel.

READING

The phrase "*A if and only if B*" is a way of writing two conditional statements at once. It means that if *A* is true, then *B* is true. It also means that if *B* is true, then *A* is true.

EXAMPLE 1 Identifying Parallel Lines

Determine which of the lines are parallel.



SOLUTION

Find the slope of each line.

Line *a*: $m = \frac{2 - 3}{1 - (-4)} = -\frac{1}{5}$

Line *b*: $m = \frac{-1 - 0}{1 - (-3)} = -\frac{1}{4}$

Line *c*: $m = \frac{-5 - (-4)}{2 - (-3)} = -\frac{1}{5}$

▶ Lines *a* and *c* have the same slope, so they are parallel.

EXAMPLE 2 Writing an Equation of a Parallel Line

Write an equation of the line that passes through (5, −4) and is parallel to the line $y = 2x + 3$.

SOLUTION

Step 1 Find the slope of the parallel line. The graph of the given equation has a slope of 2. So, the parallel line that passes through (5, −4) also has a slope of 2.

Step 2 Use the slope-intercept form to find the *y*-intercept of the parallel line.

$y = mx + b$ Write the slope-intercept form.

$-4 = 2(5) + b$ Substitute 2 for *m*, 5 for *x*, and −4 for *y*.

$-14 = b$ Solve for *b*.

▶ Using $m = 2$ and $b = -14$, an equation of the parallel line is $y = 2x - 14$.

ANOTHER WAY

You can also use the slope $m = 2$ and the point-slope form to write an equation of the line that passes through (5, −4).

$$y - y_1 = m(x - x_1)$$
$$y - (-4) = 2(x - 5)$$
$$y = 2x - 14$$

Monitoring Progress

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1. Line *a* passes through (−5, 3) and (−6, −1). Line *b* passes through (3, −2) and (2, −7). Are the lines parallel? Explain.
2. Write an equation of the line that passes through (−4, 2) and is parallel to the line $y = \frac{1}{4}x + 1$.

Identifying and Writing Equations of Perpendicular Lines

REMEMBER

The product of a nonzero number m and its negative reciprocal is -1:

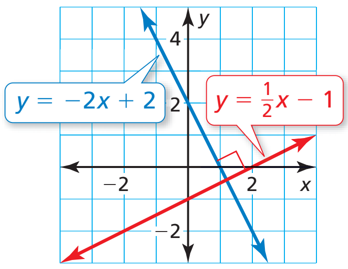
$$m\left(-\frac{1}{m}\right) = -1.$$

Core Concept

Perpendicular Lines and Slopes

Two lines in the same plane that intersect to form right angles are **perpendicular lines**. Nonvertical lines are perpendicular if and only if their slopes are negative reciprocals.

Vertical lines are perpendicular to horizontal lines.



EXAMPLE 3 Identifying Parallel and Perpendicular Lines

Determine which of the lines, if any, are parallel or perpendicular.

Line a: $y = 4x + 2$ Line b: $x + 4y = 3$ Line c: $-8y - 2x = 16$

SOLUTION

Write the equations in slope-intercept form. Then compare the slopes.

Line a: $y = 4x + 2$ Line b: $y = -\frac{1}{4}x + \frac{3}{4}$ Line c: $y = -\frac{1}{4}x - 2$

▶ Lines b and c have slopes of $-\frac{1}{4}$, so they are parallel. Line a has a slope of 4, the negative reciprocal of $-\frac{1}{4}$, so it is perpendicular to lines b and c.

EXAMPLE 4 Writing an Equation of a Perpendicular Line

Write an equation of the line that passes through $(-3, 1)$ and is perpendicular to the line $y = \frac{1}{2}x + 3$.

SOLUTION

Step 1 Find the slope of the perpendicular line. The graph of the given equation has a slope of $\frac{1}{2}$. Because the slopes of perpendicular lines are negative reciprocals, the slope of the perpendicular line that passes through $(-3, 1)$ is -2.

Step 2 Use the slope $m = -2$ and the point-slope form to write an equation of the perpendicular line that passes through $(-3, 1)$.

$y - y_1 = m(x - x_1)$ Write the point-slope form.

$y - 1 = -2[x - (-3)]$ Substitute -2 for m, -3 for x_1, and 1 for y_1.

$y - 1 = -2x - 6$ Simplify.

$y = -2x - 5$ Write in slope-intercept form.

▶ An equation of the perpendicular line is $y = -2x - 5$.

ANOTHER WAY

You can also use the slope $m = -2$ and the slope-intercept form to write an equation of the line that passes through $(-3, 1)$.

$y = mx + b$

$1 = -2(-3) + b$

$-5 = b$

So, $y = -2x - 5$.

Monitoring Progress

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3. Determine which of the lines, if any, are parallel or perpendicular. Explain.

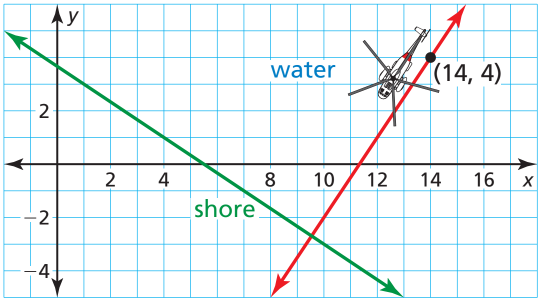
   Line a: $2x + 6y = -3$ Line b: $y = 3x - 8$ Line c: $-6y + 18x = 9$

4. Write an equation of the line that passes through $(-3, 5)$ and is perpendicular to the line $y = -3x - 1$.

Writing Equations for Real-Life Problems

EXAMPLE 5 **Writing an Equation of a Perpendicular Line**

The position of a helicopter search and rescue crew is shown in the graph. The shortest flight path to the shoreline is one that is perpendicular to the shoreline. Write an equation that represents this path.



SOLUTION

1. **Understand the Problem** You can see the line that represents the shoreline. You know the coordinates of the helicopter. You are asked to write an equation that represents the shortest flight path to the shoreline.

2. **Make a Plan** Find the slope of the line that represents the shoreline. Use the negative reciprocal of this slope, the coordinates of the helicopter, and the point-slope form to write an equation.

3. **Solve the Problem**

Step 1 Find the slope of the line that represents the shoreline. The line passes through points (1, 3) and (4, 1). So, the slope is

$$m = \frac{1 - 3}{4 - 1} = -\frac{2}{3}.$$

Because the shoreline and shortest flight path are perpendicular, the slopes of their respective graphs are negative reciprocals. So, the slope of the graph of the shortest flight path is $\frac{3}{2}$.

Step 2 Use the slope $m = \frac{3}{2}$ and the point-slope form to write an equation of the shortest flight path that passes through (14, 4).

$y - y_1 = m(x - x_1)$ — Write the point-slope form.

$y - 4 = \frac{3}{2}(x - 14)$ — Substitute $\frac{3}{2}$ for m, 14 for x_1, and 4 for y_1.

$y - 4 = \frac{3}{2}x - 21$ — Distributive Property

$y = \frac{3}{2}x - 17$ — Write in slope-intercept form.

▶ An equation that represents the shortest flight path is $y = \frac{3}{2}x - 17$.

4. **Look Back** To check that your equation is correct, verify that (14, 4) is a solution of the equation.

$4 = \frac{3}{2}(14) - 17$ ✓

Monitoring Progress

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5. In Example 5, a boat is traveling parallel to the shoreline and passes through (9, 3). Write an equation that represents the path of the boat.

4.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

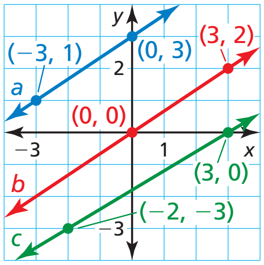
Vocabulary and Core Concept Check

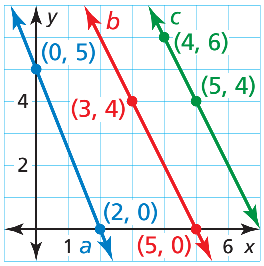
1. **COMPLETE THE SENTENCE** Two distinct nonvertical lines that have the same slope are ________.

2. **VOCABULARY** Two lines are perpendicular. The slope of one line is $-\frac{5}{7}$. What is the slope of the other line? Justify your answer.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, determine which of the lines, if any, are parallel. Explain. *(See Example 1.)*

3. 

4. 

5. Line a passes through $(-1, -2)$ and $(1, 0)$.
Line b passes through $(4, 2)$ and $(2, -2)$.
Line c passes through $(0, 2)$ and $(-1, 1)$.

6. Line a passes through $(-1, 3)$ and $(1, 9)$.
Line b passes through $(-2, 12)$ and $(-1, 14)$.
Line c passes through $(3, 8)$ and $(6, 10)$.

7. Line a: $4y + x = 8$
Line b: $2y + x = 4$
Line c: $2y = -3x + 6$

8. Line a: $3y - x = 6$
Line b: $3y = x + 18$
Line c: $3y - 2x = 9$

In Exercises 9–12, write an equation of the line that passes through the given point and is parallel to the given line. *(See Example 2.)*

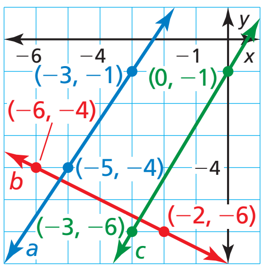
9. $(-1, 3)$; $y = 2x + 2$

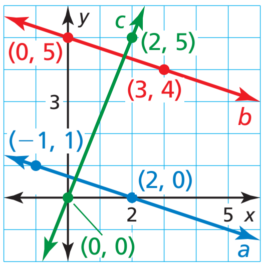
10. $(1, 2)$; $y = -5x + 4$

11. $(18, 2)$; $3y - x = -12$

12. $(2, -5)$; $2y = 3x + 10$

In Exercises 13–18, determine which of the lines, if any, are parallel or perpendicular. Explain. *(See Example 3.)*

13. 

14. 

15. Line a passes through $(-2, 1)$ and $(0, 3)$.
Line b passes through $(4, 1)$ and $(6, 4)$.
Line c passes through $(1, 3)$ and $(4, 1)$.

16. Line a passes through $(2, 10)$ and $(4, 13)$.
Line b passes through $(4, 9)$ and $(6, 12)$.
Line c passes through $(2, 10)$ and $(4, 9)$.

17. Line a: $4x - 3y = 2$
Line b: $y = \frac{4}{3}x + 2$
Line c: $4y + 3x = 4$

18. Line a: $y = 6x - 2$
Line b: $6y = -x$
Line c: $y + 6x = 1$

In Exercises 19–22, write an equation of the line that passes through the given point and is perpendicular to the given line. *(See Example 4.)*

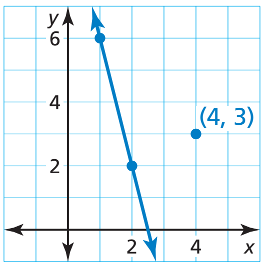
19. $(7, 10)$; $y = \frac{1}{2}x - 9$

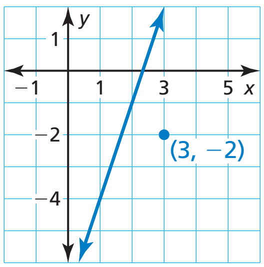
20. $(-4, -1)$; $y = \frac{4}{3}x + 6$

21. $(-3, 3)$; $2y = 8x - 6$

22. $(8, 1)$; $2y + 4x = 12$

In Exercises 23 and 24, write an equation of the line that passes through the given point and is (a) parallel and (b) perpendicular to the given line.

23. 

24. 

25. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the line that passes through $(1, 3)$ and is parallel to the line $y = \frac{1}{4}x + 2$.

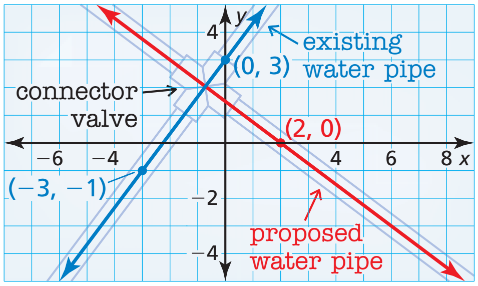
✗

$$y - y_1 = m(x - x_1)$$
$$y - 3 = -4(x - 1)$$
$$y - 3 = -4x + 4$$
$$y = -4x + 7$$

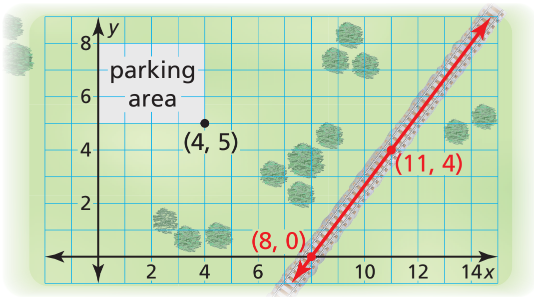
26. ERROR ANALYSIS Describe and correct the error in writing an equation of the line that passes through (4, −5) and is perpendicular to the line $y = \frac{1}{3}x + 5$.

✗
$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 3(x - 4)$$
$$y + 5 = 3x - 12$$
$$y = 3x - 17$$

27. MODELING WITH MATHEMATICS A city water department is proposing the construction of a new water pipe, as shown. The new pipe will be perpendicular to the old pipe. Write an equation that represents the new pipe. *(See Example 5.)*



28. MODELING WITH MATHEMATICS A parks and recreation department is constructing a new bike path. The path will be parallel to the railroad tracks shown and pass through the parking area at the point (4, 5). Write an equation that represents the path.



