3 Graphing Linear Functions

Submersible *(p. 140)*

Basketball *(p. 134)*

Speed of Light *(p. 125)*

Coins *(p. 116)*

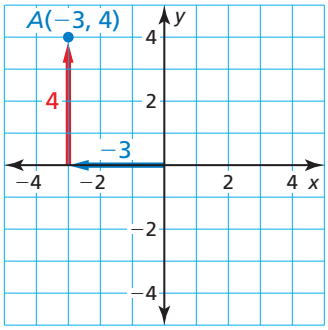
Taxi Ride *(p. 109)*

Maintaining Mathematical Proficiency

Plotting Points

Example 1 **Plot the point $A(-3, 4)$ in a coordinate plane. Describe the location of the point.**

Start at the origin. Move 3 units left and 4 units up. Then plot the point. The point is in Quadrant II.



Plot the point in a coordinate plane. Describe the location of the point.

1. $A(3, 2)$
2. $B(-5, 1)$
3. $C(0, 3)$
4. $D(-1, -4)$
5. $E(-3, 0)$
6. $F(2, -1)$

Evaluating Expressions

Example 2 **Evaluate $4x - 5$ when $x = 3$.**

$$\begin{aligned} 4x - 5 &= 4(3) - 5 && \text{Substitute 3 for } x. \\ &= 12 - 5 && \text{Multiply.} \\ &= 7 && \text{Subtract.} \end{aligned}$$

Example 3 **Evaluate $-2x + 9$ when $x = -8$.**

$$\begin{aligned} -2x + 9 &= -2(-8) + 9 && \text{Substitute } -8 \text{ for } x. \\ &= 16 + 9 && \text{Multiply.} \\ &= 25 && \text{Add.} \end{aligned}$$

Evaluate the expression for the given value of x.

7. $3x - 4; x = 7$
8. $-5x + 8; x = 3$
9. $10x + 18; x = 5$
10. $-9x - 2; x = -4$
11. $24 - 8x; x = -2$
12. $15x + 9; x = -1$

13. **ABSTRACT REASONING** Let a and b be positive real numbers. Describe how to plot (a, b), $(-a, b)$, $(a, -b)$, and $(-a, -b)$.

Mathematical Practices

Mathematically proficient students use technological tools to explore concepts.

Using a Graphing Calculator

Core Concept

Standard and Square Viewing Windows

A typical graphing calculator screen has a height to width ratio of 2 to 3. This means that when you use the *standard viewing window* of -10 to 10 (on each axis), the graph will not be in its true perspective.

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
```

This is the standard viewing window.

To see a graph in its true perspective, you need to use a *square viewing window*, in which the tick marks on the x-axis are spaced the same as the tick marks on the y-axis.