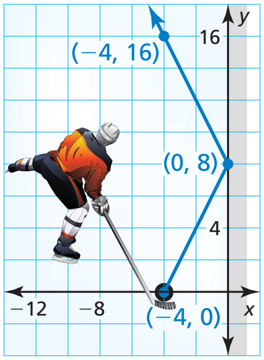
29. MATHEMATICAL CONNECTIONS The vertices of a quadrilateral are $A(2, 2)$, $B(6, 4)$, $C(8, 10)$, and $D(4, 8)$.

a. Is quadrilateral *ABCD* a parallelogram? Explain.

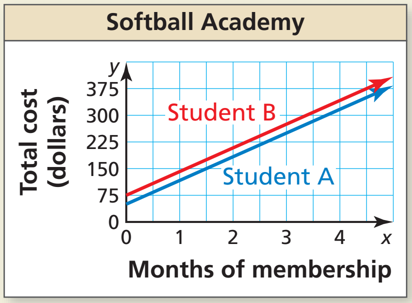
b. Is quadrilateral *ABCD* a rectangle? Explain.

30. USING STRUCTURE For what value of a are the graphs of $6y = -2x + 4$ and $2y = ax - 5$ parallel? perpendicular?

31. MAKING AN ARGUMENT A hockey puck leaves the blade of a hockey stick, bounces off a wall, and travels in a new direction, as shown. Your friend claims the path of the puck forms a right angle. Is your friend correct? Explain.



32. HOW DO YOU SEE IT? A softball academy charges students an initial registration fee plus a monthly fee. The graph shows the total amounts paid by two students over a 4-month period. The lines are parallel.



a. Did one of the students pay a greater registration fee? Explain.

b. Did one of the students pay a greater monthly fee? Explain.

REASONING In Exercises 33–35, determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

33. Two lines with positive slopes are perpendicular.

34. A vertical line is parallel to the y-axis.

35. Two lines with the same y-intercept are perpendicular.

36. THOUGHT PROVOKING You are designing a new logo for your math club. Your teacher asks you to include at least one pair of parallel lines and at least one pair of perpendicular lines. Sketch your logo in a coordinate plane. Write the equations of the parallel and perpendicular lines.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the relation is a function. Explain. *(Section 3.1)*

37. (3, 6), (4, 8), (5, 10), (6, 9), (7, 14)

38. (−1, 6), (1, 4), (−1, 2), (1, 6), (−1, 5)

4.1–4.3 What Did You Learn?

Core Vocabulary

linear model, *p. 178*
point-slope form, *p. 182*
parallel lines, *p. 188*
perpendicular lines, *p. 189*

Core Concepts

Section 4.1

Using Slope-Intercept Form, *p. 176*

Section 4.2

Using Point-Slope Form, *p. 182*

Section 4.3

Parallel Lines and Slopes, *p. 188*
Perpendicular Lines and Slopes, *p. 189*

Mathematical Practices

1. How can you explain to yourself the meaning of the graph in Exercise 36 on page 180?

2. How did you use the structure of the equations in Exercise 39 on page 186 to make a conjecture?

3. How did you use the diagram in Exercise 31 on page 192 to determine whether your friend was correct?

Study Skills

Getting Actively Involved in Class

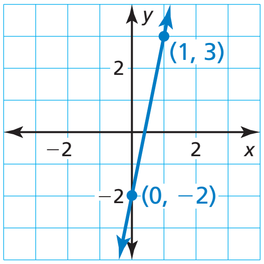
If you do not understand something at all and do not even know how to phrase a question, just ask for clarification. You might say something like, "Could you please explain the steps in this problem one more time?"

If your teacher asks for someone to go up to the board, volunteer. The student at the board often receives additional attention and instruction to complete the problem.

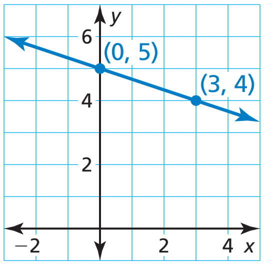
4.1–4.3 Quiz

Write an equation of the line in slope-intercept form. *(Section 4.1)*

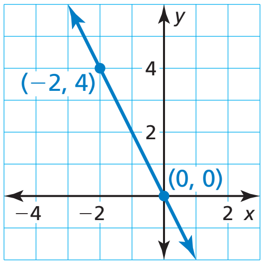
1.



2.



3.



Write an equation in point-slope form of the line that passes through the given points. *(Section 4.2)*

4. $(-2, 5), (1, -1)$

5. $(-3, -2), (2, -1)$

6. $(1, 0), (4, 4)$

Write a linear function *f* with the given values. *(Section 4.1 and Section 4.2)*

7. $f(0) = 2, f(5) = -3$

8. $f(-1) = -6, f(4) = -6$

9. $f(-3) = -2, f(-2) = 3$

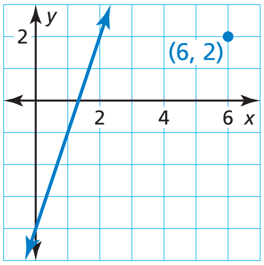
Determine which of the lines, if any, are parallel or perpendicular. Explain. *(Section 4.3)*

10. Line a passes through $(-2, 2)$ and $(2, 1)$.
Line b passes through $(1, -8)$ and $(3, 0)$.
Line c passes through $(-4, -3)$ and $(0, -2)$.

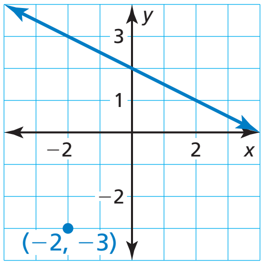
11. Line a: $2x + 6y = -12$
Line b: $y = \frac{3}{2}x - 5$
Line c: $3x - 2y = -4$

Write an equation of the line that passes through the given point and is (a) parallel and (b) perpendicular to the given line. *(Section 4.3)*

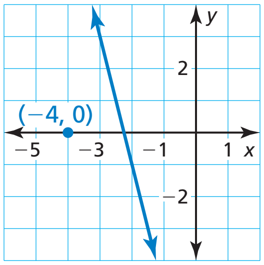
12.



13.



14.



15. A website hosting company charges an initial fee of \$48 to set up a website. The company charges \$44 per month to maintain the website. *(Section 4.1)*

a. Write a linear model that represents the total cost of setting up and maintaining a website as a function of the number of months it is maintained.

b. Find the total cost of setting up a website and maintaining it for 6 months.

c. A different website hosting company charges \$62 per month to maintain a website, but there is no initial set-up fee. You have \$620. At which company can you set up and maintain a website for the greatest amount of time? Explain.

16. The table shows the amount of water remaining in a water tank as it drains. Can the situation be modeled by a linear equation? Explain. If possible, write a linear model that represents the amount of water remaining in the tank as a function of time. *(Section 4.2)*

Time (minutes)	8	10	12	14	16
Water (gallons)	155	150	145	140	135

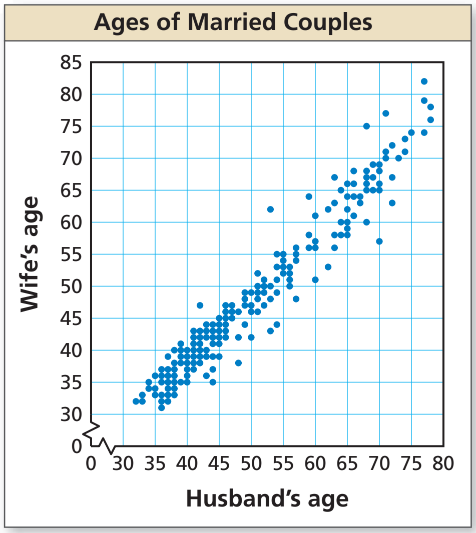
4.4 Scatter Plots and Lines of Fit

Essential Question How can you use a scatter plot and a line of fit to make conclusions about data?

A **scatter plot** is a graph that shows the relationship between two data sets. The two data sets are graphed as ordered pairs in a coordinate plane.

EXPLORATION 1 Finding a Line of Fit

Work with a partner. A survey was taken of 179 married couples. Each person was asked his or her age. The scatter plot shows the results.



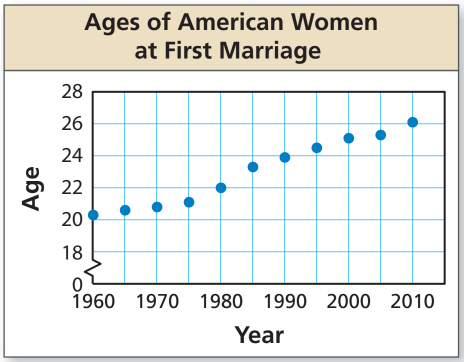
a. Draw a line that approximates the data. Write an equation of the line. Explain the method you used.

b. What conclusions can you make from the equation you wrote? Explain your reasoning.

REASONING QUANTITATIVELY
To be proficient in math, you need to make sense of quantities and their relationships in problem situations.

EXPLORATION 2 Finding a Line of Fit

Work with a partner. The scatter plot shows the median ages of American women at their first marriage for selected years from 1960 through 2010.



a. Draw a line that approximates the data. Write an equation of the line. Let x represent the number of years since 1960. Explain the method you used.

b. What conclusions can you make from the equation you wrote?

c. Use your equation to predict the median age of American women at their first marriage in the year 2020.

Communicate Your Answer

3. How can you use a scatter plot and a line of fit to make conclusions about data?

4. Use the Internet or some other reference to find a scatter plot of real-life data that is different from those given above. Then draw a line that approximates the data and write an equation of the line. Explain the method you used.

4.4 Lesson

Core Vocabulary

What You Will Learn

- Interpret scatter plots.
- Identify correlations between data sets.
- Use lines of fit to model data.

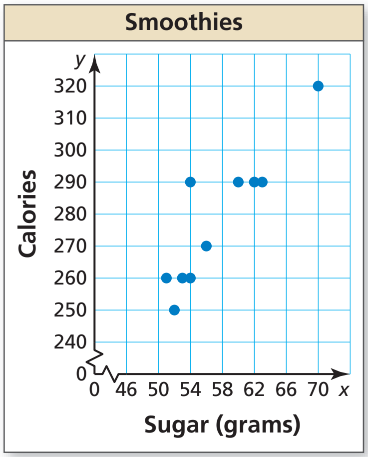
Interpreting Scatter Plots

Core Concept

Scatter Plot

A **scatter plot** is a graph that shows the relationship between two data sets. The two data sets are graphed as ordered pairs in a coordinate plane. Scatter plots can show trends in the data.

EXAMPLE 1 Interpreting a Scatter Plot



The scatter plot shows the amounts x (in grams) of sugar and the numbers y of calories in 10 smoothies.

a. How many calories are in the smoothie that contains 56 grams of sugar?

b. How many grams of sugar are in the smoothie that contains 320 calories?

c. What tends to happen to the number of calories as the number of grams of sugar increases?

SOLUTION

a. Draw a horizontal line from the point that has an x-value of 56. It crosses the y-axis at 270.

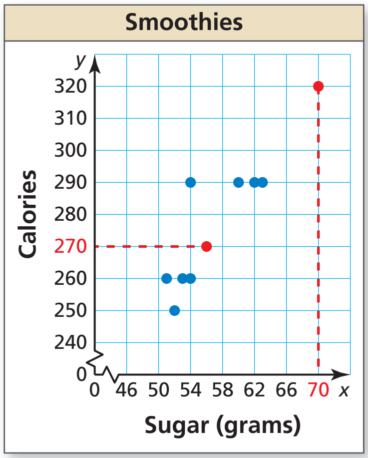
▶ So, the smoothie has 270 calories.

b. Draw a vertical line from the point that has a y-value of 320. It crosses the x-axis at 70.

▶ So, the smoothie has 70 grams of sugar.

c. Looking at the graph, the plotted points go up from left to right.

▶ So, as the number of grams of sugar increases, the number of calories increases.



Monitoring Progress

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1. How many calories are in the smoothie that contains 51 grams of sugar?

2. How many grams of sugar are in the smoothie that contains 250 calories?

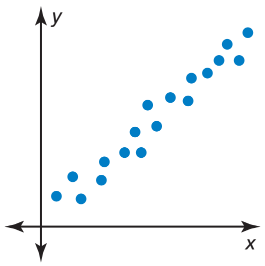
STUDY TIP

You can think of a positive correlation as having a positive slope and a negative correlation as having a negative slope.

Identifying Correlations between Data Sets

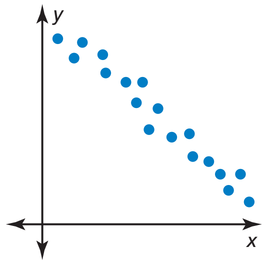
A **correlation** is a relationship between data sets. You can use a scatter plot to describe the correlation between data.

Positive Correlation



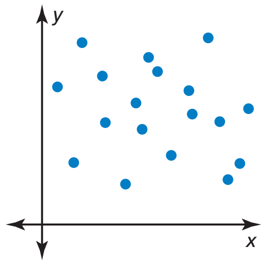
As *x* increases, *y* increases.

Negative Correlation



As *x* increases, *y* decreases.