```
WINDOW
Xmin=-9
Xmax=9
Xscl=1
Ymin=-6
Ymax=6
Yscl=1
```

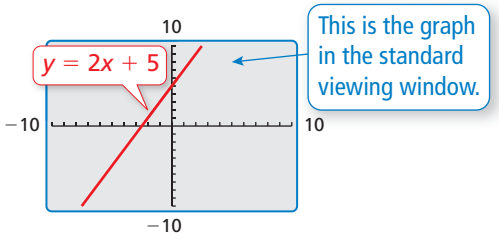
This is a square viewing window.

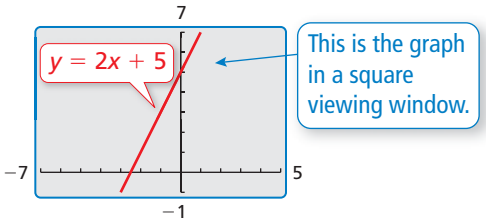
EXAMPLE 1 Using a Graphing Calculator

Use a graphing calculator to graph $y = 2x + 5$.

SOLUTION

Enter the equation $y = 2x + 5$ into your calculator. Then graph the equation. The standard viewing window does not show the graph in its true perspective. Notice that the tick marks on the y-axis are closer together than the tick marks on the x-axis. To see the graph in its true perspective, use a square viewing window.





Monitoring Progress

Determine whether the viewing window is square. Explain.

1. $-8 \le x \le 7, -3 \le y \le 7$
2. $-6 \le x \le 6, -9 \le y \le 9$
3. $-18 \le x \le 18, -12 \le y \le 12$

Use a graphing calculator to graph the equation. Use a square viewing window.

4. $y = x + 3$
5. $y = -x - 2$
6. $y = 2x - 1$
7. $y = -2x + 1$
8. $y = -\frac{1}{3}x - 4$
9. $y = \frac{1}{2}x + 2$
10. How does the appearance of the slope of a line change between a standard viewing window and a square viewing window?

3.1 Functions

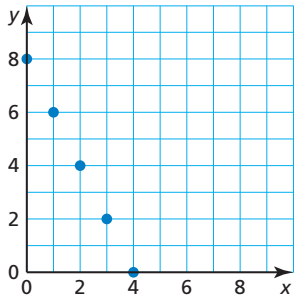
Essential Question What is a function?

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the x-coordinates are inputs and the y-coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

EXPLORATION 1 Describing a Function

Work with a partner. Functions can be described in many ways.

- by an equation
- by an input-output table
- using words
- by a graph
- as a set of ordered pairs



a. Explain why the graph shown represents a function.

b. Describe the function in two other ways.

ANALYZING RELATIONSHIPS

To be proficient in math, you need to analyze relationships mathematically to draw conclusions.

EXPLORATION 2 Identifying Functions

Work with a partner. Determine whether each relation represents a function. Explain your reasoning.

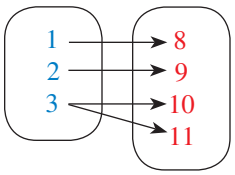
a.

Input, x	0	1	2	3	4
Output, y	8	8	8	8	8

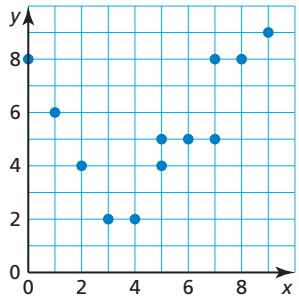
b.

Input, x	8	8	8	8	8
Output, y	0	1	2	3	4

c.



d.



e. $(-2, 5), (-1, 8), (0, 6), (1, 6), (2, 7)$

f. $(-2, 0), (-1, 0), (-1, 1), (0, 1), (1, 2), (2, 2)$

g. Each radio frequency x in a listening area has exactly one radio station y.

h. The same television station x can be found on more than one channel y.

i. $x = 2$

j. $y = 2x + 3$

Communicate Your Answer

3. What is a function? Give examples of relations, other than those in Explorations 1 and 2, that (a) are functions and (b) are not functions.

3.1 Lesson

Core Vocabulary

relation, *p. 104*
function, *p. 104*
domain, *p. 106*
range, *p. 106*
independent variable, *p. 107*
dependent variable, *p. 107*

Previous
ordered pair
mapping diagram

What You Will Learn

- Determine whether relations are functions.
- Find the domain and range of a function.
- Identify the independent and dependent variables of functions.

Determining Whether Relations Are Functions

A **relation** pairs inputs with outputs. When a relation is given as ordered pairs, the x-coordinates are inputs and the y-coordinates are outputs. A relation that pairs each input with *exactly one* output is a **function**.

EXAMPLE 1 Determining Whether Relations Are Functions

Determine whether each relation is a function. Explain.

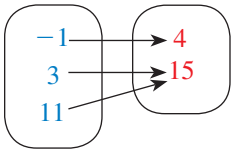
a. $(-2, 2), (-1, 2), (0, 2), (1, 0), (2, 0)$

b. $(4, 0), (8, 7), (6, 4), (4, 3), (5, 2)$

c.

Input, x	-2	-1	0	0	1	2
Output, y	3	4	5	6	7	8

d. Input, x Output, y



REMEMBER
A relation can be represented by a mapping diagram.

SOLUTION

a. Every input has exactly one output.

▶ So, the relation is a function.

b. The input 4 has two outputs, 0 and 3.

▶ So, the relation is *not* a function.

c. The input 0 has two outputs, 5 and 6.

▶ So, the relation is *not* a function.

d. Every input has exactly one output.

▶ So, the relation is a function.

Monitoring Progress

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Determine whether the relation is a function. Explain.

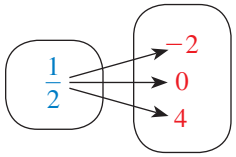
1. $(-5, 0), (0, 0), (5, 0), (5, 10)$

2. $(-4, 8), (-1, 2), (2, -4), (5, -10)$

3.

Input, x	Output, y
2	2.6
4	5.2
6	7.8

4. Input, x Output, y

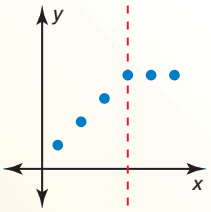


Core Concept

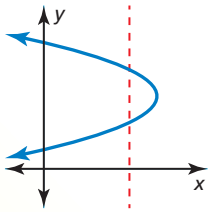
Vertical Line Test

Words A graph represents a function when no vertical line passes through more than one point on the graph.

Examples Function



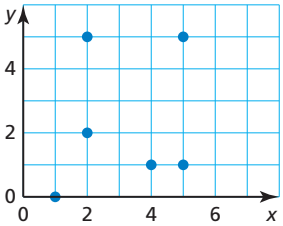
Not a function



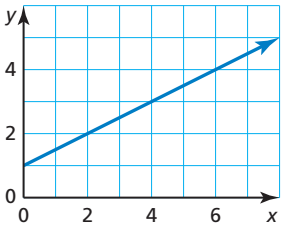
EXAMPLE 2 Using the Vertical Line Test

Determine whether each graph represents a function. Explain.

a.



b.



SOLUTION

a. You can draw a vertical line through (2, 2) and (2, 5).

▶ So, the graph does *not* represent a function.

b. No vertical line can be drawn through more than one point on the graph.

▶ So, the graph represents a function.

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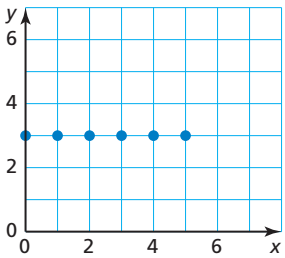
Determine whether the graph represents a function. Explain.

5.



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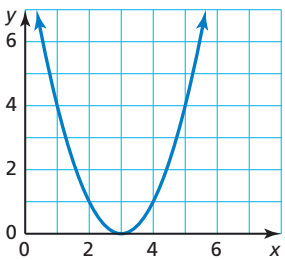

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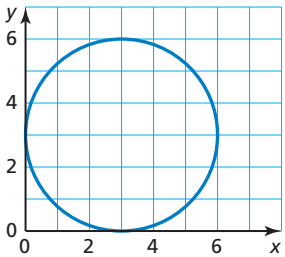

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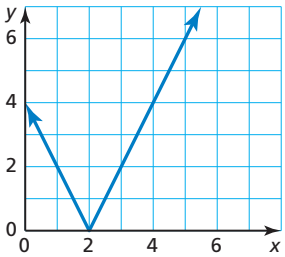


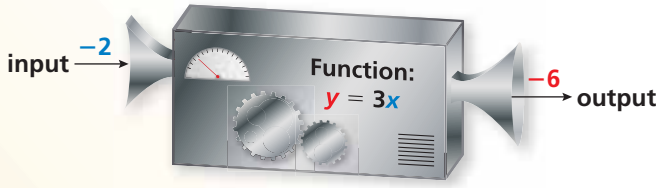
Finding the Domain and Range of a Function

Core Concept

The Domain and Range of a Function

The **domain** of a function is the set of all possible input values.

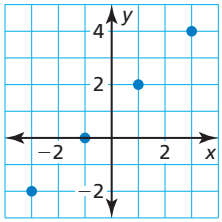
The **range** of a function is the set of all possible output values.



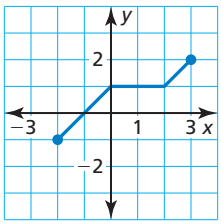
EXAMPLE 3 Finding the Domain and Range from a Graph

Find the domain and range of the function represented by the graph.

a.

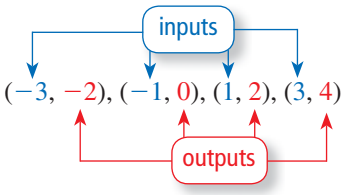


b.



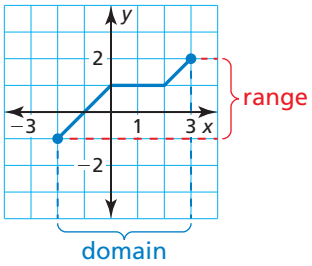
SOLUTION

a. Write the ordered pairs. Identify the inputs and outputs.



▶ The domain is −3, −1, 1, and 3.
The range is −2, 0, 2, and 4.

b. Identify the x- and y-values represented by the graph.



▶ The domain is $-2 \le x \le 3$.
The range is $-1 \le y \le 2$.

STUDY TIP

A relation also has a domain and a range.

Monitoring Progress

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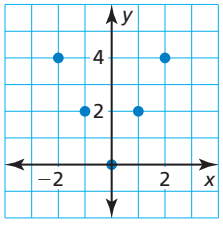
Find the domain and range of the function represented by the graph.

9.



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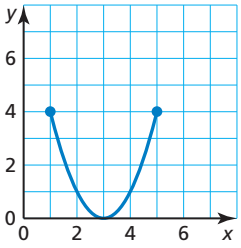

Identifying Independent and Dependent Variables

The variable that represents the input values of a function is the **independent variable** because it can be *any* value in the domain. The variable that represents the output values of a function is the **dependent variable** because it *depends* on the value of the independent variable. When an equation represents a function, the dependent variable is defined in terms of the independent variable. The statement "y is a function of x" means that y varies depending on the value of x.

$$y = -x + 10$$

dependent variable, y — independent variable, x

EXAMPLE 4 Identifying Independent and Dependent Variables

The function $y = -3x + 12$ represents the amount y (in fluid ounces) of juice remaining in a bottle after you take x gulps.

a. Identify the independent and dependent variables.

b. The domain is 0, 1, 2, 3, and 4. What is the range?

SOLUTION

a. The amount y of juice remaining depends on the number x of gulps.

▶ So, y is the dependent variable, and x is the independent variable.

b. Make an input-output table to find the range.

Input, x	$-3x + 12$	Output, y
0	$-3(0) + 12$	12
1	$-3(1) + 12$	9
2	$-3(2) + 12$	6
3	$-3(3) + 12$	3
4	$-3(4) + 12$	0

▶ The range is 12, 9, 6, 3, and 0.

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11. The function $a = -4b + 14$ represents the number a of avocados you have left after making b batches of guacamole.

a. Identify the independent and dependent variables.

b. The domain is 0, 1, 2, and 3. What is the range?

12. The function $t = 19m + 65$ represents the temperature t (in degrees Fahrenheit) of an oven after preheating for m minutes.

a. Identify the independent and dependent variables.

b. A recipe calls for an oven temperature of 350°F. Describe the domain and range of the function.

3.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. WRITING How are independent variables and dependent variables different?

2. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

Find the range of the function represented by the table.

Find the inputs of the function represented by the table.

x	-1	0	1
y	7	5	-1

Find the x-values of the function represented by $(-1, 7)$, $(0, 5)$, and $(1, -1)$.

Find the domain of the function represented by $(-1, 7)$, $(0, 5)$, and $(1, -1)$.

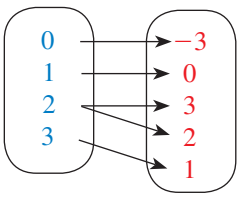
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, determine whether the relation is a function. Explain. *(See Example 1.)*

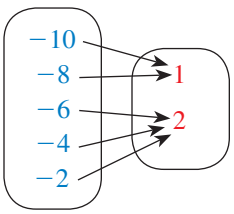
3. $(1, -2)$, $(2, 1)$, $(3, 6)$, $(4, 13)$, $(5, 22)$

4. $(7, 4)$, $(5, -1)$, $(3, -8)$, $(1, -5)$, $(3, 6)$

5. Input, x Output, y



6. Input, x Output, y



7.

Input, x	16	1	0	1	16
Output, y	-2	-1	0	1	2

8.

Input, x	-3	0	3	6	9
Output, y	11	5	-1	-7	-13

In Exercises 9–12, determine whether the graph represents a function. Explain. *(See Example 2.)*

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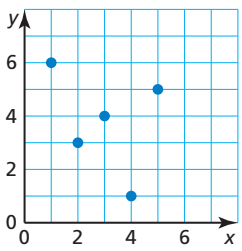

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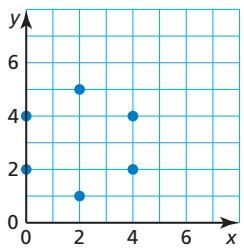

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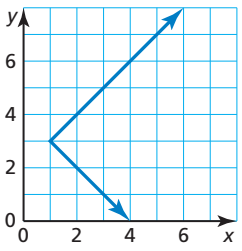

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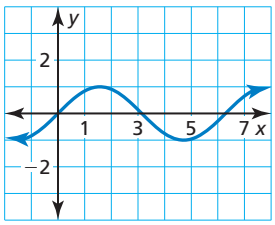

In Exercises 13–16, find the domain and range of the function represented by the graph. *(See Example 3.)*

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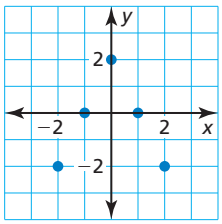

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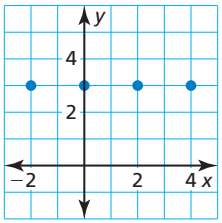

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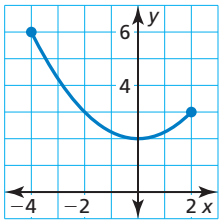

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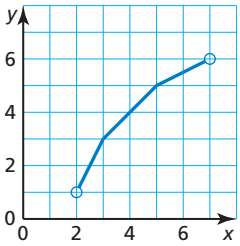


17. MODELING WITH MATHEMATICS The function $y = 25x + 500$ represents your monthly rent y (in dollars) when you pay x days late. *(See Example 4.)*

a. Identify the independent and dependent variables.

b. The domain is 0, 1, 2, 3, 4, and 5. What is the range?

18. MODELING WITH MATHEMATICS The function $y = 3.5x + 2.8$ represents the cost y (in dollars) of a taxi ride of x miles.

a. Identify the independent and dependent variables.

b. You have enough money to travel at most 20 miles in the taxi. Find the domain and range of the function.

ERROR ANALYSIS In Exercises 19 and 20, describe and correct the error in the statement about the relation shown in the table.

Input, x	1	2	3	4	5
Output, y	6	7	8	6	9

19. ✗ The relation is *not* a function. One output is paired with two inputs.

20. ✗ The relation is a function. The range is 1, 2, 3, 4, and 5.

ANALYZING RELATIONSHIPS In Exercises 21 and 22, identify the independent and dependent variables.

21. The number of quarters you put into a parking meter affects the amount of time you have on the meter.

22. The battery power remaining on your MP3 player is based on the amount of time you listen to it.

23. MULTIPLE REPRESENTATIONS The balance y (in dollars) of your savings account is a function of the month x.

Month, x	0	1	2	3	4
Balance (dollars), y	100	125	150	175	200

a. Describe this situation in words.

b. Write the function as a set of ordered pairs.

c. Plot the ordered pairs in a coordinate plane.

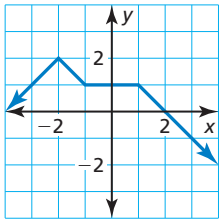
24. MULTIPLE REPRESENTATIONS The function $1.5x + 0.5y = 12$ represents the number of hardcover books x and softcover books y you can buy at a used book sale.

a. Solve the equation for y.

b. Make an input-output table to find ordered pairs for the function.

c. Plot the ordered pairs in a coordinate plane.

25. ATTENDING TO PRECISION The graph represents a function. Find the input value corresponding to an output of 2.



26. OPEN-ENDED Fill in the table so that when t is the independent variable, the relation is a function, and when t is the dependent variable, the relation is not a function.

t				
v				

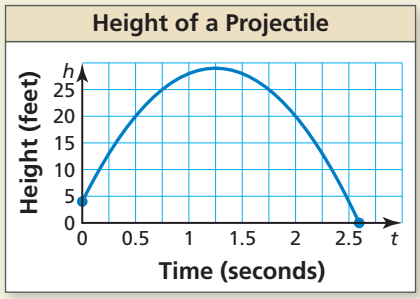
27. ANALYZING RELATIONSHIPS You select items in a vending machine by pressing one letter and then one number.

a. Explain why the relation that pairs letter-number combinations with food or drink items is a function.

b. Identify the independent and dependent variables.

c. Find the domain and range of the function.

28. HOW DO YOU SEE IT? The graph represents the height h of a projectile after t seconds.



a. Explain why h is a function of t.

b. Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

c. Approximate the domain of the function.

d. Is t a function of h? Explain.

29. MAKING AN ARGUMENT Your friend says that a line always represents a function. Is your friend correct? Explain.

30. THOUGHT PROVOKING Write a function in which the inputs and/or the outputs are not numbers. Identify the independent and dependent variables. Then find the domain and range of the function.

ATTENDING TO PRECISION In Exercises 31–34, determine whether the statement uses the word *function* in a way that is mathematically correct. Explain your reasoning.

31. The selling price of an item is a function of the cost of making the item.

32. The sales tax on a purchased item in a given state is a function of the selling price.

33. A function pairs each student in your school with a homeroom teacher.

34. A function pairs each chaperone on a school trip with 10 students.

REASONING In Exercises 35–38, tell whether the statement is true or false. If it is false, explain why.

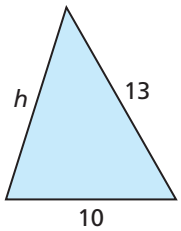
35. Every function is a relation.

36. Every relation is a function.

37. When you switch the inputs and outputs of any function, the resulting relation is a function.

38. When the domain of a function has an infinite number of values, the range always has an infinite number of values.

39. MATHEMATICAL CONNECTIONS Consider the triangle shown.



a. Write a function that represents the perimeter of the triangle.

b. Identify the independent and dependent variables.

c. Describe the domain and range of the function. (*Hint:* The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.)

REASONING In Exercises 40–43, find the domain and range of the function.

40. $y = |x|$

41. $y = -|x|$

42. $y = |x| - 6$

43. $y = 4 - |x|$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the sentence as an inequality. *(Section 2.1)*

44. A number y is less than 16.

45. Three is no less than a number x.

46. Seven is at most the quotient of a number d and -5.

47. The sum of a number w and 4 is more than -12.

Evaluate the expression. *(Skills Review Handbook)*

48. 11^2

49. $(-3)^4$

50. -5^2

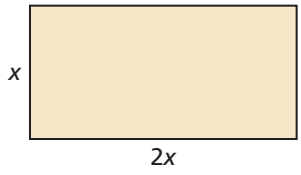
51. 2^5

3.2 Linear Functions

Essential Question How can you determine whether a function is linear or nonlinear?

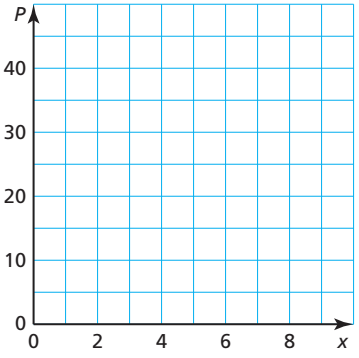
EXPLORATION 1 Finding Patterns for Similar Figures

Work with a partner. Copy and complete each table for the sequence of similar figures. (In parts (a) and (b), use the rectangle shown.) Graph the data in each table. Decide whether each pattern is linear or nonlinear. Justify your conclusion.



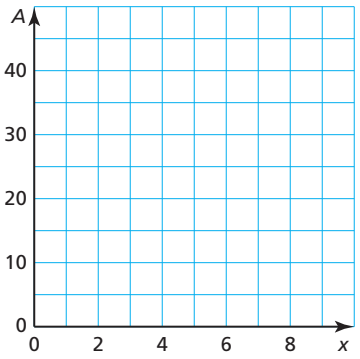
a. perimeters of similar rectangles

x	1	2	3	4	5
P					



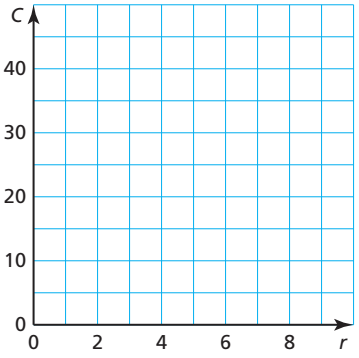
b. areas of similar rectangles

x	1	2	3	4	5
A					



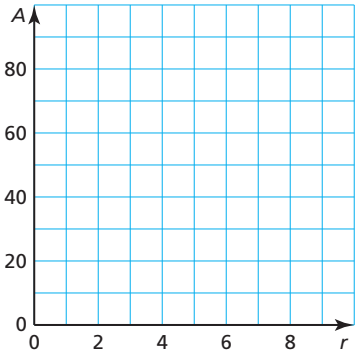
c. circumferences of circles of radius r

r	1	2	3	4	5
C					



d. areas of circles of radius r

r	1	2	3	4	5
A					



USING TOOLS STRATEGICALLY

To be proficient in math, you need to identify relationships using tools, such as tables and graphs.

Communicate Your Answer

2. How do you know that the patterns you found in Exploration 1 represent functions?
3. How can you determine whether a function is linear or nonlinear?
4. Describe two real-life patterns: one that is linear and one that is nonlinear. Use patterns that are different from those described in Exploration 1.

3.2 Lesson

Core Vocabulary

linear equation in two variables, *p. 112*
linear function, *p. 112*
nonlinear function, *p. 112*
solution of a linear equation in two variables, *p. 114*
discrete domain, *p. 114*
continuous domain, *p. 114*

Previous
whole number

What You Will Learn

- Identify linear functions using graphs, tables, and equations.
- Graph linear functions using discrete and continuous data.
- Write real-life problems to fit data.

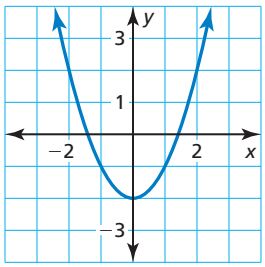
Identifying Linear Functions

A **linear equation in two variables**, x and y, is an equation that can be written in the form $y = mx + b$, where m and b are constants. The graph of a linear equation is a line. Likewise, a **linear function** is a function whose graph is a nonvertical line. A linear function has a constant rate of change and can be represented by a linear equation in two variables. A **nonlinear function** does not have a constant rate of change. So, its graph is *not* a line.

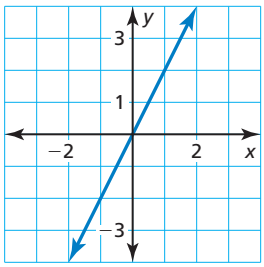
EXAMPLE 1 Identifying Linear Functions Using Graphs

Does the graph represent a *linear* or *nonlinear* function? Explain.

a.



b.



SOLUTION

a. The graph is *not* a line.

▶ So, the function is nonlinear.

b. The graph is a line.

▶ So, the function is linear.

EXAMPLE 2 Identifying Linear Functions Using Tables

Does the table represent a *linear* or *nonlinear* function? Explain.

a.

x	3	6	9	12
y	36	30	24	18

b.

x	1	3	5	7
y	2	9	20	35

SOLUTION

a.

	+3	+3	+3	
x	3	6	9	12
y	36	30	24	18
	−6	−6	−6	

As x increases by 3, y decreases by 6. The rate of change is constant.

▶ So, the function is linear.

b.

	+2	+2	+2	
x	1	3	5	7
y	2	9	20	35
	+7	+11	+15	

As x increases by 2, y increases by different amounts. The rate of change is *not* constant.

▶ So, the function is nonlinear.

REMEMBER

A constant rate of change describes a quantity that changes by equal amounts over equal intervals.

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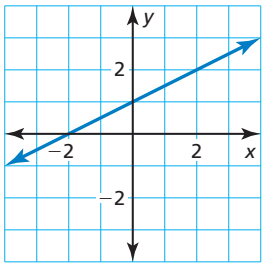
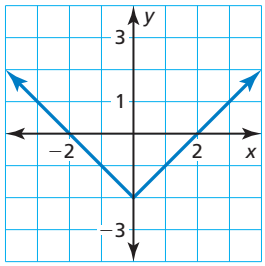
Does the graph or table represent a *linear* or *nonlinear* function? Explain.

1.



2.



3.

x	0	1	2	3
y	3	5	7	9

4.

x	1	2	3	4
y	16	8	4	2

EXAMPLE 3 Identifying Linear Functions Using Equations

Which of the following equations represent linear functions? Explain.