No Correlation



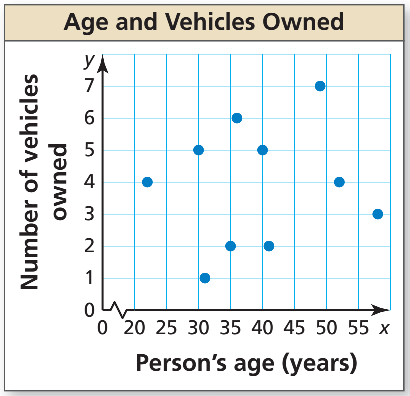
The points show no pattern.

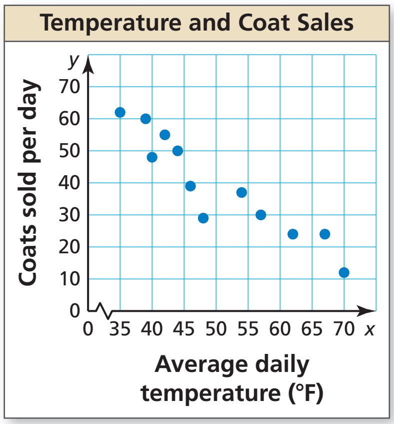
EXAMPLE 2 Identifying Correlations

Tell whether the data show a *positive*, a *negative*, or *no* correlation.

a. age and vehicles owned

b. temperature and coat sales at a store





SOLUTION

a. The points show no pattern. The number of vehicles owned does not depend on a person's age.

▶ So, the scatter plot shows no correlation.

b. As the average temperature increases, the number of coats sold decreases.

▶ So, the scatter plot shows a negative correlation.

Monitoring Progress

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Make a scatter plot of the data. Tell whether the data show a *positive*, a *negative*, or *no* correlation.

3.

Temperature (°F), *x*	82	78	68	87	75	71	92	84
Attendees (thousands), *y*	4.5	4.0	1.7	5.5	3.8	2.9	4.7	5.3

4.

Age of a car (years), *x*	1	2	3	4	5	6	7	8
Value (thousands), *y*	\$24	\$21	\$19	\$18	\$15	\$12	\$8	\$7

Using Lines of Fit to Model Data

When data show a positive or negative correlation, you can model the *trend* in the data using a line of fit. A **line of fit** is a line drawn on a scatter plot that is close to most of the data points.

STUDY TIP

A line of fit is also called a *trend line*.

Core Concept

Using a Line of Fit to Model Data

Step 1 Make a scatter plot of the data.

Step 2 Decide whether the data can be modeled by a line.

Step 3 Draw a line that appears to fit the data closely. There should be approximately as many points above the line as below it.

Step 4 Write an equation using two points on the line. The points do not have to represent actual data pairs, but they must lie on the line of fit.

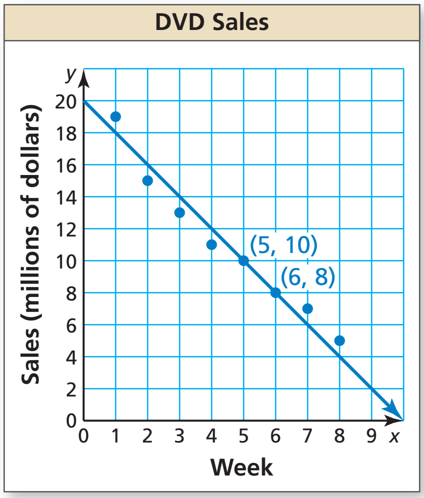
EXAMPLE 3 Finding a Line of Fit

The table shows the weekly sales of a DVD and the number of weeks since its release. Write an equation that models the DVD sales as a function of the number of weeks since its release. Interpret the slope and y-intercept of the line of fit.

Week, x	1	2	3	4	5	6	7	8
Sales (millions), y	\$19	\$15	\$13	\$11	\$10	\$8	\$7	\$5

SOLUTION

Step 1 Make a scatter plot of the data.



Step 2 Decide whether the data can be modeled by a line. Because the scatter plot shows a negative correlation, you can fit a line to the data.

Step 3 Draw a line that appears to fit the data closely.

Step 4 Write an equation using two points on the line. Use (5, 10) and (6, 8).

The slope of the line is $m = \frac{8 - 10}{6 - 5} = -2$.

Use the slope $m = -2$ and the point (6, 8) to write an equation of the line.

$y - y_1 = m(x - x_1)$ Write the point-slope form.

$y - 8 = -2(x - 6)$ Substitute -2 for m, 6 for x_1, and 8 for y_1.

$y = -2x + 20$ Solve for y.

▶ An equation of the line of fit is $y = -2x + 20$. The slope of the line is -2. This means the sales are decreasing by about \$2 million each week. The y-intercept is 20. The y-intercept has no meaning in this context because there are no sales in week 0.

Monitoring Progress

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5. The following data pairs show the monthly income x (in dollars) and the monthly car payment y (in dollars) of six people: (2100, 410), (1650, 315), (1950, 405), (1500, 295), (2250, 440), and (1800, 375). Write an equation that models the monthly car payment as a function of the monthly income. Interpret the slope and y-intercept of the line of fit.

4.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** When data show a positive correlation, the dependent variable tends to ____________ as the independent variable increases.

2. **VOCABULARY** What is a line of fit?

Monitoring Progress and Modeling with Mathematics

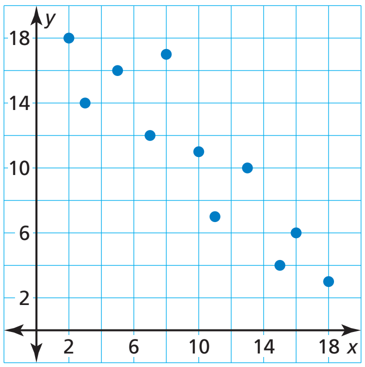
In Exercises 3–6, use the scatter plot to fill in the missing coordinate of the ordered pair.

3. (16, ▭)

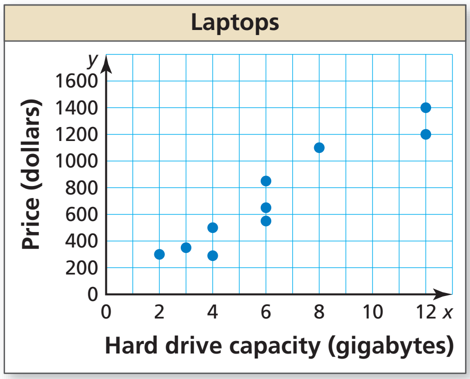
4. (3, ▭)

5. (▭, 12)

6. (▭, 17)



7. **INTERPRETING A SCATTER PLOT** The scatter plot shows the hard drive capacities (in gigabytes) and the prices (in dollars) of 10 laptops. *(See Example 1.)*

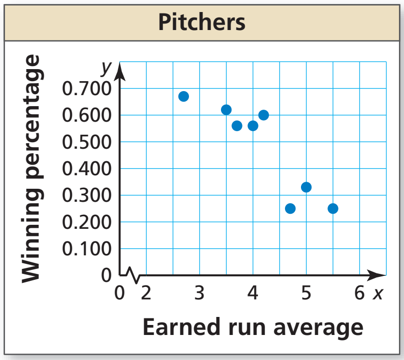


a. What is the price of the laptop with a hard drive capacity of 8 gigabytes?

b. What is the hard drive capacity of the $1200 laptop?

c. What tends to happen to the price as the hard drive capacity increases?

8. **INTERPRETING A SCATTER PLOT** The scatter plot shows the earned run averages and the winning percentages of eight pitchers on a baseball team.



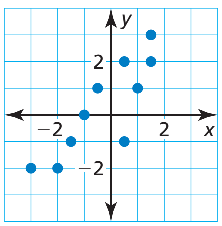
a. What is the winning percentage of the pitcher with an earned run average of 4.2?

b. What is the earned run average of the pitcher with a winning percentage of 0.33?

c. What tends to happen to the winning percentage as the earned run average increases?

In Exercises 9–12, tell whether *x* and *y* show a *positive*, a *negative*, or *no* correlation. *(See Example 2.)*

9.



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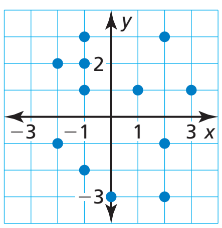

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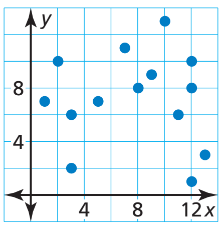
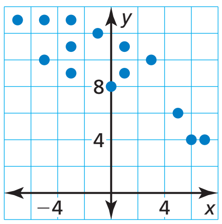


12.



In Exercises 13 and 14, make a scatter plot of the data. Tell whether x and y show a *positive*, a *negative*, or *no* correlation.

13.

x	3.1	2.2	2.5	3.7	3.9	1.5	2.7	2.0
y	1	0	1	2	0	2	3	2

14.

x	3	4	5	6	7	8	9	10
y	67	67	50	33	25	21	19	4

15. MODELING WITH MATHEMATICS The table shows the world birth rates y (number of births per 1000 people) x years since 1960. *(See Example 3.)*

x	0	10	20	30	40	50
y	35.4	33.6	28.3	27.0	22.4	20.0

a. Write an equation that models the birthrate as a function of the number of years since 1960.

b. Interpret the slope and y-intercept of the line of fit.

16. MODELING WITH MATHEMATICS The table shows the total earnings y (in dollars) of a food server who works x hours.

x	0	1	2	3	4	5	6
y	0	18	40	62	77	85	113

a. Write an equation that models the server's earnings as a function of the number of hours the server works.

b. Interpret the slope and y-intercept of the line of fit.

17. OPEN-ENDED Give an example of a real-life data set that shows a negative correlation.

18. MAKING AN ARGUMENT Your friend says that the data in the table show a negative correlation because the dependent variable y is decreasing. Is your friend correct? Explain.

x	14	12	10	8	6	4	2
y	4	1	0	−1	−2	−4	−5

19. USING TOOLS Use a ruler or a yardstick to find the heights and arm spans of five people.

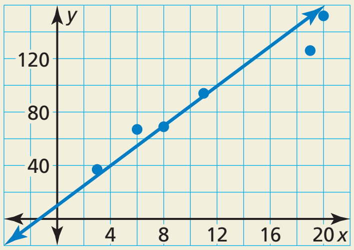
a. Make a scatter plot using the data you collected. Then draw a line of fit for the data.

b. Interpret the slope and y-intercept of the line of fit.

20. THOUGHT PROVOKING A line of fit for a scatter plot is given by the equation $y = 5x + 20$. Describe a real-life data set that could be represented by the scatter plot.

21. WRITING When is data best displayed in a scatter plot, rather than another type of display, such as a bar graph or circle graph?

22. HOW DO YOU SEE IT? The scatter plot shows part of a data set and a line of fit for the data set. Four data points are missing. Choose possible coordinates for these data points.



23. REASONING A data set has no correlation. Is it possible to find a line of fit for the data? Explain.

24. ANALYZING RELATIONSHIPS Make a scatter plot of the data in the tables. Describe the relationship between the variables. Is it possible to fit a line to the data? If so, write an equation of the line. If not, explain why.

x	−12	−9	−7	−4	−3	−1
y	150	76	50	15	10	1

x	2	5	6	7	9	15
y	5	22	37	52	90	226

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the function when $x = -3$, 0, and 4. *(Section 3.3)*

25. $g(x) = 6x$

26. $h(x) = -10x$

27. $f(x) = 5x - 8$

28. $v(x) = 14 - 3x$

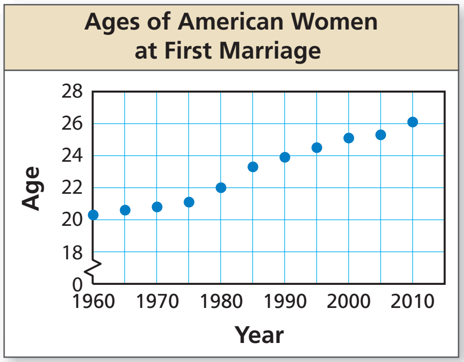
4.5 Analyzing Lines of Fit

Essential Question

How can you *analytically* find a line of best fit for a scatter plot?

EXPLORATION 1 Finding a Line of Best Fit

Work with a partner. The scatter plot shows the median ages of American women at their first marriage for selected years from 1960 through 2010. In Exploration 2 in Section 4.4, you approximated a line of fit graphically. To find the line of *best* fit, you can use a computer, spreadsheet, or graphing calculator that has a *linear regression* feature.



a. The data from the scatter plot is shown in the table. Note that 0, 5, 10, and so on represent the numbers of years since 1960. What does the ordered pair (25, 23.3) represent?

L1	L2	L3
0	20.3	
5	20.6	
10	20.8	
15	21.1	
20	22	
25	23.3	
30	23.9	
35	24.5	
40	25.1	
45	25.3	
50	26.1	

L1(55)=

b. Use the *linear regression* feature to find an equation of the line of best fit. You should obtain results such as those shown below.

```
LinReg
 y=ax+b
 a=.1261818182
 b=19.84545455
 r²=.9738676804
 r=.986847344
```

c. Write an equation of the line of best fit. Compare your result with the equation you obtained in Exploration 2 in Section 4.4.

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to reason inductively about data.