$$y = 3.8,\ y = \sqrt{x},\ y = 3^x,\ y = \frac{2}{x},\ y = 6(x - 1),\ \text{and}\ x^2 - y = 0$$

SOLUTION

You cannot rewrite the equations $y = \sqrt{x}$, $y = 3^x$, $y = \frac{2}{x}$, and $x^2 - y = 0$ in the form $y = mx + b$. So, these equations cannot represent linear functions.

You can rewrite the equation $y = 3.8$ as $y = 0x + 3.8$ and the equation $y = 6(x - 1)$ as $y = 6x - 6$. So, they represent linear functions.

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Does the equation represent a *linear* or *nonlinear* function? Explain.

5. $y = x + 9$

6. $y = \frac{3x}{5}$

7. $y = 5 - 2x^2$

Concept Summary

Representations of Functions

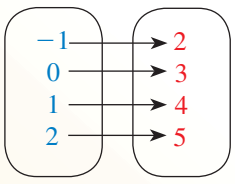
Words An output is 3 more than the input.

Equation $y = x + 3$

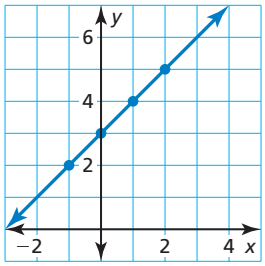
Input-Output Table

Input, x	Output, y
−1	2
0	3
1	4
2	5

Mapping Diagram



Graph



Graphing Linear Functions

A **solution of a linear equation in two variables** is an ordered pair (x, y) that makes the equation true. The graph of a linear equation in two variables is the set of points (x, y) in a coordinate plane that represents all solutions of the equation. Sometimes the points are distinct, and other times the points are connected.

Core Concept

Discrete and Continuous Domains

A **discrete domain** is a set of input values that consists of only certain numbers in an interval.

Example: Integers from 1 to 5

A **continuous domain** is a set of input values that consists of all numbers in an interval.

Example: All numbers from 1 to 5

EXAMPLE 4 Graphing Discrete Data

The linear function $y = 15.95x$ represents the cost y (in dollars) of x tickets for a museum. Each customer can buy a maximum of four tickets.

a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

STUDY TIP

The domain of a function depends on the real-life context of the function, not just the equation that represents the function.

SOLUTION

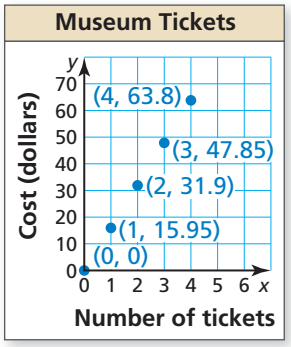
a. You cannot buy part of a ticket, only a certain number of tickets. Because x represents the number of tickets, it must be a whole number. The maximum number of tickets a customer can buy is four.

▶ So, the domain is 0, 1, 2, 3, and 4, and it is discrete.

b. Step 1 Make an input-output table to find the ordered pairs.

Input, x	$15.95x$	Output, y	(x, y)
0	15.95(0)	0	(0, 0)
1	15.95(1)	15.95	(1, 15.95)
2	15.95(2)	31.9	(2, 31.9)
3	15.95(3)	47.85	(3, 47.85)
4	15.95(4)	63.8	(4, 63.8)

Step 2 Plot the ordered pairs. The domain is discrete. So, the graph consists of individual points.



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8. The linear function $m = 50 - 9d$ represents the amount m (in dollars) of money you have after buying d DVDs. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain.

EXAMPLE 5 Graphing Continuous Data

A cereal bar contains 130 calories. The number c of calories consumed is a function of the number b of bars eaten.

a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

STUDY TIP

When the domain of a linear function is not specified or cannot be obtained from a real-life context, it is understood to be all real numbers.

SOLUTION

a. As b increases by 1, c increases by 130. The rate of change is constant.

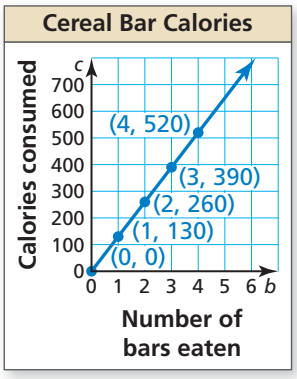
▶ So, this situation represents a linear function.

b. You can eat part of a cereal bar. The number b of bars eaten can be any value greater than or equal to 0.

▶ So, the domain is $b \geq 0$, and it is continuous.

c. **Step 1** Make an input-output table to find ordered pairs.

Input, b	Output, c	(b, c)
0	0	(0, 0)
1	130	(1, 130)
2	260	(2, 260)
3	390	(3, 390)
4	520	(4, 520)



Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points. The line should start at (0, 0) and continue to the right. Use an arrow to indicate that the line continues without end, as shown. The domain is continuous. So, the graph is a line with a domain of $b \geq 0$.

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9. Is the domain discrete or continuous? Explain.

Input Number of stories, x	1	2	3
Output Height of building (feet), y	12	24	36

10. A 20-gallon bathtub is draining at a rate of 2.5 gallons per minute. The number g of gallons remaining is a function of the number m of minutes.

a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

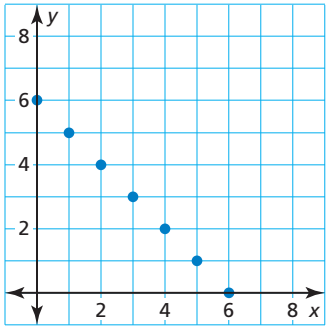
c. Graph the function using its domain.

Writing Real-Life Problems

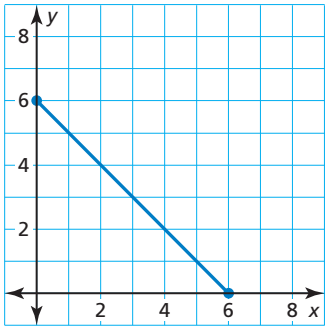
EXAMPLE 6 Writing Real-Life Problems

Write a real-life problem to fit the data shown in each graph. Is the domain of each function *discrete* or *continuous*? Explain.

a.



b.



SOLUTION

a. You want to think of a real-life situation in which there are two variables, x and y. Using the graph, notice that the sum of the variables is always 6, and the value of each variable must be a whole number from 0 to 6.

x	0	1	2	3	4	5	6
y	6	5	4	3	2	1	0

Discrete domain

▶ One possibility is two people bidding against each other on six coins at an auction. Each coin will be purchased by one of the two people. Because it is not possible to purchase part of a coin, the domain is discrete.

b. You want to think of a real-life situation in which there are two variables, x and y. Using the graph, notice that the sum of the variables is always 6, and the value of each variable can be any real number from 0 to 6.

$x + y = 6$ or $y = -x + 6$ Continuous domain

▶ One possibility is two people bidding against each other on 6 ounces of gold dust at an auction. All the dust will be purchased by the two people. Because it is possible to purchase any portion of the dust, the domain is continuous.

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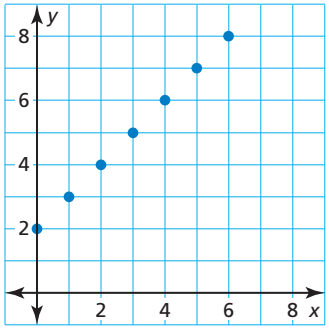
Write a real-life problem to fit the data shown in the graph. Is the domain of the function *discrete* or *continuous*? Explain.

11.



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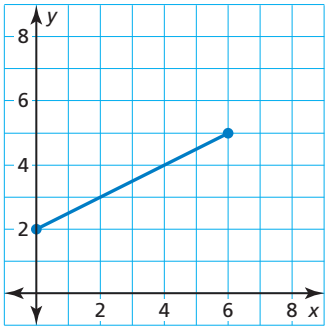


3.2 Exercises

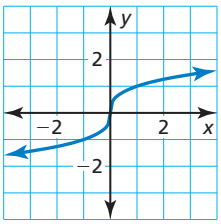
Dynamic Solutions available at *BigIdeasMath.com*

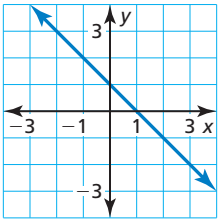
Vocabulary and Core Concept Check

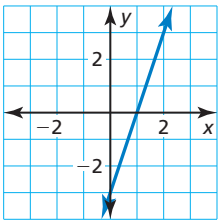
1. **COMPLETE THE SENTENCE** A linear equation in two variables is an equation that can be written in the form _____, where m and b are constants.
2. **VOCABULARY** Compare linear functions and nonlinear functions.
3. **VOCABULARY** Compare discrete domains and continuous domains.
4. **WRITING** How can you tell whether a graph shows a discrete domain or a continuous domain?

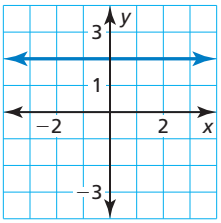
Monitoring Progress and Modeling with Mathematics

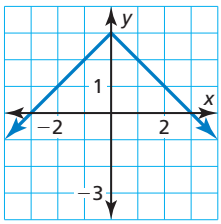
In Exercises 5–10, determine whether the graph represents a *linear* or *nonlinear* function. Explain. *(See Example 1.)*

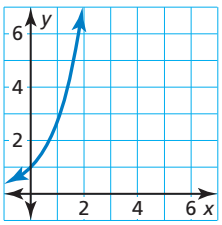
5. 

6. 

7. 

8. 

9. 

10. 

In Exercises 11–14, determine whether the table represents a *linear* or *nonlinear* function. Explain. *(See Example 2.)*

11.

x	1	2	3	4
y	5	10	15	20

12.

x	5	7	9	11
y	−9	−3	−1	3

13.

x	4	8	12	16
y	16	12	7	1

14.

x	−1	0	1	2
y	35	20	5	−10

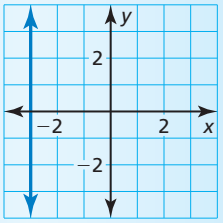
ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in determining whether the table or graph represents a linear function.**

15. ✗

	+2	+2	+2	
x	2	4	6	8
y	4	16	64	256
	×4	×4	×4	

As x increases by 2, y increases by a constant factor of 4. So, the function is linear.

16. ✗



The graph is a line. So, the graph represents a linear function.

In Exercises 17–24, determine whether the equation represents a *linear* or *nonlinear* function. Explain. *(See Example 3.)*

17. $y = x^2 + 13$

18. $y = 7 - 3x$

19. $y = \sqrt[3]{8} - x$

20. $y = 4x(8 - x)$

21. $2 + \frac{1}{6}y = 3x + 4$

22. $y - x = 2x - \frac{2}{3}y$

23. $18x - 2y = 26$

24. $2x + 3y = 9xy$

25. CLASSIFYING FUNCTIONS Which of the following equations *do not* represent linear functions? Explain.

Ⓐ $12 = 2x^2 + 4y^2$

Ⓑ $y - x + 3 = x$

Ⓒ $x = 8$

Ⓓ $x = 9 - \frac{3}{4}y$

Ⓔ $y = \frac{5x}{11}$

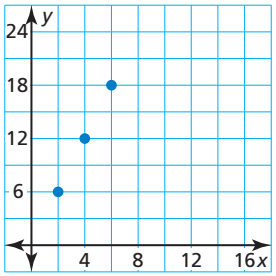
Ⓕ $y = \sqrt{x} + 3$

26. USING STRUCTURE Fill in the table so it represents a linear function.

x	5	10	15	20	25
y	−1				11

In Exercises 27 and 28, find the domain of the function represented by the graph. Determine whether the domain is *discrete* or *continuous*. Explain.

27.



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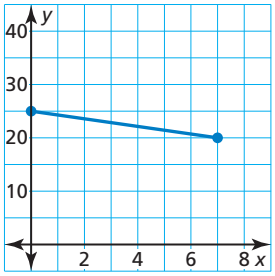


In Exercises 29–32, determine whether the domain is *discrete* or *continuous*. Explain.

29.

Input Bags, x	2	4	6
Output Marbles, y	20	40	60

30.

Input Years, x	1	2	3
Output Height of tree (feet), y	6	9	12

31.

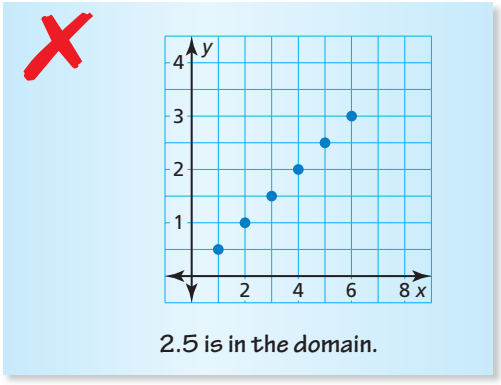
Input Time (hours), x	3	6	9
Output Distance (miles), y	150	300	450

32.

Input Relay teams, x	0	1	2
Output Athletes, y	0	4	8

ERROR ANALYSIS In Exercises 33 and 34, describe and correct the error in the statement about the domain.

33.



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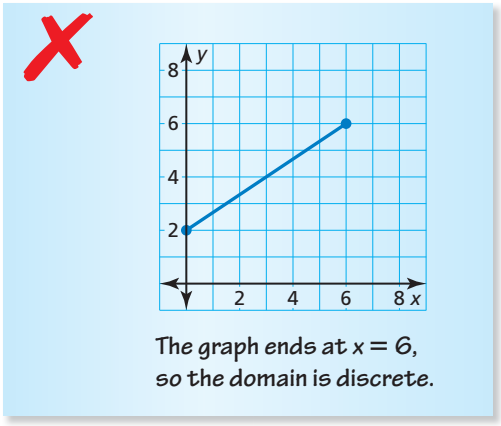


35. MODELING WITH MATHEMATICS The linear function $m = 55 - 8.5b$ represents the amount m (in dollars) of money that you have after buying b books. *(See Example 4.)*

a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

36. MODELING WITH MATHEMATICS The number y of calories burned after x hours of rock climbing is represented by the linear function $y = 650x$.

a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

37. MODELING WITH MATHEMATICS You are researching the speed of sound waves in dry air at 86°F. The table shows the distances d (in miles) sound waves travel in t seconds. *(See Example 5.)*

Time (seconds), t	Distance (miles), d
2	0.434
4	0.868
6	1.302
8	1.736
10	2.170

a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

38. MODELING WITH MATHEMATICS The function $y = 30 + 5x$ represents the cost y (in dollars) of having your dog groomed and buying x extra services.

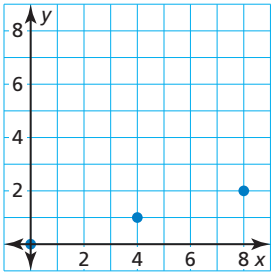
a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

WRITING In Exercises 39–42, write a real-life problem to fit the data shown in the graph. Determine whether the domain of the function is *discrete* or *continuous*. Explain. *(See Example 6.)*

39.



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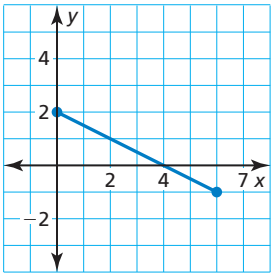

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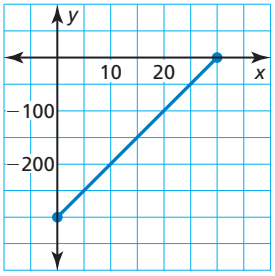

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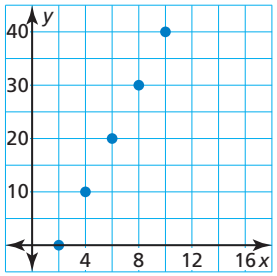


43. USING STRUCTURE The table shows your earnings y (in dollars) for working x hours.

a. What is the missing y-value that makes the table represent a linear function?

b. What is your hourly pay rate?

Time (hours), x	Earnings (dollars), y
4	40.80
5	
6	61.20
7	71.40

44. MAKING AN ARGUMENT The linear function $d = 50t$ represents the distance d (in miles) Car A is from a car rental store after t hours. The table shows the distances Car B is from the rental store.

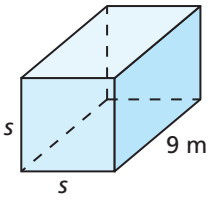
Time (hours), t	Distance (miles), d
1	60
3	180
5	310

a. Does the table represent a linear or nonlinear function? Explain.

b. Your friend claims Car B is moving at a faster rate. Is your friend correct? Explain.

MATHEMATICAL CONNECTIONS **In Exercises 45–48, tell whether the volume of the solid is a linear or nonlinear function of the missing dimension(s). Explain.**

45.



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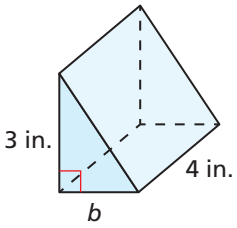

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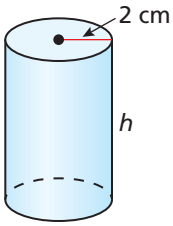

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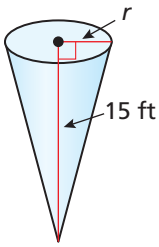


49. **REASONING** A water company fills two different-sized jugs. The first jug can hold x gallons of water. The second jug can hold y gallons of water. The company fills A jugs of the first size and B jugs of the second size. What does each expression represent? Does each expression represent a set of discrete or continuous values?

a. $x + y$

b. $A + B$

c. Ax

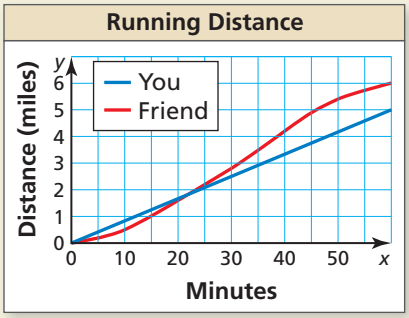
d. $Ax + By$

50. **THOUGHT PROVOKING** You go to a farmer's market to buy tomatoes. Graph a function that represents the cost of buying tomatoes. Explain your reasoning.

51. **CLASSIFYING A FUNCTION** Is the function represented by the ordered pairs linear or nonlinear? Explain your reasoning.

(0, 2), (3, 14), (5, 22), (9, 38), (11, 46)

52. **HOW DO YOU SEE IT?** You and your friend go running. The graph shows the distances you and your friend run.



a. Describe your run and your friend's run. Who runs at a constant rate? How do you know? Why might a person not run at a constant rate?

b. Find the domain of each function. Describe the domains using the context of the problem.

WRITING **In Exercises 53 and 54, describe a real-life situation for the constraints.**

53. The function has at least one negative number in the domain. The domain is continuous.

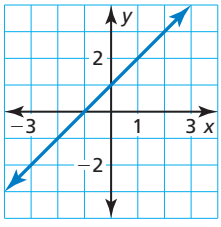
54. The function gives at least one negative number as an output. The domain is discrete.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Tell whether x and y show direct variation. Explain your reasoning. *(Skills Review Handbook)*

55.



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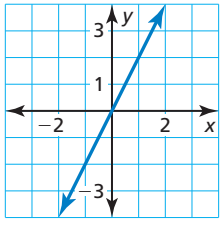

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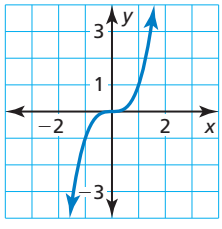


Evaluate the expression when $x = 2$. *(Skills Review Handbook)*

58. $6x + 8$

59. $10 - 2x + 8$

60. $4(x + 2 - 5x)$

61. $\frac{x}{2} + 5x - 7$

3.3 Function Notation

Essential Question How can you use function notation to represent a function?

The notation $f(x)$, called **function notation**, is another name for y. This notation is read as "the value of f at x" or "f of x." The parentheses do not imply multiplication. You can use letters other than f to name a function. The letters g, h, j, and k are often used to name functions.

EXPLORATION 1 Matching Functions with Their Graphs

Work with a partner. Match each function with its graph.

a. $f(x) = 2x - 3$

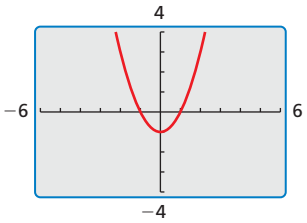
b. $g(x) = -x + 2$

c. $h(x) = x^2 - 1$

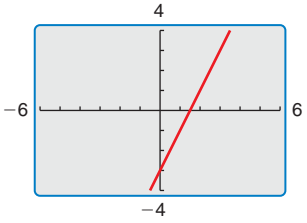
d. $j(x) = 2x^2 - 3$

ATTENDING TO PRECISION
To be proficient in math, you need to use clear definitions and state the meanings of the symbols you use.

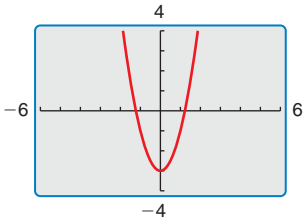
A.



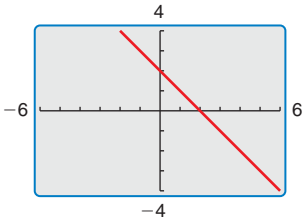
B.



C.



D.

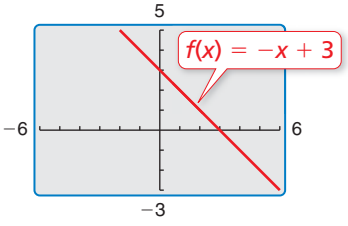


EXPLORATION 2 Evaluating a Function

Work with a partner. Consider the function

$$f(x) = -x + 3.$$

Locate the points $(x, f(x))$ on the graph. Explain how you found each point.



a. $(-1, f(-1))$

b. $(0, f(0))$

c. $(1, f(1))$

d. $(2, f(2))$

Communicate Your Answer

3. How can you use function notation to represent a function? How are standard notation and function notation similar? How are they different?

Standard Notation	*Function Notation*
$y = 2x + 5$	$f(x) = 2x + 5$

3.3 Lesson

Core Vocabulary

function notation, *p. 122*

Previous
linear function
quadrant

What You Will Learn

- Use function notation to evaluate and interpret functions.
- Use function notation to solve and graph functions.
- Solve real-life problems using function notation.

Using Function Notation to Evaluate and Interpret

You know that a linear function can be written in the form $y = mx + b$. By naming a linear function f, you can also write the function using **function notation**.

$f(x) = mx + b$ Function notation

The notation $f(x)$ is another name for y. If f is a function, and x is in its domain, then $f(x)$ represents the output of f corresponding to the input x. You can use letters other than f to name a function, such as g or h.

READING

The notation $f(x)$ is read as "the value of f at x" or "f of x." It does not mean "f times x."

EXAMPLE 1 Evaluating a Function

Evaluate $f(x) = -4x + 7$ when $x = 2$ and $x = -2$.

SOLUTION

$f(x) = -4x + 7$	Write the function.	$f(x) = -4x + 7$
$f(2) = -4(2) + 7$	Substitute for x.	$f(-2) = -4(-2) + 7$
$= -8 + 7$	Multiply.	$= 8 + 7$
$= -1$	Add.	$= 15$

▶ When $x = 2$, $f(x) = -1$, and when $x = -2$, $f(x) = 15$.

EXAMPLE 2 Interpreting Function Notation

Let $f(t)$ be the outside temperature (°F) t hours after 6 A.M. Explain the meaning of each statement.

a. $f(0) = 58$ **b.** $f(6) = n$ **c.** $f(3) < f(9)$

SOLUTION

a. The initial value of the function is 58. So, the temperature at 6 A.M. is 58°F.

b. The output of f when $t = 6$ is n. So, the temperature at noon (6 hours after 6 A.M.) is n°F.

c. The output of f when $t = 3$ is less than the output of f when $t = 9$. So, the temperature at 9 A.M. (3 hours after 6 A.M.) is less than the temperature at 3 P.M. (9 hours after 6 A.M.).

Monitoring Progress

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Evaluate the function when $x = -4$, 0, and 3.

1. $f(x) = 2x - 5$ **2.** $g(x) = -x - 1$

3. WHAT IF? In Example 2, let $f(t)$ be the outside temperature (°F) t hours after 9 A.M. Explain the meaning of each statement.

a. $f(4) = 75$ **b.** $f(m) = 70$ **c.** $f(2) = f(9)$ d. $f(6) > f(0)$

Using Function Notation to Solve and Graph

EXAMPLE 3 Solving for the Independent Variable

For $h(x) = \frac{2}{3}x - 5$, find the value of x for which $h(x) = -7$.

SOLUTION

$h(x) = \frac{2}{3}x - 5$ Write the function.

$-7 = \frac{2}{3}x - 5$ Substitute –7 for $h(x)$.

$+5 \quad +5$ Add 5 to each side.

$-2 = \frac{2}{3}x$ Simplify.

$\frac{3}{2} \cdot (-2) = \frac{3}{2} \cdot \frac{2}{3}x$ Multiply each side by $\frac{3}{2}$.

$-3 = x$ Simplify.

▶ When $x = -3$, $h(x) = -7$.

EXAMPLE 4 Graphing a Linear Function in Function Notation

Graph $f(x) = 2x + 5$.

SOLUTION

Step 1 Make an input-output table to find ordered pairs.

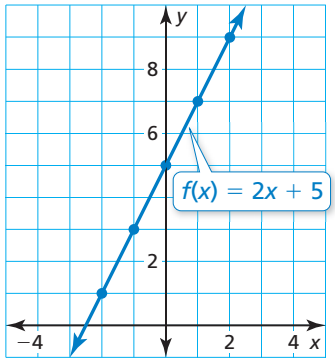
x	-2	-1	0	1	2
$f(x)$	1	3	5	7	9

Step 2 Plot the ordered pairs.

Step 3 Draw a line through the points.

STUDY TIP

The graph of $y = f(x)$ consists of the points $(x, f(x))$.



Monitoring Progress

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Find the value of x so that the function has the given value.

4. $f(x) = 6x + 9$; $f(x) = 21$

5. $g(x) = -\frac{1}{2}x + 3$; $g(x) = -1$

Graph the linear function.

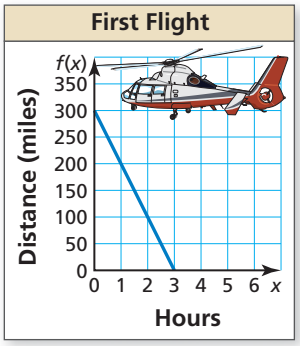
6. $f(x) = 3x - 2$

7. $g(x) = -x + 4$

8. $h(x) = -\frac{3}{4}x - 1$

Solving Real-Life Problems

EXAMPLE 5 **Modeling with Mathematics**



The graph shows the number of miles a helicopter is from its destination after x hours on its first flight. On its second flight, the helicopter travels 50 miles farther and increases its speed by 25 miles per hour. The function $f(x) = 350 - 125x$ represents the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