Communicate Your Answer

2. How can you *analytically* find a line of best fit for a scatter plot?

3. The data set relates the number of chirps per second for striped ground crickets and the outside temperature in degrees Fahrenheit. Make a scatter plot of the data. Then find an equation of the line of best fit. Use your result to estimate the outside temperature when there are 19 chirps per second.

Chirps per second	20.0	16.0	19.8	18.4	17.1
Temperature (°F)	88.6	71.6	93.3	84.3	80.6

Chirps per second	14.7	15.4	16.2	15.0	14.4
Temperature (°F)	69.7	69.4	83.3	79.6	76.3

4.5 Lesson

What You Will Learn

- Use residuals to determine how well lines of fit model data.
- Use technology to find lines of best fit.
- Distinguish between correlation and causation.

Core Vocabulary

residual, *p. 202*
linear regression, *p. 203*
line of best fit, *p. 203*
correlation coefficient, *p. 203*
interpolation, *p. 205*
extrapolation, *p. 205*
causation, *p. 205*

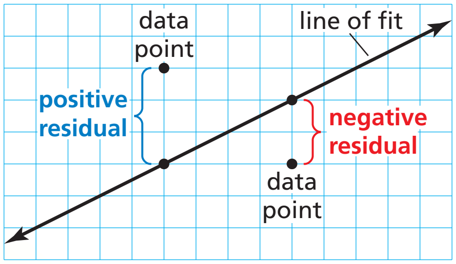
Analyzing Residuals

One way to determine how well a line of fit models a data set is to analyze *residuals*.

Core Concept

Residuals

A **residual** is the difference of the y-value of a data point and the corresponding y-value found using the line of fit. A residual can be positive, negative, or zero.



A scatter plot of the residuals shows how well a model fits a data set. If the model is a good fit, then the absolute values of the residuals are relatively small, and the residual points will be more or less evenly dispersed about the horizontal axis. If the model is not a good fit, then the residual points will form some type of pattern that suggests the data are not linear. Wildly scattered residual points suggest that the data might have no correlation.

EXAMPLE 1 Using Residuals

Week, x	Sales (millions), y
1	\$19
2	\$15
3	\$13
4	\$11
5	\$10
6	\$8
7	\$7
8	\$5

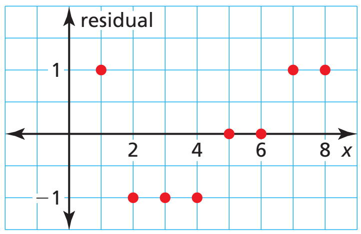
In Example 3 in Section 4.4, the equation $y = -2x + 20$ models the data in the table shown. Is the model a good fit?

SOLUTION

Step 1 Calculate the residuals. Organize your results in a table.

Step 2 Use the points (x, residual) to make a scatter plot.

x	y	y-Value from model	Residual
1	19	18	$19 - 18 = 1$
2	15	16	$15 - 16 = -1$
3	13	14	$13 - 14 = -1$
4	11	12	$11 - 12 = -1$
5	10	10	$10 - 10 = 0$
6	8	8	$8 - 8 = 0$
7	7	6	$7 - 6 = 1$
8	5	4	$5 - 4 = 1$



- The points are evenly dispersed about the horizontal axis. So, the equation $y = -2x + 20$ is a good fit.

EXAMPLE 2 Using Residuals

The table shows the ages x and salaries y (in thousands of dollars) of eight employees at a company. The equation $y = 0.2x + 38$ models the data. Is the model a good fit?

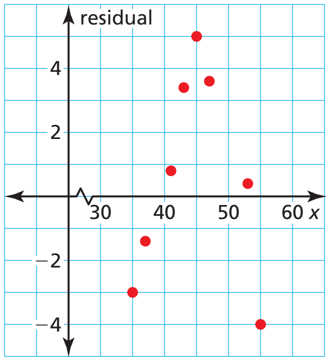
Age, x	35	37	41	43	45	47	53	55
Salary, y	42	44	47	50	52	51	49	45

SOLUTION

Step 1 Calculate the residuals. Organize your results in a table.

Step 2 Use the points (x, residual) to make a scatter plot.

x	y	y-Value from model	Residual
35	42	45.0	$42 - 45.0 = -3.0$
37	44	45.4	$44 - 45.4 = -1.4$
41	47	46.2	$47 - 46.2 = 0.8$
43	50	46.6	$50 - 46.6 = 3.4$
45	52	47.0	$52 - 47.0 = 5.0$
47	51	47.4	$51 - 47.4 = 3.6$
53	49	48.6	$49 - 48.6 = 0.4$
55	45	49.0	$45 - 49.0 = -4.0$



▶ The residual points form a ∩-shaped pattern, which suggests the data are not linear. So, the equation $y = 0.2x + 38$ does not model the data well.

Monitoring Progress

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1. The table shows the attendances y (in thousands) at an amusement park from 2005 to 2014, where $x = 0$ represents the year 2005. The equation $y = -9.8x + 850$ models the data. Is the model a good fit?

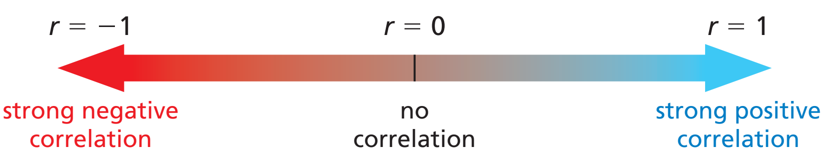
Year, x	0	1	2	3	4	5	6	7	8	9
Attendance, y	850	845	828	798	800	792	785	781	775	760

STUDY TIP

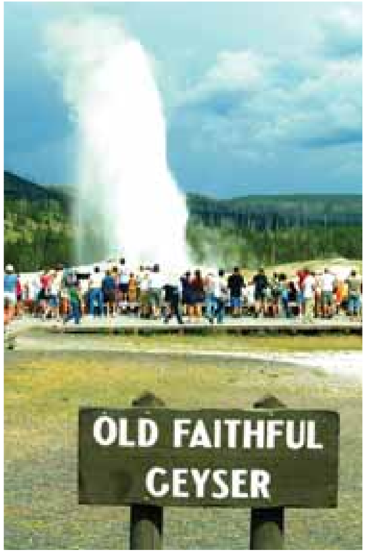
You know how to use two points to find an equation of a line of fit. When finding an equation of the line of best fit, every point in the data set is used.

Finding Lines of Best Fit

Graphing calculators use a method called **linear regression** to find a precise line of fit called a **line of best fit**. This line best models a set of data. A calculator often gives a value r, called the **correlation coefficient**. This value tells whether the correlation is positive or negative and how closely the equation models the data. Values of r range from -1 to 1. When r is close to 1 or -1, there is a strong correlation between the variables. As r, gets closer to 0, the correlation becomes weaker.



EXAMPLE 3 Finding a Line of Best Fit Using Technology

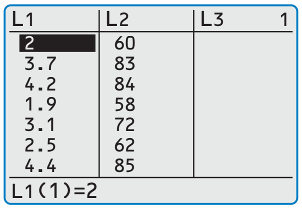


The table shows the durations x (in minutes) of several eruptions of the geyser Old Faithful and the times y (in minutes) until the next eruption. (a) Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window. (b) Identify and interpret the correlation coefficient. (c) Interpret the slope and y-intercept of the line of best fit.

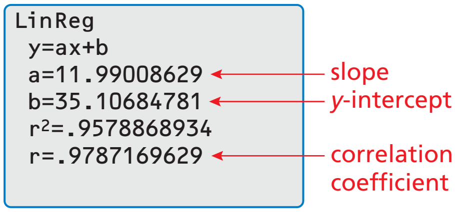
Duration, x	2.0	3.7	4.2	1.9	3.1	2.5	4.4	3.9
Time, y	60	83	84	58	72	62	85	85

SOLUTION

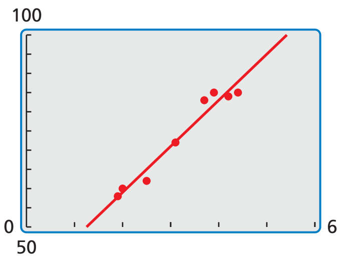
a. Step 1 Enter the data from the table into two lists.



Step 2 Use the *linear regression* feature. The values in the equation can be rounded to obtain $y = 12.0x + 35$.



Step 3 Enter the equation $y = 12.0x + 35$ into the calculator. Then plot the data and graph the equation in the same viewing window.



PRECISION

Be sure to analyze the data values to select an appropriate viewing window for your graph.

b. The correlation coefficient is about 0.979. This means that the relationship between the durations and the times until the next eruption has a strong positive correlation and the equation closely models the data, as shown in the graph.

c. The slope of the line is 12. This means the time until the next eruption increases by about 12 minutes for each minute the duration increases. The y-intercept is 35, but it has no meaning in this context because the duration cannot be 0 minutes.

Monitoring Progress

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2. Use the data in Monitoring Progress Question 1. (a) Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window. (b) Identify and interpret the correlation coefficient. (c) Interpret the slope and y-intercept of the line of best fit.

Using a graph or its equation to *approximate* a value between two known values is called **interpolation**. Using a graph or its equation to *predict* a value outside the range of known values is called **extrapolation**. In general, the farther removed a value is from the known values, the less confidence you can have in the accuracy of the prediction.

STUDY TIP

To approximate or predict an unknown value, you can evaluate the model algebraically or graph the model with a graphing calculator and use the *trace* feature.

EXAMPLE 4 Interpolating and Extrapolating Data

Refer to Example 3. Use the equation of the line of best fit.

a. Approximate the duration before a time of 77 minutes.

b. Predict the time after an eruption lasting 5.0 minutes.

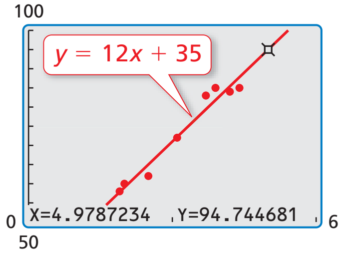
SOLUTION

a. $y = 12.0x + 35$ Write the equation.

$77 = 12.0x + 35$ Substitute 77 for y.

$3.5 = x$ Solve for x.

▶ An eruption lasts about 3.5 minutes before a time of 77 minutes.



b. Use a graphing calculator to graph the equation. Use the *trace* feature to find the value of y when $x \approx 5.0$, as shown.

▶ A time of about 95 minutes will follow an eruption of 5.0 minutes.

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3. Refer to Monitoring Progress Question 2. Use the equation of the line of best fit to predict the attendance at the amusement park in 2017.

Correlation and Causation

When a change in one variable causes a change in another variable, it is called **causation**. Causation produces a strong correlation between the two variables. The converse is *not* true. In other words, correlation does not imply causation.

READING

A causal relationship exists when one variable causes a change in another variable.

EXAMPLE 5 Identifying Correlation and Causation

Tell whether a correlation is likely in the situation. If so, tell whether there is a causal relationship. Explain your reasoning.

a. time spent exercising and the number of calories burned

b. the number of banks and the population of a city

SOLUTION

a. There is a positive correlation and a causal relationship because the more time you spend exercising, the more calories you burn.

b. There may be a positive correlation but no causal relationship. Building more banks will not cause the population to increase.

Monitoring Progress

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4. Is there a correlation between time spent playing video games and grade point average? If so, is there a causal relationship? Explain your reasoning.

4.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** When is a residual positive? When is it negative?

2. **WRITING** Explain how you can use residuals to determine how well a line of fit models a data set.

3. **VOCABULARY** Compare interpolation and extrapolation.

4. **WHICH ONE DOESN'T BELONG?** Which correlation coefficient does *not* belong with the other three? Explain your reasoning.

$r = -0.98$ $r = 0.96$ $r = -0.09$ $r = 0.97$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, use residuals to determine whether the model is a good fit for the data in the table. Explain. *(See Examples 1 and 2.)*

5. $y = 4x - 5$

x	−4	−3	−2	−1	0	1	2	3	4
y	−18	−13	−10	−7	−2	0	6	10	15

6. $y = 6x + 4$

x	1	2	3	4	5	6	7	8	9
y	13	14	23	26	31	42	45	52	62

7. $y = -1.3x + 1$

x	−8	−6	−4	−2	0	2	4	6	8
y	9	10	5	8	−1	1	−4	−12	−7

8. $y = -0.5x - 2$

x	4	6	8	10	12	14	16	18	20
y	−1	−3	−6	−8	−10	−10	−10	−9	−9

9. **ANALYZING RESIDUALS** The table shows the growth y (in inches) of an elk's antlers during week x. The equation $y = -0.7x + 6.8$ models the data. Is the model a good fit? Explain.

Week, x	1	2	3	4	5
Growth, y	6.0	5.5	4.7	3.9	3.3

10. **ANALYZING RESIDUALS** The table shows the approximate numbers y (in thousands) of movie tickets sold from January to June for a theater. In the table, $x = 1$ represents January. The equation $y = 1.3x + 27$ models the data. Is the model a good fit? Explain.

Month, x	Ticket sales, y
1	27
2	28
3	36
4	28
5	32
6	35

In Exercises 11–14, use a graphing calculator to find an equation of the line of best fit for the data. Identify and interpret the correlation coefficient.