SOLUTION

1. **Understand the Problem** You are given a graph of the first flight and an equation of the second flight. You are asked to compare the flight times to determine which flight takes less time.

2. **Make a Plan** Graph the function that represents the second flight. Compare the graph to the graph of the first flight. The x-value that corresponds to $f(x) = 0$ represents the flight time.

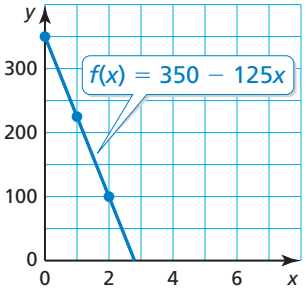
3. **Solve the Problem** Graph $f(x) = 350 - 125x$.

   Step 1 Make an input-output table to find the ordered pairs.

x	0	1	2	3
$f(x)$	350	225	100	−25

   Step 2 Plot the ordered pairs.

   Step 3 Draw a line through the points. Note that the function only makes sense when x and $f(x)$ are positive. So, only draw the line in the first quadrant.



   ▶ From the graph of the first flight, you can see that when $f(x) = 0$, $x = 3$. From the graph of the second flight, you can see that when $f(x) = 0$, x is slightly less than 3. So, the second flight takes less time.

4. **Look Back** You can check that your answer is correct by finding the value of x for which $f(x) = 0$.

$$f(x) = 350 - 125x \quad \text{Write the function.}$$
$$0 = 350 - 125x \quad \text{Substitute 0 for } f(x).$$
$$-350 = -125x \quad \text{Subtract 350 from each side.}$$
$$2.8 = x \quad \text{Divide each side by } -125.$$

   So, the second flight takes 2.8 hours, which is less than 3.

Monitoring Progress

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9. **WHAT IF?** Let $f(x) = 250 - 75x$ represent the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after x hours. Which flight takes less time? Explain.

3.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** When you write the function $y = 2x + 10$ as $f(x) = 2x + 10$, you are using _____.

2. **REASONING** Your height can be represented by a function h, where the input is your age. What does $h(14)$ represent?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, evaluate the function when $x = -2$, 0, and 5. *(See Example 1.)*

3. $f(x) = x + 6$
4. $g(x) = 3x$
5. $h(x) = -2x + 9$
6. $r(x) = -x - 7$
7. $p(x) = -3 + 4x$
8. $b(x) = 18 - 0.5x$
9. $v(x) = 12 - 2x - 5$
10. $n(x) = -1 - x + 4$

11. **INTERPRETING FUNCTION NOTATION** Let $c(t)$ be the number of customers in a restaurant t hours after 8 A.M. Explain the meaning of each statement. *(See Example 2.)*

    a. $c(0) = 0$
    b. $c(3) = c(8)$
    c. $c(n) = 29$
    d. $c(13) < c(12)$

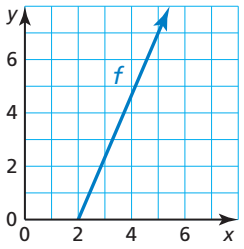
12. **INTERPRETING FUNCTION NOTATION** Let $H(x)$ be the percent of U.S. households with Internet use x years after 1980. Explain the meaning of each statement.

    a. $H(23) = 55$
    b. $H(4) = k$
    c. $H(27) \geq 61$
    d. $H(17) + H(21) \approx H(29)$

In Exercises 13–18, find the value of x so that the function has the given value. *(See Example 3.)*

13. $h(x) = -7x$; $h(x) = 63$
14. $t(x) = 3x$; $t(x) = 24$
15. $m(x) = 4x + 15$; $m(x) = 7$
16. $k(x) = 6x - 12$; $k(x) = 18$
17. $q(x) = \frac{1}{2}x - 3$; $q(x) = -4$
18. $j(x) = -\frac{4}{5}x + 7$; $j(x) = -5$

In Exercises 19 and 20, find the value of x so that $f(x) = 7$.

19.


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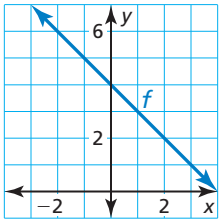


21. **MODELING WITH MATHEMATICS** The function $C(x) = 17.5x - 10$ represents the cost (in dollars) of buying x tickets to the orchestra with a \$10 coupon.

    a. How much does it cost to buy five tickets?
    b. How many tickets can you buy with \$130?

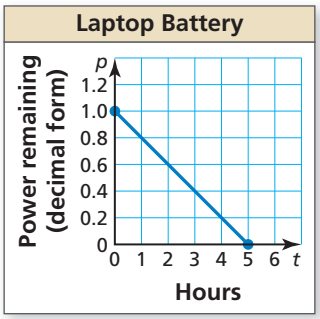
22. **MODELING WITH MATHEMATICS** The function $d(t) = 300{,}000t$ represents the distance (in kilometers) that light travels in t seconds.

    a. How far does light travel in 15 seconds?
    b. How long does it take light to travel 12 million kilometers?

In Exercises 23–28, graph the linear function. *(See Example 4.)*

23. $p(x) = 4x$
24. $h(x) = -5$
25. $d(x) = -\frac{1}{2}x - 3$
26. $w(x) = \frac{3}{5}x + 2$
27. $g(x) = -4 + 7x$
28. $f(x) = 3 - 6x$

29. PROBLEM SOLVING The graph shows the percent p (in decimal form) of battery power remaining in a laptop computer after t hours of use. A tablet computer initially has 75% of its battery power remaining and loses 12.5% per hour. Which computer's battery will last longer? Explain. *(See Example 5.)*



30. PROBLEM SOLVING The function $C(x) = 25x + 50$ represents the labor cost (in dollars) for Certified Remodeling to build a deck, where x is the number of hours of labor. The table shows sample labor costs from its main competitor, Master Remodeling. The deck is estimated to take 8 hours of labor. Which company would you hire? Explain.

Hours	Cost
2	$130
4	$160
6	$190

31. MAKING AN ARGUMENT Let $P(x)$ be the number of people in the U.S. who own a cell phone x years after 1990. Your friend says that $P(x + 1) > P(x)$ for any x because $x + 1$ is always greater than x. Is your friend correct? Explain.

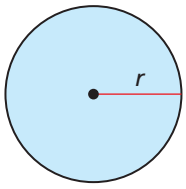
32. THOUGHT PROVOKING Let $B(t)$ be your bank account balance after t days. Describe a situation in which $B(0) < B(4) < B(2)$.

33. MATHEMATICAL CONNECTIONS Rewrite each geometry formula using function notation. Evaluate each function when $r = 5$ feet. Then explain the meaning of the result.

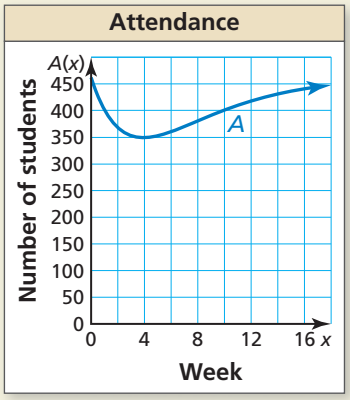
a. Diameter, $d = 2r$

b. Area, $A = \pi r^2$

c. Circumference, $C = 2\pi r$



34. HOW DO YOU SEE IT? The function $y = A(x)$ represents the attendance at a high school x weeks after a flu outbreak. The graph of the function is shown.



a. What happens to the school's attendance after the flu outbreak?

b. Estimate $A(13)$ and explain its meaning.

c. Use the graph to estimate the solution(s) of the equation $A(x) = 400$. Explain the meaning of the solution(s).

d. What was the least attendance? When did that occur?

e. How many students do you think are enrolled at this high school? Explain your reasoning.

35. INTERPRETING FUNCTION NOTATION Let f be a function. Use each statement to find the coordinates of a point on the graph of f.

a. $f(5)$ is equal to 9.

b. A solution of the equation $f(n) = -3$ is 5.

36. REASONING Given a function f, tell whether the statement

$$f(a + b) = f(a) + f(b)$$

is true or false for all inputs a and b. If it is false, explain why.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution. *(Section 2.5)*

37. $-2 \leq x - 11 \leq 6$

38. $5a < -35$ *or* $a - 14 > 1$

39. $-16 < 6k + 2 < 0$

40. $2d + 7 < -9$ *or* $4d - 1 > -3$

41. $5 \leq 3y + 8 < 17$

42. $4v + 9 \leq 5$ *or* $-3v \geq -6$

3.1–3.3 What Did You Learn?

Core Vocabulary

relation, *p. 104*
function, *p. 104*
domain, *p. 106*
range, *p. 106*
independent variable, *p. 107*
dependent variable, *p. 107*
linear equation in two variables, *p. 112*
linear function, *p. 112*
nonlinear function, *p. 112*
solution of a linear equation in two variables, *p. 114*
discrete domain, *p. 114*
continuous domain, *p. 114*
function notation, *p. 122*

Core Concepts

Section 3.1

Determining Whether Relations Are Functions, *p. 104*
Vertical Line Test, *p. 105*
The Domain and Range of a Function, *p. 106*
Independent and Dependent Variables, *p. 107*

Section 3.2

Linear and Nonlinear Functions, *p. 112*
Representations of Functions, *p. 113*
Discrete and Continuous Domains, *p. 114*

Section 3.3

Using Function Notation, *p. 122*

Mathematical Practices

1. How can you use technology to confirm your answers in Exercises 40–43 on page 110?
2. How can you use patterns to solve Exercise 43 on page 119?
3. How can you make sense of the quantities in the function in Exercise 21 on page 125?

Study Skills

Staying Focused during Class

As soon as class starts, quickly review your notes from the previous class and start thinking about math.

Repeat what you are writing in your head.

When a particular topic is difficult, ask for another example.

3.1–3.3 Quiz

Determine whether the relation is a function. Explain. *(Section 3.1)*

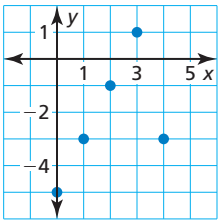
1.

Input, x	-1	0	1	2	3
Output, y	0	1	4	4	8

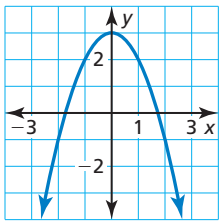
2. $(-10, 2), (-8, 3), (-6, 5), (-8, 8), (-10, 6)$

Find the domain and range of the function represented by the graph. *(Section 3.1)*

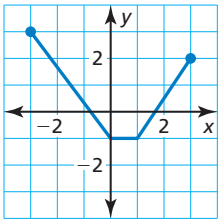
3.



4.

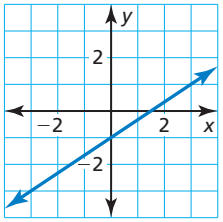


5.



Determine whether the graph, table, or equation represents a *linear* or *nonlinear* function. Explain. *(Section 3.2)*

6.



7.

x	y
-5	3
0	7
5	10

8. $y = x(2 - x)$

Determine whether the domain is *discrete* or *continuous*. Explain. *(Section 3.2)*

9.

Depth (feet), x	33	66	99
Pressure (ATM), y	2	3	4

10.

Hats, x	2	3	4
Cost (dollars), y	36	54	72

11. For $w(x) = -2x + 7$, find the value of x for which $w(x) = -3$. *(Section 3.3)*

Graph the linear function. *(Section 3.3)*

12. $g(x) = x + 3$

13. $p(x) = -3x - 1$

14. $m(x) = \frac{2}{3}x$

15. The function $m = 30 - 3r$ represents the amount m (in dollars) of money you have after renting r video games. *(Section 3.1 and Section 3.2)*

a. Identify the independent and dependent variables.

b. Find the domain and range of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

16. The function $d(x) = 1375 - 110x$ represents the distance (in miles) a high-speed train is from its destination after x hours. *(Section 3.3)*