11.

x	0	1	2	3	4	5	6	7
y	−8	−5	−2	−1	−1	2	5	8

12.

x	−4	−2	0	2	4	6	8	10
y	17	7	8	1	5	−2	2	−8

13.

x	−15	−10	−5	0	5	10	15	20
y	−4	2	7	16	22	30	37	43

14.

x	5	6	7	8	9	10	11	12
y	12	−2	8	3	−1	−4	6	0

ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in interpreting the graphing calculator display.**

```
LinReg
 y=ax+b
 a=-4.47
 b=23.16
 r²=.9989451055
 r=-.9994724136
```

15. ✗ An equation of the line of best fit is $y = 23.16x - 4.47$.

16. ✗ The data have a strong positive correlation.

17. **MODELING WITH MATHEMATICS** The table shows the total numbers y of people who reported an earthquake x minutes after it ended. *(See Example 3.)*

Minutes, x	People, y
1	10
2	100
3	400
4	900
5	1400
6	1800
7	2100

a. Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window.

b. Identify and interpret the correlation coefficient.

c. Interpret the slope and y-intercept of the line of best fit.

18. **MODELING WITH MATHEMATICS** The table shows the numbers y of people who volunteer at an animal shelter on each day x.

Day, x	1	2	3	4	5	6	7	8
People, y	9	5	13	11	10	11	19	12

a. Use a graphing calculator to find an equation of the line of best fit. Then plot the data and graph the equation in the same viewing window.

b. Identify and interpret the correlation coefficient.

c. Interpret the slope and y-intercept of the line of best fit.

19. **MODELING WITH MATHEMATICS** The table shows the mileages x (in thousands of miles) and the selling prices y (in thousands of dollars) of several used automobiles of the same year and model. *(See Example 4.)*

Mileage, x	22	14	18	30	8	24
Price, y	16	17	17	14	18	15

a. Use a graphing calculator to find an equation of the line of best fit.

b. Identify and interpret the correlation coefficient.

c. Interpret the slope and y-intercept of the line of best fit.

d. Approximate the mileage of an automobile that costs \$15,500.

e. Predict the price of an automobile with 6000 miles.

20. **MODELING WITH MATHEMATICS** The table shows the lengths x and costs y of several sailboats.

Length (feet), x	Cost (thousands of dollars), y
27	94
18	56
25	58
32	123
18	60
26	87
36	145

a. Use a graphing calculator to find an equation of the line of best fit.

b. Identify and interpret the correlation coefficient.

c. Interpret the slope and y-intercept of the line of best fit.

d. Approximate the cost of a sailboat that is 20 feet long.

e. Predict the length of a sailboat that costs \$147,000.

In Exercises 21–24, tell whether a correlation is likely in the situation. If so, tell whether there is a causal relationship. Explain your reasoning. *(See Example 5.)*

21. the amount of time spent talking on a cell phone and the remaining battery life

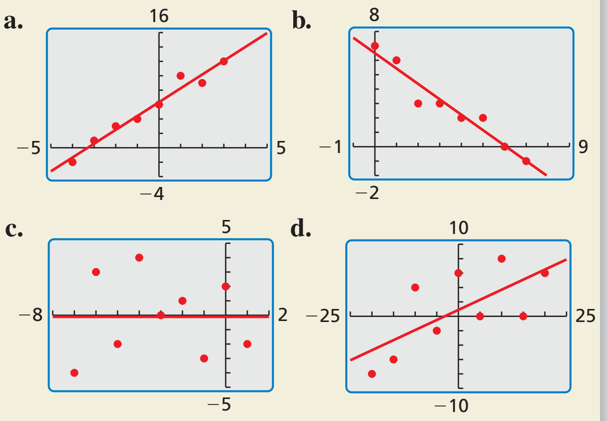
22. the height of a toddler and the size of the toddler's vocabulary

23. the number of hats you own and the size of your head

24. the weight of a dog and the length of its tail

25. **OPEN-ENDED** Describe a data set that has a strong correlation but does not have a causal relationship.

26. **HOW DO YOU SEE IT?** Match each graph with its correlation coefficient. Explain your reasoning.



A. $r = 0$ **B.** $r = 0.98$

C. $r = -0.97$ **D.** $r = 0.69$

27. **ANALYZING RELATIONSHIPS** The table shows the grade point averages y of several students and the numbers x of hours they spend watching television each week.

Hours, x	Grade point average, y
10	3.0
5	3.4
3	3.5
12	2.7
20	2.1
15	2.8
8	3.0
4	3.7
16	2.5

a. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

b. Interpret the slope and y-intercept of the line of best fit.

c. Another student watches about 14 hours of television each week. Approximate the student's grade point average.

d. Do you think there is a causal relationship between time spent watching television and grade point average? Explain.

28. **MAKING AN ARGUMENT** A student spends 2 hours watching television each week and has a grade point average of 2.4. Your friend says including this information in the data set in Exercise 27 will weaken the correlation. Is your friend correct? Explain.

29. **USING MODELS** Refer to Exercise 17.

a. Predict the total numbers of people who reported an earthquake 9 minutes and 15 minutes after it ended.

b. The table shows the actual data. Describe the accuracy of your extrapolations in part (a).

Minutes, x	9	15
People, y	2750	3200

30. **THOUGHT PROVOKING** A data set consists of the numbers x of people at Beach 1 and the numbers y of people at Beach 2 recorded daily for 1 week. Sketch a possible graph of the data set. Describe the situation shown in the graph and give a possible correlation coefficient. Determine whether there is a causal relationship. Explain.

31. **COMPARING METHODS** The table shows the numbers y (in billions) of text messages sent each year in a five-year period, where $x = 1$ represents the first year in the five-year period.

Year, x	1	2	3	4	5
Text messages (billions), y	241	601	1360	1806	2206

a. Use a graphing calculator to find an equation of the line of best fit. Identify and interpret the correlation coefficient.

b. Is there a causal relationship? Explain your reasoning.

c. Calculate the residuals. Then make a scatter plot of the residuals and interpret the results.

d. Compare the methods you used in parts (a) and (c) to determine whether the model is a good fit. Which method do you prefer? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether the table represents a *linear* or *nonlinear* function. Explain. *(Section 3.2)*

32.

x	5	6	7	8
y	−4	4	−4	4

33.

x	2	4	6	8
y	13	8	3	−2

4.6 Arithmetic Sequences

Essential Question

How can you use an arithmetic sequence to describe a pattern?

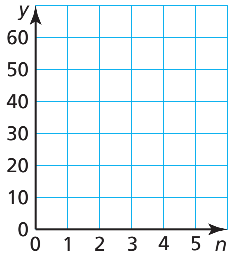
An **arithmetic sequence** is an ordered list of numbers in which the difference between each pair of consecutive **terms**, or numbers in the list, is the same.

EXPLORATION 1 Describing a Pattern

Work with a partner. Use the figures to complete the table. Plot the points given by your completed table. Describe the pattern of the *y*-values.

a. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$

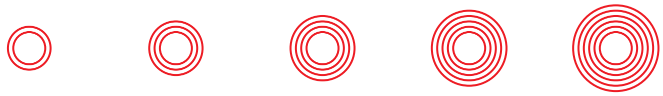
Number of stars, *n*	1	2	3	4	5
Number of sides, *y*					



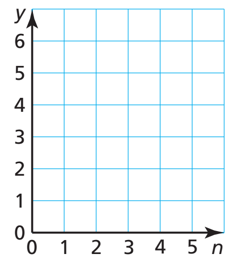
LOOKING FOR A PATTERN

To be proficient in math, you need to look closely to discern patterns and structure.

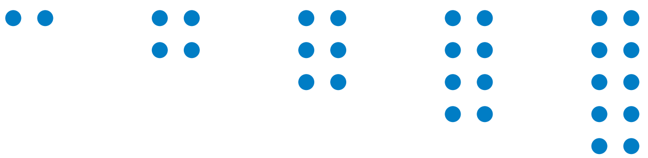
b. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



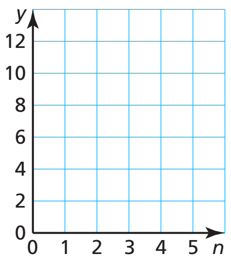
n	1	2	3	4	5
Number of circles, *y*					



c. $n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



Number of rows, *n*	1	2	3	4	5
Number of dots, *y*					

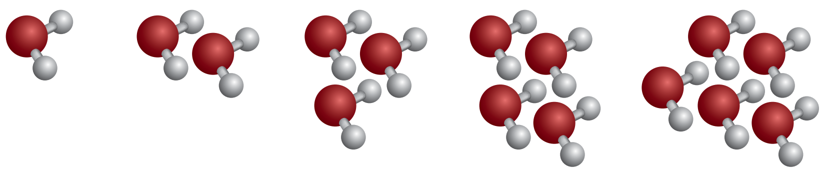


Communicate Your Answer

2. How can you use an arithmetic sequence to describe a pattern? Give an example from real life.

3. In chemistry, water is called H_2O because each molecule of water has two hydrogen atoms and one oxygen atom. Describe the pattern shown below. Use the pattern to determine the number of atoms in 23 molecules.

$n = 1$ $n = 2$ $n = 3$ $n = 4$ $n = 5$



4.6 Lesson

What You Will Learn

- Write the terms of arithmetic sequences.
- Graph arithmetic sequences.
- Write arithmetic sequences as functions.

Core Vocabulary

sequence, *p. 210*
term, *p. 210*
arithmetic sequence, *p. 210*
common difference, *p. 210*

Previous
point-slope form
function notation

Writing the Terms of Arithmetic Sequences

A **sequence** is an ordered list of numbers. Each number in a sequence is called a **term**. Each term a_n has a specific position n in the sequence.

$5, \; 10, \; 15, \; 20, \; 25, \ldots, a_n, \ldots$

1st position (5); 3rd position (15); *n*th position (a_n)

Core Concept

Arithmetic Sequence

In an **arithmetic sequence**, the difference between each pair of consecutive terms is the same. This difference is called the **common difference**. Each term is found by adding the common difference to the previous term.

$5, \; 10, \; 15, \; 20, \ldots$ Terms of an arithmetic sequence

$+5 \quad +5 \quad +5$ ← common difference

READING

An ellipsis (. . .) is a series of dots that indicates an intentional omission of information. In mathematics, the . . . notation means "and so forth." The ellipsis indicates that there are more terms in the sequence that are not shown.

EXAMPLE 1 Extending an Arithmetic Sequence

Write the next three terms of the arithmetic sequence.

$-7, -14, -21, -28, \ldots$

SOLUTION

Use a table to organize the terms and find the pattern.

Position	1	2	3	4
Term	−7	−14	−21	−28

$+(-7) \quad +(-7) \quad +(-7)$

Each term is 7 less than the previous term. So, the common difference is −7.

Add −7 to a term to find the next term.

Position	1	2	3	4	5	6	7
Term	−7	−14	−21	−28	−35	−42	−49

$+(-7) \quad +(-7) \quad +(-7)$

▶ The next three terms are −35, −42, and −49.

Monitoring Progress

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Write the next three terms of the arithmetic sequence.

1. $-12, 0, 12, 24, \ldots$ **2.** $0.2, 0.6, 1, 1.4, \ldots$ **3.** $4, 3\frac{3}{4}, 3\frac{1}{2}, 3\frac{1}{4}, \ldots$

Graphing Arithmetic Sequences

To graph a sequence, let a term's position number n in the sequence be the x-value. The term a_n is the corresponding y-value. Plot the ordered pairs (n, a_n).

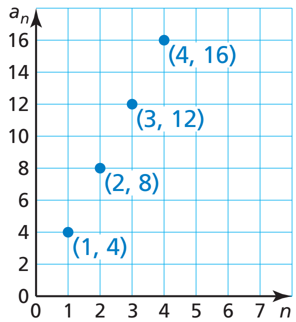
EXAMPLE 2 Graphing an Arithmetic Sequence

Graph the arithmetic sequence 4, 8, 12, 16, ... What do you notice?

SOLUTION

Make a table. Then plot the ordered pairs (n, a_n).

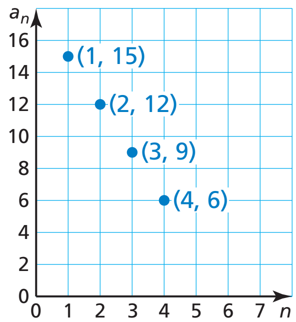
Position, n	Term, a_n
1	4
2	8
3	12
4	16



▶ The points lie on a line.

EXAMPLE 3 Identifying an Arithmetic Sequence from a Graph

Does the graph represent an arithmetic sequence? Explain.



SOLUTION

Make a table to organize the ordered pairs. Then determine whether there is a common difference.

Position, n	1	2	3	4
Term, a_n	15	12	9	6

$+(-3)$ $+(-3)$ $+(-3)$

Each term is 3 less than the previous term. So, the common difference is -3.

▶ Consecutive terms have a common difference of -3. So, the graph represents the arithmetic sequence 15, 12, 9, 6, ...

Monitoring Progress

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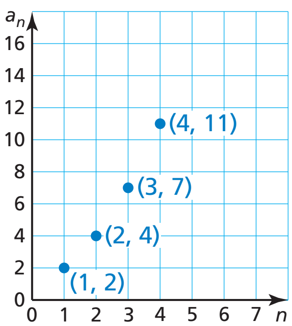
Graph the arithmetic sequence. What do you notice?

4. 3, 6, 9, 12, . . .

5. 4, 2, 0, -2, . . .

6. 1, 0.8, 0.6, 0.4, . . .