a. How far is the train from its destination after 8 hours?

b. How long does the train travel before reaching its destination?

3.4 Graphing Linear Equations in Standard Form

Essential Question How can you describe the graph of the equation $Ax + By = C$?

EXPLORATION 1 Using a Table to Plot Points

Work with a partner. You sold a total of \$16 worth of tickets to a fundraiser. You lost track of how many of each type of ticket you sold. Adult tickets are \$4 each. Child tickets are \$2 each.

$$\frac{\square}{\text{adult}} \cdot \text{Number of adult tickets} + \frac{\square}{\text{child}} \cdot \text{Number of child tickets} = \square$$

a. Let x represent the number of adult tickets. Let y represent the number of child tickets. Use the verbal model to write an equation that relates x and y.

b. Copy and complete the table to show the different combinations of tickets you might have sold.

x					
y					

c. Plot the points from the table. Describe the pattern formed by the points.

d. If you remember how many adult tickets you sold, can you determine how many child tickets you sold? Explain your reasoning.

FINDING AN ENTRY POINT

To be proficient in math, you need to find an entry point into the solution of a problem. Determining what information you know, and what you can do with that information, can help you find an entry point.

EXPLORATION 2 Rewriting and Graphing an Equation

Work with a partner. You sold a total of \$48 worth of cheese. You forgot how many pounds of each type of cheese you sold. Swiss cheese costs \$8 per pound. Cheddar cheese costs \$6 per pound.

$$\frac{\square}{\text{pound}} \cdot \text{Pounds of Swiss} + \frac{\square}{\text{pound}} \cdot \text{Pounds of cheddar} = \square$$

a. Let x represent the number of pounds of Swiss cheese. Let y represent the number of pounds of cheddar cheese. Use the verbal model to write an equation that relates x and y.

b. Solve the equation for y. Then use a graphing calculator to graph the equation. Given the real-life context of the problem, find the domain and range of the function.

c. The ***x*-intercept** of a graph is the x-coordinate of a point where the graph crosses the x-axis. The ***y*-intercept** of a graph is the y-coordinate of a point where the graph crosses the y-axis. Use the graph to determine the x- and y-intercepts.

d. How could you use the equation you found in part (a) to determine the x- and y-intercepts? Explain your reasoning.

e. Explain the meaning of the intercepts in the context of the problem.

Communicate Your Answer

3. How can you describe the graph of the equation $Ax + By = C$?

4. Write a real-life problem that is similar to those shown in Explorations 1 and 2.

3.4 Lesson

What You Will Learn

- Graph equations of horizontal and vertical lines.
- Graph linear equations in standard form using intercepts.
- Use linear equations in standard form to solve real-life problems.

Core Vocabulary

standard form, *p. 130*
x-intercept, *p. 131*
y-intercept, *p. 131*

Previous
ordered pair
quadrant

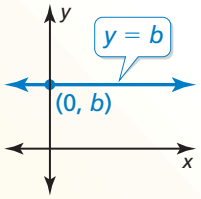
Horizontal and Vertical Lines

The **standard form** of a linear equation is $Ax + By = C$, where A, B, and C are real numbers and A and B are not both zero.

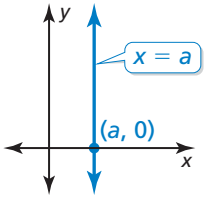
Consider what happens when $A = 0$ or when $B = 0$. When $A = 0$, the equation becomes $By = C$, or $y = \frac{C}{B}$. Because $\frac{C}{B}$ is a constant, you can write $y = b$. Similarly, when $B = 0$, the equation becomes $Ax = C$, or $x = \frac{C}{A}$, and you can write $x = a$.

Core Concept

Horizontal and Vertical Lines



The graph of $y = b$ is a horizontal line. The line passes through the point $(0, b)$.



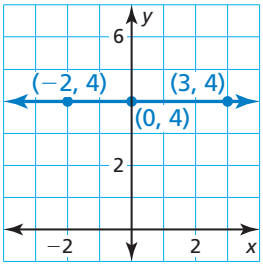
The graph of $x = a$ is a vertical line. The line passes through the point $(a, 0)$.

EXAMPLE 1 Horizontal and Vertical Lines

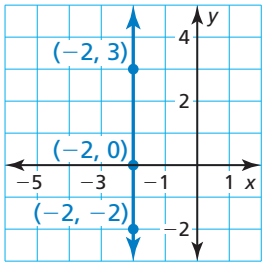
Graph (a) $y = 4$ and (b) $x = -2$.

SOLUTION

a. For every value of x, the value of y is 4. The graph of the equation $y = 4$ is a horizontal line 4 units above the x-axis.



b. For every value of y, the value of x is -2. The graph of the equation $x = -2$ is a vertical line 2 units to the left of the y-axis.



STUDY TIP

For every value of x, the ordered pair $(x, 4)$ is a solution of $y = 4$.

Monitoring Progress

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Graph the linear equation.

1. $y = -2.5$

2. $x = 5$

Using Intercepts to Graph Linear Equations

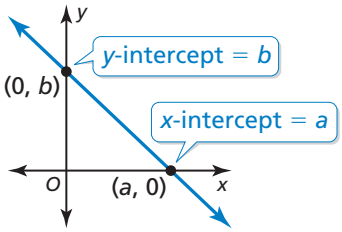
You can use the fact that two points determine a line to graph a linear equation. Two convenient points are the points where the graph crosses the axes.

Core Concept

Using Intercepts to Graph Equations

The **x-intercept** of a graph is the x-coordinate of a point where the graph crosses the x-axis. It occurs when $y = 0$.

The **y-intercept** of a graph is the y-coordinate of a point where the graph crosses the y-axis. It occurs when $x = 0$.



To graph the linear equation $Ax + By = C$, find the intercepts and draw the line that passes through the two intercepts.

- To find the x-intercept, let $y = 0$ and solve for x.
- To find the y-intercept, let $x = 0$ and solve for y.

EXAMPLE 2 Using Intercepts to Graph a Linear Equation

Use intercepts to graph the equation $3x + 4y = 12$.

SOLUTION

Step 1 Find the intercepts.

To find the x-intercept, substitute 0 for y and solve for x.

$3x + 4y = 12$	Write the original equation.
$3x + 4(0) = 12$	Substitute 0 for y.
$x = 4$	Solve for x.

To find the y-intercept, substitute 0 for x and solve for y.

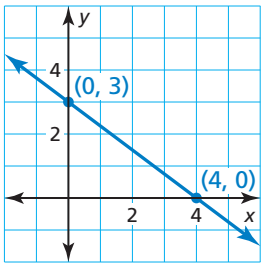
$3x + 4y = 12$	Write the original equation.
$3(0) + 4y = 12$	Substitute 0 for x.
$y = 3$	Solve for y.

STUDY TIP

As a check, you can find a third solution of the equation and verify that the corresponding point is on the graph. To find a third solution, substitute any value for one of the variables and solve for the other variable.

Step 2 Plot the points and draw the line.

The x-intercept is 4, so plot the point (4, 0).
The y-intercept is 3, so plot the point (0, 3).
Draw a line through the points.



Monitoring Progress

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Use intercepts to graph the linear equation. Label the points corresponding to the intercepts.

3. $2x - y = 4$

4. $x + 3y = -9$

Solving Real-Life Problems

EXAMPLE 3 Modeling with Mathematics

You are planning an awards banquet for your school. You need to rent tables to seat 180 people. Tables come in two sizes. Small tables seat 6 people, and large tables seat 10 people. The equation $6x + 10y = 180$ models this situation, where x is the number of small tables and y is the number of large tables.

a. Graph the equation. Interpret the intercepts.

b. Find four possible solutions in the context of the problem.

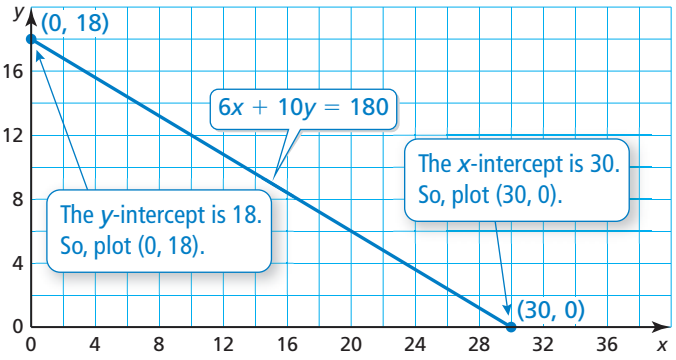
SOLUTION

1. Understand the Problem You know the equation that models the situation. You are asked to graph the equation, interpret the intercepts, and find four solutions.

2. Make a Plan Use intercepts to graph the equation. Then use the graph to interpret the intercepts and find other solutions.

3. Solve the Problem

a. Use intercepts to graph the equation. Neither x nor y can be negative, so only graph the equation in the first quadrant.



▶ The x-intercept shows that you can rent 30 small tables when you do not rent any large tables. The y-intercept shows that you can rent 18 large tables when you do not rent any small tables.

b. Only whole-number values of x and y make sense in the context of the problem. Besides the intercepts, it appears that the line passes through the points (10, 12) and (20, 6). To verify that these points are solutions, check them in the equation, as shown.

▶ So, four possible combinations of tables that will seat 180 people are 0 small and 18 large, 10 small and 12 large, 20 small and 6 large, and 30 small and 0 large.

4. Look Back The graph shows that as the number x of small tables increases, the number y of large tables decreases. This makes sense in the context of the problem. So, the graph is reasonable.

STUDY TIP

Although x and y represent whole numbers, it is convenient to draw a line segment that includes points whose coordinates are not whole numbers.

Check

$6x + 10y = 180$

$6(10) + 10(12) \stackrel{?}{=} 180$

$180 = 180$ ✓

$6x + 10y = 180$

$6(20) + 10(6) \stackrel{?}{=} 180$

$180 = 180$ ✓

Monitoring Progress

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5. WHAT IF? You decide to rent tables from a different company. The situation can be modeled by the equation $4x + 6y = 180$, where x is the number of small tables and y is the number of large tables. Graph the equation and interpret the intercepts.

3.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How are x-intercepts and y-intercepts alike? How are they different?

2. **WHICH ONE DOESN'T BELONG?** Which point does not belong with the other three? Explain your reasoning.

$(0, -3)$ $(0, 0)$ $(4, -3)$ $(4, 0)$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, graph the linear equation. *(See Example 1.)*

3. $x = 4$
4. $y = 2$
5. $y = -3$
6. $x = -1$

In Exercises 7–12, find the x- and y-intercepts of the graph of the linear equation.

7. $2x + 3y = 12$
8. $3x + 6y = 24$
9. $-4x + 8y = -16$
10. $-6x + 9y = -18$
11. $3x - 6y = 2$
12. $-x + 8y = 4$