7. Does the graph shown represent an arithmetic sequence? Explain.



Writing Arithmetic Sequences as Functions

Because consecutive terms of an arithmetic sequence have a common difference, the sequence has a constant rate of change. So, the points represented by any arithmetic sequence lie on a line. You can use the first term and the common difference to write a linear function that describes an arithmetic sequence. Let $a_1 = 4$ and $d = 3$.

Position, n	Term, a_n	Written using a_1 and d	Numbers
1	first term, a_1	a_1	4
2	second term, a_2	$a_1 + d$	$4 + 3 = 7$
3	third term, a_3	$a_1 + 2d$	$4 + 2(3) = 10$
4	fourth term, a_4	$a_1 + 3d$	$4 + 3(3) = 13$
⋮	⋮	⋮	⋮
n	nth term, a_n	$a_1 + (n - 1)d$	$4 + (n - 1)(3)$

ANOTHER WAY

An *arithmetic sequence* is a linear function whose domain is the set of positive integers. You can think of d as the slope and $(1, a_1)$ as a point on the graph of the function. An equation in point-slope form for the function is

$$a_n - a_1 = d(n - 1).$$

This equation can be rewritten as

$$a_n = a_1 + (n - 1)d.$$

Core Concept

Equation for an Arithmetic Sequence

Let a_n be the nth term of an arithmetic sequence with first term a_1 and common difference d. The nth term is given by

$$a_n = a_1 + (n - 1)d.$$

EXAMPLE 4 Finding the nth Term of an Arithmetic Sequence

Write an equation for the nth term of the arithmetic sequence 14, 11, 8, 5, ... Then find a_{50}.

STUDY TIP

Notice that the equation in Example 4 is of the form $y = mx + b$, where y is replaced by a_n and x is replaced by n.

SOLUTION

The first term is 14, and the common difference is -3.

$a_n = a_1 + (n - 1)d$ Equation for an arithmetic sequence

$a_n = 14 + (n - 1)(-3)$ Substitute 14 for a_1 and -3 for d.

$a_n = -3n + 17$ Simplify.

Use the equation to find the 50th term.

$a_n = -3n + 17$ Write the equation.

$a_{50} = -3(50) + 17$ Substitute 50 for n.

$= -133$ Simplify.

▶ The 50th term of the arithmetic sequence is -133.

Monitoring Progress

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Write an equation for the nth term of the arithmetic sequence. Then find a_{25}.

8. 4, 5, 6, 7, . . .

9. 8, 16, 24, 32, . . .

10. 1, 0, -1, -2, . . .

You can rewrite the equation for an arithmetic sequence with first term a_1 and common difference d in function notation by replacing a_n with $f(n)$.

$$f(n) = a_1 + (n - 1)d$$

The domain of the function is the set of positive integers.

EXAMPLE 5 Writing Real-Life Functions

Online bidding for a purse increases by $5 for each bid after the $60 initial bid.

Bid number	1	2	3	4
Bid amount	$60	$65	$70	$75

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. The winning bid is $105. How many bids were there?

SOLUTION

a. The first term is 60, and the common difference is 5.

$f(n) = a_1 + (n - 1)d$ — Function for an arithmetic sequence

$f(n) = 60 + (n - 1)5$ — Substitute 60 for a_1 and 5 for d.

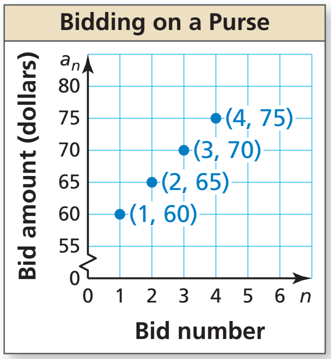
$f(n) = 5n + 55$ — Simplify.

▶ The function $f(n) = 5n + 55$ represents the arithmetic sequence.

b. Make a table. Then plot the ordered pairs (n, a_n).

REMEMBER
The domain is the set of positive integers.

Bid number, n	Bid amount, a_n
1	60
2	65
3	70
4	75



c. Use the function to find the value of n for which $f(n) = 105$.

$f(n) = 5n + 55$ — Write the function.

$105 = 5n + 55$ — Substitute 105 for $f(n)$.

$10 = n$ — Solve for n.

▶ There were 10 bids.

Monitoring Progress

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11. A carnival charges $2 for each game after you pay a $5 entry fee.

Games	Total cost
1	$7
2	$9
3	$11
4	$13

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. How many games can you play when you take $29 to the carnival?

4.6 Exercises

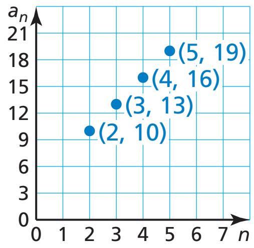
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Describe the graph of an arithmetic sequence.

2. **DIFFERENT WORDS, SAME QUESTION** Consider the arithmetic sequence represented by the graph. Which is different? Find "both" answers.

Find the slope of the linear function.	Find the difference between consecutive terms of the arithmetic sequence.
Find the difference between the terms a_2 and a_4.	Find the common difference of the arithmetic sequence.



Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, write the next three terms of the arithmetic sequence.

3. First term: 2
   Common difference: 13

4. First term: 18
   Common difference: –6

In Exercises 5–10, find the common difference of the arithmetic sequence.

5. 13, 18, 23, 28, . . .
6. 175, 150, 125, 100, . . .
7. $-16, -12, -8, -4, \ldots$
8. $4, 3\frac{2}{3}, 3\frac{1}{3}, 3, \ldots$
9. 6.5, 5, 3.5, 2, . . .
10. $-16, -7, 2, 11, \ldots$

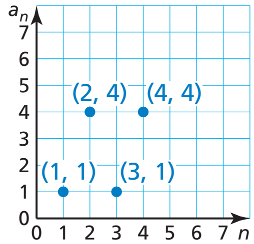
In Exercises 11–16, write the next three terms of the arithmetic sequence. *(See Example 1.)*

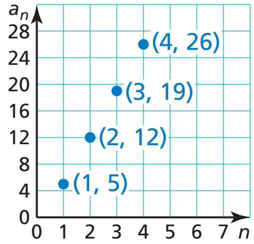
11. 19, 22, 25, 28, . . .
12. 1, 12, 23, 34, . . .
13. 16, 21, 26, 31, . . .
14. $60, 30, 0, -30, \ldots$
15. 1.3, 1, 0.7, 0.4, . . .
16. $\frac{5}{6}, \frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \ldots$

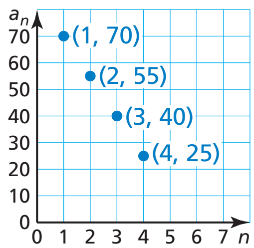
In Exercises 17–22, graph the arithmetic sequence. *(See Example 2.)*

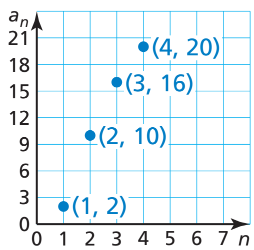
17. 4, 12, 20, 28, . . .
18. $-15, 0, 15, 30, \ldots$
19. –1, –3, –5, –7, . . .
20. 2, 19, 36, 53, . . .
21. $0, 4\frac{1}{2}, 9, 13\frac{1}{2}, \ldots$
22. 6, 5.25, 4.5, 3.75, . . .

In Exercises 23–26, determine whether the graph represents an arithmetic sequence. Explain. *(See Example 3.)*

23. 

24. 

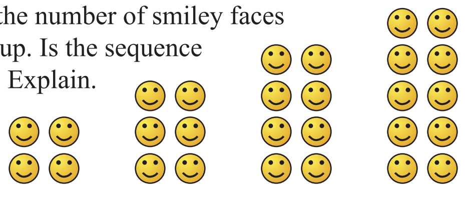
25. 

26. 

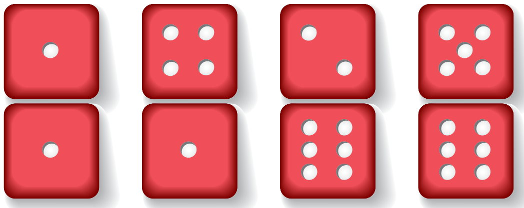
In Exercises 27–30, determine whether the sequence is arithmetic. If so, find the common difference.

27. 13, 26, 39, 52, . . .
28. 5, 9, 14, 20, . . .
29. 48, 24, 12, 6, . . .
30. 87, 81, 75, 69, . . .

31. **FINDING A PATTERN** Write a sequence that represents the number of smiley faces in each group. Is the sequence arithmetic? Explain.



32. FINDING A PATTERN Write a sequence that represents the sum of the numbers in each roll. Is the sequence arithmetic? Explain.



In Exercises 33–38, write an equation for the nth term of the arithmetic sequence. Then find a_{10}. *(See Example 4.)*

33. $-5, -4, -3, -2, \ldots$

34. $-6, -9, -12, -15, \ldots$

35. $\frac{1}{2}, 1, 1\frac{1}{2}, 2, \ldots$

36. $100, 110, 120, 130, \ldots$

37. $10, 0, -10, -20, \ldots$

38. $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \ldots$

39. ERROR ANALYSIS Describe and correct the error in finding the common difference of the arithmetic sequence.

✗ $2, \quad 1, \quad 0, \quad -1, \ldots$

$-1 \quad -1 \quad -1$

The common difference is 1.

40. ERROR ANALYSIS Describe and correct the error in writing an equation for the nth term of the arithmetic sequence.

✗ $14, 22, 30, 38, \ldots$

$a_n = a_1 + nd$

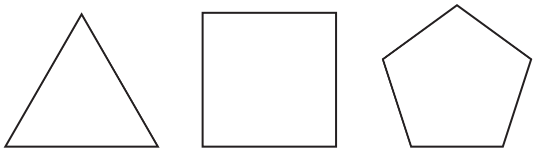
$a_n = 14 + 8n$

41. NUMBER SENSE The first term of an arithmetic sequence is 3. The common difference of the sequence is 1.5 times the first term. Write the next three terms of the sequence. Then graph the sequence.

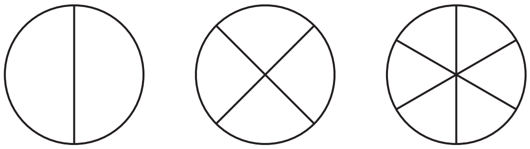
42. NUMBER SENSE The first row of a dominoes display has 10 dominoes. Each row after the first has two more dominoes than the row before it. Write the first five terms of the sequence that represents the number of dominoes in each row. Then graph the sequence.

REPEATED REASONING In Exercises 43 and 44, (a) draw the next three figures in the sequence and (b) describe the 20th figure in the sequence.

43.



44.



45. MODELING WITH MATHEMATICS The total number of babies born in a country each minute after midnight January 1st can be estimated by the sequence shown in the table. *(See Example 5.)*

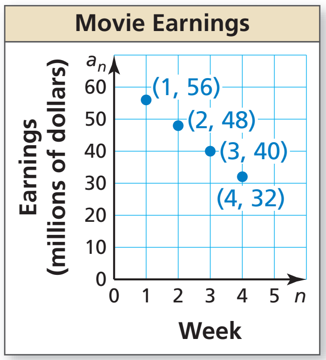
Minutes after midnight January 1st	1	2	3	4
Total babies born	5	10	15	20

a. Write a function that represents the arithmetic sequence.

b. Graph the function.

c. Estimate how many minutes after midnight January 1st it takes for 100 babies to be born.

46. MODELING WITH MATHEMATICS The amount of money a movie earns each week after its release can be approximated by the sequence shown in the graph.



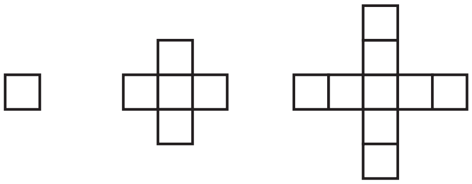
a. Write a function that represents the arithmetic sequence.

b. In what week does the movie earn $16 million?

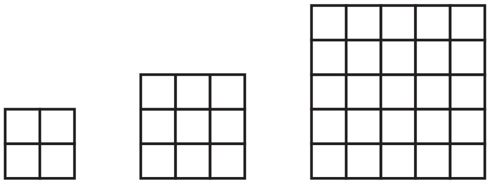
c. How much money does the movie earn overall?

MATHEMATICAL CONNECTIONS **In Exercises 47 and 48, each small square represents 1 square inch. Determine whether the areas of the figures form an arithmetic sequence. If so, write a function f that represents the arithmetic sequence and find $f(30)$.**

47.



48.



49. **REASONING** Is the domain of an arithmetic sequence discrete or continuous? Is the range of an arithmetic sequence discrete or continuous?

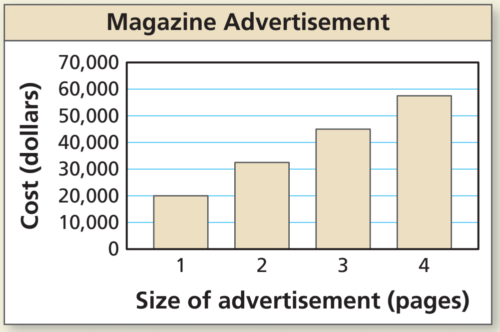
50. **MAKING AN ARGUMENT** Your friend says that the range of a function that represents an arithmetic sequence always contains only positive numbers or only negative numbers. Your friend claims this is true because the domain is the set of positive integers and the output values either constantly increase or constantly decrease. Is your friend correct? Explain.

51. **OPEN-ENDED** Write the first four terms of two different arithmetic sequences with a common difference of -3. Write an equation for the nth term of each sequence.

52. **THOUGHT PROVOKING** Describe an arithmetic sequence that models the numbers of people in a real-life situation.

53. **REPEATED REASONING** Firewood is stacked in a pile. The bottom row has 20 logs, and the top row has 14 logs. Each row has one more log than the row above it. How many logs are in the pile?

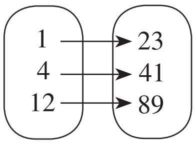
54. **HOW DO YOU SEE IT?** The bar graph shows the costs of advertising in a magazine.



a. Does the graph represent an arithmetic sequence? Explain.

b. Explain how you would estimate the cost of a six-page advertisement in the magazine.

55. **REASONING** Write a function f that represents the arithmetic sequence shown in the mapping diagram.



56. **PROBLEM SOLVING** A train stops at a station every 12 minutes starting at 6:00 A.M. You arrive at the station at 7:29 A.M. How long must you wait for the train?

57. **ABSTRACT REASONING** Let x be a constant. Determine whether each sequence is an arithmetic sequence. Explain.

a. $x + 6, 3x + 6, 5x + 6, 7x + 6, \dots$

b. $x + 1, 3x + 1, 9x + 1, 27x + 1, \dots$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution. *(Section 2.2)*

58. $x + 8 \geq -9$ **59.** $15 < b - 4$ **60.** $t - 21 < -12$ **61.** $7 + y \leq 3$

Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range. *(Section 3.7)*

62. $h(x) = 3|x|$ **63.** $v(x) = |x - 5|$

64. $g(x) = |x| + 1$ **65.** $r(x) = -2|x|$

4.7 Piecewise Functions

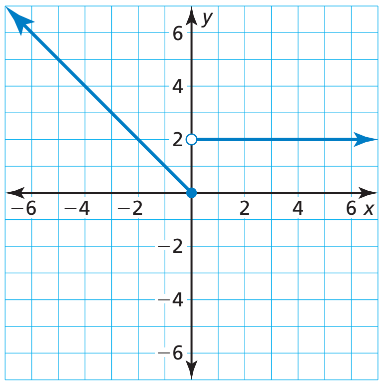
Essential Question How can you describe a function that is represented by more than one equation?

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to justify your conclusions and communicate them to others.

EXPLORATION 1 Writing Equations for a Function

Work with a partner.



a. Does the graph represent y as a function of x? Justify your conclusion.

b. What is the value of the function when $x = 0$? How can you tell?

c. Write an equation that represents the values of the function when $x \leq 0$.

$f(x) =$ ______, if $x \leq 0$

d. Write an equation that represents the values of the function when $x > 0$.

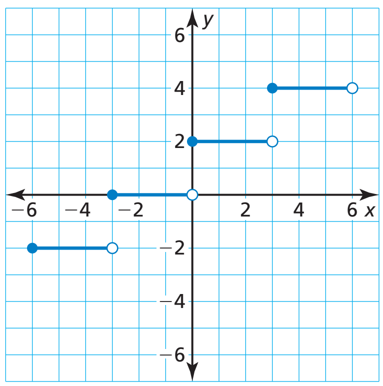
$f(x) =$ ______, if $x > 0$

e. Combine the results of parts (c) and (d) to write a single description of the function.

$$f(x) = \begin{cases} \_\_\_\_\_\_, & \text{if } x \leq 0 \\ \_\_\_\_\_\_, & \text{if } x > 0 \end{cases}$$

EXPLORATION 2 Writing Equations for a Function

Work with a partner.



a. Does the graph represent y as a function of x? Justify your conclusion.

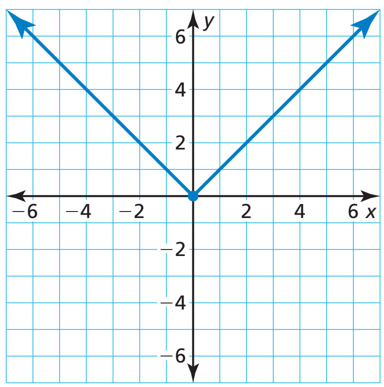
b. Describe the values of the function for the following intervals.

$$f(x) = \begin{cases} \_\_\_\_\_\_, & \text{if } -6 \leq x < -3 \\ \_\_\_\_\_\_, & \text{if } -3 \leq x < 0 \\ \_\_\_\_\_\_, & \text{if } 0 \leq x < 3 \\ \_\_\_\_\_\_, & \text{if } 3 \leq x < 6 \end{cases}$$

Communicate Your Answer

3. How can you describe a function that is represented by more than one equation?

4. Use two equations to describe the function represented by the graph.



4.7 Lesson

Core Vocabulary

piecewise function, *p. 218*
step function, *p. 220*

Previous
absolute value function
vertex form
vertex

What You Will Learn

- Evaluate piecewise functions.
- Graph and write piecewise functions.
- Graph and write step functions.
- Write absolute value functions.

Evaluating Piecewise Functions

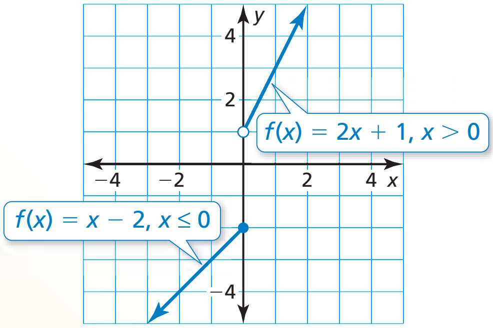
Core Concept

Piecewise Function

A **piecewise function** is a function defined by two or more equations. Each "piece" of the function applies to a different part of its domain. An example is shown below.

$$f(x) = \begin{cases} x - 2, & \text{if } x \le 0 \\ 2x + 1, & \text{if } x > 0 \end{cases}$$

- The expression $x - 2$ represents the value of f when x is less than or equal to 0.
- The expression $2x + 1$ represents the value of f when x is greater than 0.



EXAMPLE 1 Evaluating a Piecewise Function

Evaluate the function f above when (a) $x = 0$ and (b) $x = 4$.

SOLUTION

a. $f(x) = x - 2$ — Because $0 \le 0$, use the first equation.

$f(0) = 0 - 2$ — Substitute 0 for x.

$f(0) = -2$ — Simplify.

▶ The value of f is -2 when $x = 0$.

b. $f(x) = 2x + 1$ — Because $4 > 0$, use the second equation.

$f(4) = 2(4) + 1$ — Substitute 4 for x.

$f(4) = 9$ — Simplify.

▶ The value of f is 9 when $x = 4$.

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Evaluate the function.

$$f(x) = \begin{cases} 3, & \text{if } x < -2 \\ x + 2, & \text{if } -2 \le x \le 5 \\ 4x, & \text{if } x > 5 \end{cases}$$

1. $f(-8)$
2. $f(-2)$
3. $f(0)$
4. $f(3)$
5. $f(5)$
6. $f(10)$

Graphing and Writing Piecewise Functions

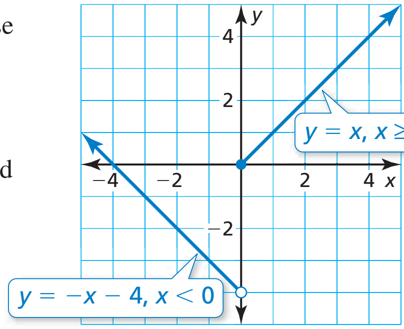
EXAMPLE 2 Graphing a Piecewise Function

Graph $y = \begin{cases} -x - 4, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$. Describe the domain and range.

SOLUTION

Step 1 Graph $y = -x - 4$ for $x < 0$. Because x is not equal to 0, use an open circle at $(0, -4)$.

Step 2 Graph $y = x$ for $x \geq 0$. Because x is greater than or equal to 0, use a closed circle at $(0, 0)$.



▶ The domain is all real numbers. The range is $y > -4$.

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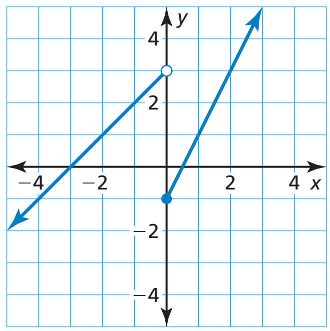
Graph the function. Describe the domain and range.

7. $y = \begin{cases} x + 1, & \text{if } x \leq 0 \\ -x, & \text{if } x > 0 \end{cases}$

8. $y = \begin{cases} x - 2, & \text{if } x < 0 \\ 4x, & \text{if } x \geq 0 \end{cases}$

EXAMPLE 3 Writing a Piecewise Function

Write a piecewise function for the graph.



SOLUTION

Each "piece" of the function is linear.

Left Piece When $x < 0$, the graph is the line given by $y = x + 3$.

Right Piece When $x \geq 0$, the graph is the line given by $y = 2x - 1$.

▶ So, a piecewise function for the graph is

$$f(x) = \begin{cases} x + 3, & \text{if } x < 0 \\ 2x - 1, & \text{if } x \geq 0 \end{cases}.$$

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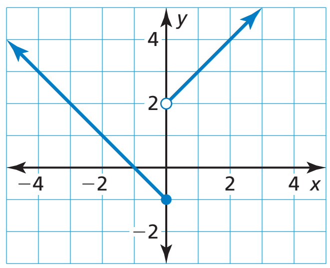
Write a piecewise function for the graph.

9.



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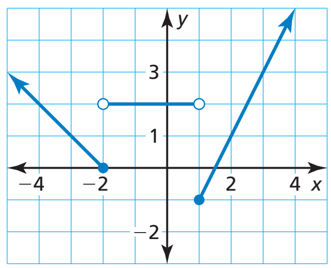
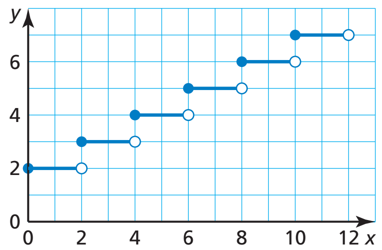


Graphing and Writing Step Functions

A **step function** is a piecewise function defined by a constant value over each part of its domain. The graph of a step function consists of a series of line segments.

STUDY TIP

The graph of a step function looks like a staircase.



$$f(x) = \begin{cases} 2, & \text{if } 0 \le x < 2 \\ 3, & \text{if } 2 \le x < 4 \\ 4, & \text{if } 4 \le x < 6 \\ 5, & \text{if } 6 \le x < 8 \\ 6, & \text{if } 8 \le x < 10 \\ 7, & \text{if } 10 \le x < 12 \end{cases}$$

EXAMPLE 4 Graphing and Writing a Step Function

You rent a karaoke machine for 5 days. The rental company charges \$50 for the first day and \$25 for each additional day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the karaoke machine.

SOLUTION

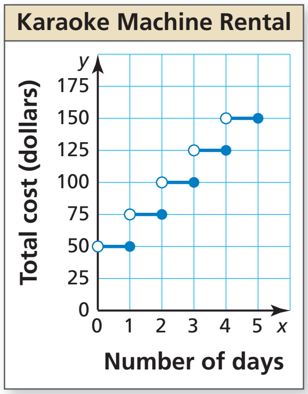
Step 1 Use a table to organize the information.

Number of days	Total cost (dollars)
$0 < x \le 1$	50
$1 < x \le 2$	75
$2 < x \le 3$	100
$3 < x \le 4$	125
$4 < x \le 5$	150

Step 2 Write the step function.

$$f(x) = \begin{cases} 50, & \text{if } 0 < x \le 1 \\ 75, & \text{if } 1 < x \le 2 \\ 100, & \text{if } 2 < x \le 3 \\ 125, & \text{if } 3 < x \le 4 \\ 150, & \text{if } 4 < x \le 5 \end{cases}$$

Step 3 Graph the step function.



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11. A landscaper rents a wood chipper for 4 days. The rental company charges \$100 for the first day and \$50 for each additional day. Write and graph a step function that represents the relationship between the number x of days and the total cost y (in dollars) of renting the chipper.

Writing Absolute Value Functions

The absolute value function $f(x) = |x|$ can be written as a piecewise function.

$$f(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Similarly, the vertex form of an absolute value function $g(x) = a|x - h| + k$ can be written as a piecewise function.

$$g(x) = \begin{cases} a[-(x - h)] + k, & \text{if } x - h < 0 \\ a(x - h) + k, & \text{if } x - h \geq 0 \end{cases}$$

REMEMBER

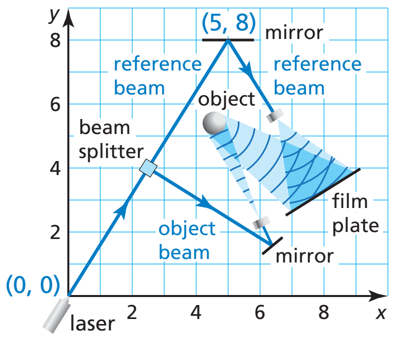
The vertex form of an absolute value function is $g(x) = a|x - h| + k$, where $a \neq 0$. The vertex of the graph of g is (h, k).

EXAMPLE 5 Writing an Absolute Value Function

In holography, light from a laser beam is split into two beams, a reference beam and an object beam. Light from the object beam reflects off an object and is recombined with the reference beam to form images on film that can be used to create three-dimensional images.

a. Write an absolute value function that represents the path of the reference beam.

b. Write the function in part (a) as a piecewise function.



SOLUTION

a. The vertex of the path of the reference beam is (5, 8). So, the function has the form $g(x) = a|x - 5| + 8$. Substitute the coordinates of the point (0, 0) into the equation and solve for a.

$g(x) = a|x - 5| + 8$ — Vertex form of the function

$0 = a|0 - 5| + 8$ — Substitute 0 for x and 0 for $g(x)$.

$-1.6 = a$ — Solve for a.

▶ So, the function $g(x) = -1.6|x - 5| + 8$ represents the path of the reference beam.

b. Write $g(x) = -1.6|x - 5| + 8$ as a piecewise function.

$$g(x) = \begin{cases} -1.6[-(x - 5)] + 8, & \text{if } x - 5 < 0 \\ -1.6(x - 5) + 8, & \text{if } x - 5 \geq 0 \end{cases}$$

Simplify each expression and solve the inequalities.

▶ So, a piecewise function for $g(x) = -1.6|x - 5| + 8$ is

$$g(x) = \begin{cases} 1.6x, & \text{if } x < 5 \\ -1.6x + 16, & \text{if } x \geq 5 \end{cases}.$$

STUDY TIP

Recall that the graph of an absolute value function is symmetric about the line $x = h$. So, it makes sense that the piecewise definition "splits" the function at $x = 5$.

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12. WHAT IF? The reference beam originates at (3, 0) and reflects off a mirror at (5, 4).

a. Write an absolute value function that represents the path of the reference beam.

b. Write the function in part (a) as a piecewise function.

4.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Compare piecewise functions and step functions.

2. **WRITING** Use a graph to explain why you can write the absolute value function $y = |x|$ as a piecewise function.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, evaluate the function. *(See Example 1.)*

$$f(x) = \begin{cases} 5x - 1, & \text{if } x < -2 \\ x + 3, & \text{if } x \geq -2 \end{cases}$$

$$g(x) = \begin{cases} -x + 4, & \text{if } x \leq -1 \\ 3, & \text{if } -1 < x < 2 \\ 2x - 5, & \text{if } x \geq 2 \end{cases}$$

3. $f(-3)$

4. $f(-2)$

5. $f(0)$

6. $f(5)$

7. $g(-4)$

8. $g(-1)$

9. $g(0)$

10. $g(1)$

11. $g(2)$

12. $g(5)$

13. **MODELING WITH MATHEMATICS** On a trip, the total distance (in miles) you travel in x hours is represented by the piecewise function

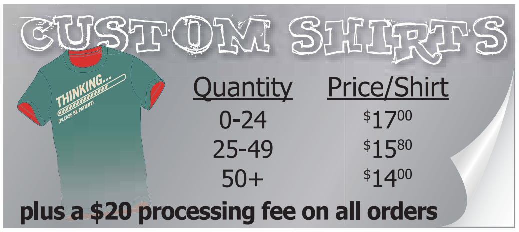
$$d(x) = \begin{cases} 55x, & \text{if } 0 \leq x \leq 2 \\ 65x - 20, & \text{if } 2 < x \leq 5 \end{cases}.$$

How far do you travel in 4 hours?

14. **MODELING WITH MATHEMATICS** The total cost (in dollars) of ordering x custom shirts is represented by the piecewise function

$$c(x) = \begin{cases} 17x + 20, & \text{if } 0 \leq x < 25 \\ 15.80x + 20, & \text{if } 25 \leq x < 50 \\ 14x + 20, & \text{if } x \geq 50 \end{cases}.$$

Determine the total cost of ordering 26 shirts.



In Exercises 15–20, graph the function. Describe the domain and range. *(See Example 2.)*

15. $y = \begin{cases} -x, & \text{if } x < 2 \\ x - 6, & \text{if } x \geq 2 \end{cases}$

16. $y = \begin{cases} 2x, & \text{if } x \leq -3 \\ -2x, & \text{if } x > -3 \end{cases}$

17. $y = \begin{cases} -3x - 2, & \text{if } x \leq -1 \\ x + 2, & \text{if } x > -1 \end{cases}$

18. $y = \begin{cases} x + 8, & \text{if } x < 4 \\ 4x - 4, & \text{if } x \geq 4 \end{cases}$

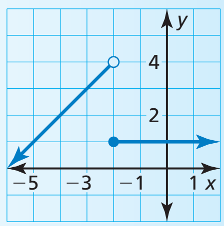
19. $y = \begin{cases} 1, & \text{if } x < -3 \\ x - 1, & \text{if } -3 \leq x \leq 3 \\ -2x + 4, & \text{if } x > 3 \end{cases}$

20. $y = \begin{cases} 2x + 1, & \text{if } x \leq -1 \\ -x + 2, & \text{if } -1 < x < 2 \\ -3, & \text{if } x \geq 2 \end{cases}$

21. **ERROR ANALYSIS** Describe and correct the error in finding $f(5)$ when $f(x) = \begin{cases} 2x - 3, & \text{if } x < 5 \\ x + 8, & \text{if } x \geq 5 \end{cases}$.

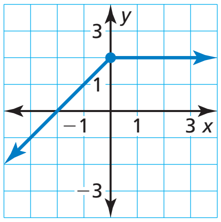
$$f(5) = 2(5) - 3$$
$$= 7$$

22. **ERROR ANALYSIS** Describe and correct the error in graphing $y = \begin{cases} x + 6, & \text{if } x \leq -2 \\ 1, & \text{if } x > -2 \end{cases}$.



In Exercises 23–30, write a piecewise function for the graph. *(See Example 3.)*

23.



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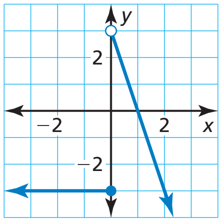

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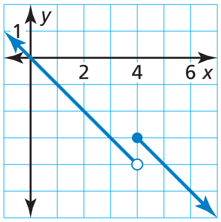

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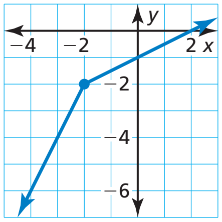

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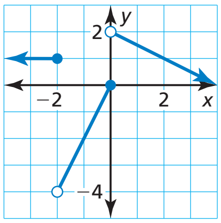

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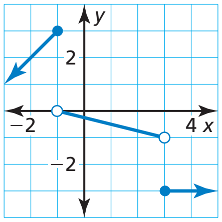

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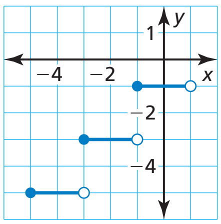

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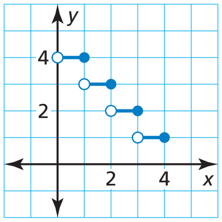


In Exercises 31–34, graph the step function. Describe the domain and range.

31. $f(x) = \begin{cases} 3, & \text{if } 0 \le x < 2 \\ 4, & \text{if } 2 \le x < 4 \\ 5, & \text{if } 4 \le x < 6 \\ 6, & \text{if } 6 \le x < 8 \end{cases}$

32. $f(x) = \begin{cases} -4, & \text{if } 1 < x \le 2 \\ -6, & \text{if } 2 < x \le 3 \\ -8, & \text{if } 3 < x \le 4 \\ -10, & \text{if } 4 < x \le 5 \end{cases}$

33. $f(x) = \begin{cases} 9, & \text{if } 1 < x \le 2 \\ 6, & \text{if } 2 < x \le 4 \\ 5, & \text{if } 4 < x \le 9 \\ 1, & \text{if } 9 < x \le 12 \end{cases}$

34. $f(x) = \begin{cases} -2, & \text{if } -6 \le x < -5 \\ -1, & \text{if } -5 \le x < -3 \\ 0, & \text{if } -3 \le x < -2 \\ 1, & \text{if } -2 \le x < 0 \end{cases}$

35. **MODELING WITH MATHEMATICS** The cost to join an intramural sports league is $180 per team and includes the first five team members. For each additional team member, there is a $30 fee. You plan to have nine people on your team. Write and graph a step function that represents the relationship between the number p of people on your team and the total cost of joining the league. *(See Example 4.)*

36. **MODELING WITH MATHEMATICS** The rates for a parking garage are shown. Write and graph a step function that represents the relationship between the number x of hours a car is parked in the garage and the total cost of parking in the garage for 1 day.

In Exercises 37–46, write the absolute value function as a piecewise function.

37. $y = |x| + 1$

38. $y = |x| - 3$

39. $y = |x - 2|$

40. $y = |x + 5|$

41. $y = 2|x + 3|$

42. $y = 4|x - 1|$

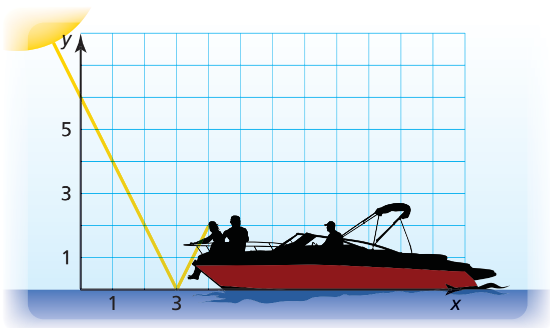
43. $y = -5|x - 8|$

44. $y = -3|x + 6|$

45. $y = -|x - 3| + 2$

46. $y = 7|x + 1| - 5$

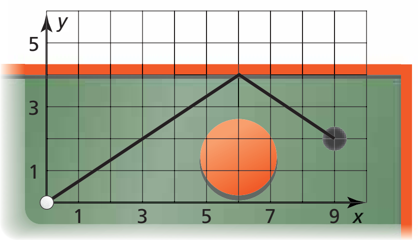
47. **MODELING WITH MATHEMATICS** You are sitting on a boat on a lake. You can get a sunburn from the sunlight that hits you directly and also from the sunlight that reflects off the water. *(See Example 5.)*



a. Write an absolute value function that represents the path of the sunlight that reflects off the water.

b. Write the function in part (a) as a piecewise function.

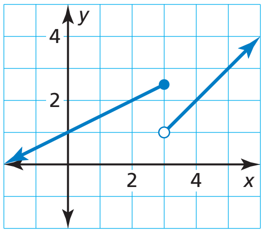
48. MODELING WITH MATHEMATICS You are trying to make a hole in one on the miniature golf green.



a. Write an absolute value function that represents the path of the golf ball.

b. Write the function in part (a) as a piecewise function.

49. REASONING The piecewise function f consists of two linear "pieces." The graph of f is shown.



a. What is the value of $f(-10)$?

b. What is the value of $f(8)$?

50. CRITICAL THINKING Describe how the graph of each piecewise function changes when < is replaced with ≤ and ≥ is replaced with >. Do the domain and range change? Explain.

a. $f(x) = \begin{cases} x + 2, & \text{if } x < 2 \\ -x - 1, & \text{if } x \geq 2 \end{cases}$

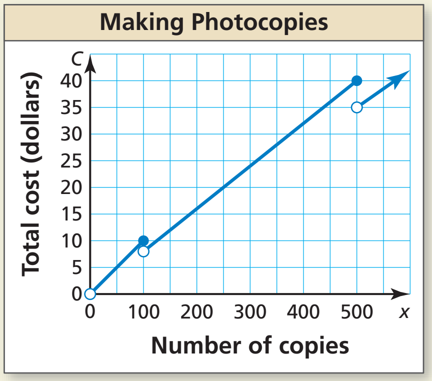
b. $f(x) = \begin{cases} \frac{1}{2}x + \frac{3}{2}, & \text{if } x < 1 \\ -x + 3, & \text{if } x \geq 1 \end{cases}$

51. USING STRUCTURE Graph

$$y = \begin{cases} -x + 2, & \text{if } x \leq -2 \\ |x|, & \text{if } x > -2 \end{cases}.$$

Describe the domain and range.

52. HOW DO YOU SEE IT? The graph shows the total cost C of making x photocopies at a copy shop.



a. Does it cost more money to make 100 photocopies or 101 photocopies? Explain.

b. You have $40 to make photocopies. Can you buy more than 500 photocopies? Explain.

53. USING STRUCTURE The output y of the *greatest integer function* is the greatest integer less than or equal to the input value x. This function is written as $f(x) = [\![x]\!]$. Graph the function for $-4 \leq x < 4$. Is it a piecewise function? a step function? Explain.

54. THOUGHT PROVOKING Explain why

$$y = \begin{cases} 2x - 2, & \text{if } x \leq 3 \\ -3, & \text{if } x \geq 3 \end{cases}$$

does not represent a function. How can you redefine y so that it does represent a function?

55. MAKING AN ARGUMENT During a 9-hour snowstorm, it snows at a rate of 1 inch per hour for the first 2 hours, 2 inches per hour for the next 6 hours, and 1 inch per hour for the final hour.

a. Write and graph a piecewise function that represents the depth of the snow during the snowstorm.

b. Your friend says 12 inches of snow accumulated during the storm. Is your friend correct? Explain.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. Graph the inequality. *(Section 2.5)*

56. A number r is greater than -12 and no more than 13.

57. A number t is less than or equal to 4 or no less than 18.

Graph f and h. Describe the transformations from the graph of f to the graph of h. *(Section 3.6)*

58. $f(x) = x$; $h(x) = 4x + 3$

59. $f(x) = x$; $h(x) = -x - 8$

60. $f(x) = x$; $h(x) = -\frac{1}{2}x + 5$