In Exercises 13–22, use intercepts to graph the linear equation. Label the points corresponding to the intercepts. *(See Example 2.)*

13. $5x + 3y = 30$
14. $4x + 6y = 12$
15. $-12x + 3y = 24$
16. $-2x + 6y = 18$
17. $-4x + 3y = -30$
18. $-2x + 7y = -21$
19. $-x + 2y = 7$
20. $3x - y = -5$
21. $-\frac{5}{2}x + y = 10$
22. $-\frac{1}{2}x + y = -4$

23. **MODELING WITH MATHEMATICS** A football team has an away game, and the bus breaks down. The coaches decide to drive the players to the game in cars and vans. Four players can ride in each car. Six players can ride in each van. There are 48 players on the team. The equation $4x + 6y = 48$ models this situation, where x is the number of cars and y is the number of vans. *(See Example 3.)*

   a. Graph the equation. Interpret the intercepts.

   b. Find four possible solutions in the context of the problem.

24. **MODELING WITH MATHEMATICS** You are ordering shirts for the math club at your school. Short-sleeved shirts cost \$10 each. Long-sleeved shirts cost \$12 each. You have a budget of \$300 for the shirts. The equation $10x + 12y = 300$ models the total cost, where x is the number of short-sleeved shirts and y is the number of long-sleeved shirts.

   a. Graph the equation. Interpret the intercepts.

   b. Twelve students decide they want short-sleeved shirts. How many long-sleeved shirts can you order?

ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in finding the intercepts of the graph of the equation.**

25. ✗

$3x + 12y = 24$	$3x + 12y = 24$
$3x + 12(0) = 24$	$3(0) + 12y = 24$
$3x = 24$	$12y = 24$
$x = 8$	$y = 2$

The intercept is at $(8, 2)$.

26. ✗

$4x + 10y = 20$	$4x + 10y = 20$
$4x + 10(0) = 20$	$4(0) + 10y = 20$
$4x = 20$	$10y = 20$
$x = 5$	$y = 2$

The x-intercept is at $(0, 5)$, and the y-intercept is at $(2, 0)$.

27. MAKING AN ARGUMENT You overhear your friend explaining how to find intercepts to a classmate. Your friend says, "When you want to find the x-intercept, just substitute 0 for x and continue to solve the equation." Is your friend's explanation correct? Explain.

28. ANALYZING RELATIONSHIPS You lose track of how many 2-point baskets and 3-point baskets a team makes in a basketball game. The team misses all the 1-point baskets and still scores 54 points. The equation $2x + 3y = 54$ models the total points scored, where x is the number of 2-point baskets made and y is the number of 3-point baskets made.

a. Find and interpret the intercepts.

b. Can the number of 3-point baskets made be odd? Explain your reasoning.

c. Graph the equation. Find two more possible solutions in the context of the problem.

MULTIPLE REPRESENTATIONS In Exercises 29–32, match the equation with its graph.

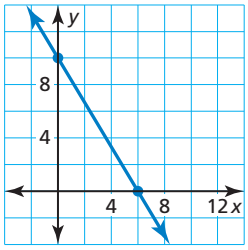
29. $5x + 3y = 30$

30. $5x + 3y = -30$

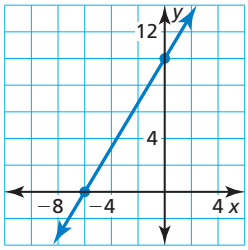
31. $5x - 3y = 30$

32. $5x - 3y = -30$

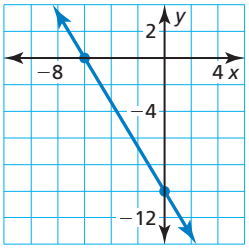
A.



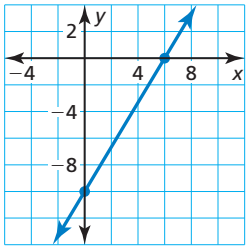
B.



C.

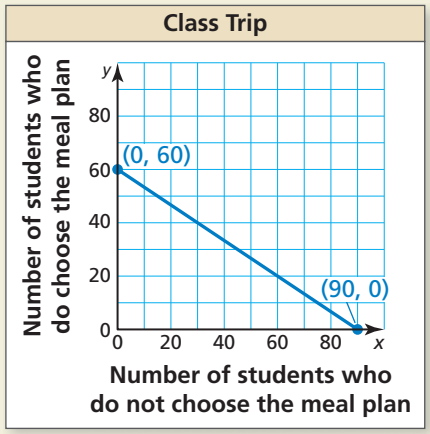


D.



33. MATHEMATICAL CONNECTIONS Graph the equations $x = 5$, $x = 2$, $y = -2$, and $y = 1$. What enclosed shape do the lines form? Explain your reasoning.

34. HOW DO YOU SEE IT? You are organizing a class trip to an amusement park. The cost to enter the park is \$30. The cost to enter with a meal plan is \$45. You have a budget of \$2700 for the trip. The equation $30x + 45y = 2700$ models the total cost for the class to go on the trip, where x is the number of students who do not choose the meal plan and y is the number of students who do choose the meal plan.



a. Interpret the intercepts of the graph.

b. Describe the domain and range in the context of the problem.

35. REASONING Use the values to fill in the equation $\square x + \square y = 30$ so that the x-intercept of the graph is -10 and the y-intercept of the graph is 5.

-10	-3	1	5	6

36. THOUGHT PROVOKING Write an equation in standard form of a line whose intercepts are integers. Explain how you know the intercepts are integers.

37. WRITING Are the equations of horizontal and vertical lines written in standard form? Explain your reasoning.

38. ABSTRACT REASONING The x- and y-intercepts of the graph of the equation $3x + 5y = k$ are integers. Describe the values of k. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. *(Skills Review Handbook)*

39. $\dfrac{2 - (-2)}{4 - (-4)}$

40. $\dfrac{14 - 18}{0 - 2}$

41. $\dfrac{-3 - 9}{8 - (-7)}$

42. $\dfrac{12 - 17}{-5 - (-2)}$

3.5 Graphing Linear Equations in Slope-Intercept Form

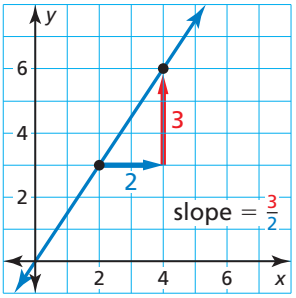
Essential Question

How can you describe the graph of the equation $y = mx + b$?

Slope is the rate of change between any two points on a line. It is the measure of the *steepness* of the line.

To find the slope of a line, find the ratio of the change in y (vertical change) to the change in x (horizontal change).

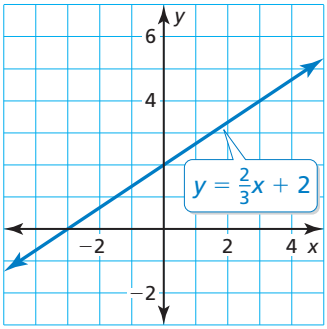
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



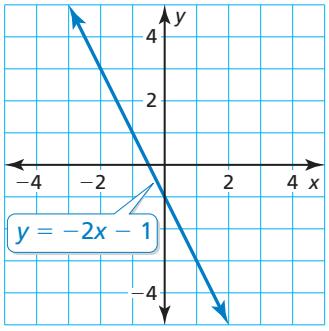
EXPLORATION 1 Finding Slopes and y-Intercepts

Work with a partner. Find the slope and y-intercept of each line.

a.



b.



MAKING CONJECTURES

To be proficient in math, you first need to collect and organize data. Then make conjectures about the patterns you observe in the data.

EXPLORATION 2 Writing a Conjecture

Work with a partner. Graph each equation. Then copy and complete the table. Use the completed table to write a conjecture about the relationship between the graph of $y = mx + b$ and the values of m and b.

Equation	Description of graph	Slope of graph	y-Intercept
a. $y = -\frac{2}{3}x + 3$	Line	$-\frac{2}{3}$	3
b. $y = 2x - 2$			
c. $y = -x + 1$			
d. $y = x - 4$			

Communicate Your Answer

3. How can you describe the graph of the equation $y = mx + b$?

a. How does the value of m affect the graph of the equation?

b. How does the value of b affect the graph of the equation?

c. Check your answers to parts (a) and (b) by choosing one equation from Exploration 2 and (1) varying only m and (2) varying only b.

3.5 Lesson

What You Will Learn

- Find the slope of a line.
- Use the slope-intercept form of a linear equation.
- Use slopes and *y*-intercepts to solve real-life problems.

Core Vocabulary

slope, *p. 136*
rise, *p. 136*
run, *p. 136*
slope-intercept form, *p. 138*
constant function, *p. 138*

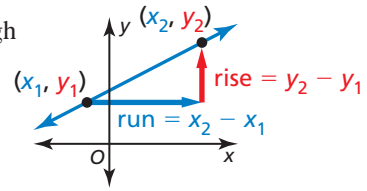
Previous
dependent variable
independent variable

The Slope of a Line

Core Concept

Slope

The **slope** m of a nonvertical line passing through two points (x_1, y_1) and (x_2, y_2) is the ratio of the **rise** (change in y) to the **run** (change in x).



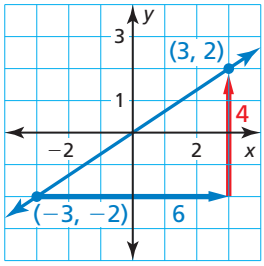
$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

When the line rises from left to right, the slope is positive. When the line falls from left to right, the slope is negative.

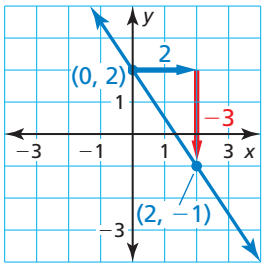
EXAMPLE 1 Finding the Slope of a Line

Describe the slope of each line. Then find the slope.

a.



b.



SOLUTION

a. The line rises from left to right. So, the slope is positive.
Let $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (3, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$$

b. The line falls from left to right. So, the slope is negative.
Let $(x_1, y_1) = (0, 2)$ and $(x_2, y_2) = (2, -1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{2 - 0} = \frac{-3}{2} = -\frac{3}{2}$$

STUDY TIP

When finding slope, you can label either point as (x_1, y_1) and the other point as (x_2, y_2). The result is the same.

READING

In the slope formula, x_1 is read as "x sub one" and y_2 is read as "y sub two." The numbers 1 and 2 in x_1 and y_2 are called *subscripts*.

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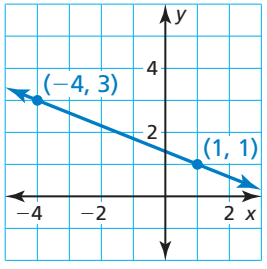
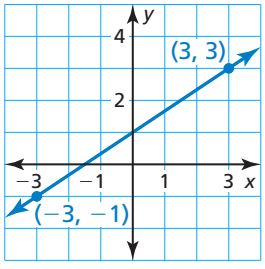
Describe the slope of the line. Then find the slope.

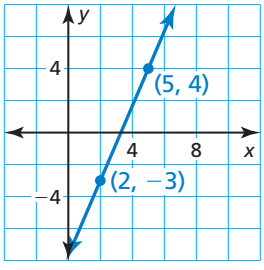
1.



2.



3.



EXAMPLE 2 Finding Slope from a Table

The points represented by each table lie on a line. How can you find the slope of each line from the table? What is the slope of each line?

a.

x	y
4	20
7	14
10	8
13	2

b.

x	y
−1	2
1	2
3	2
5	2

c.

x	y
−3	−3
−3	0
−3	6
−3	9

STUDY TIP

As a check, you can plot the points represented by the table to verify that the line through them has a slope of −2.

SOLUTION

a. Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (4, 20)$ and $(x_2, y_2) = (7, 14)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 20}{7 - 4} = \frac{-6}{3}, \text{ or } -2$$

▶ The slope is −2.

b. Note that there is no change in y. Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (5, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{5 - (-1)} = \frac{0}{6}, \text{ or } 0$$

The change in y is 0.

▶ The slope is 0.

c. Note that there is no change in x. Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (-3, 0)$ and $(x_2, y_2) = (-3, 6)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{-3 - (-3)} = \frac{6}{0}$$ ✗

The change in x is 0.

▶ Because division by zero is undefined, the slope of the line is undefined.

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The points represented by the table lie on a line. How can you find the slope of the line from the table? What is the slope of the line?

4.

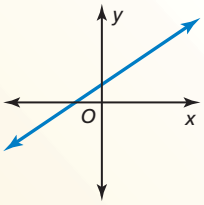
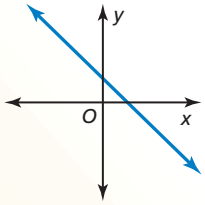
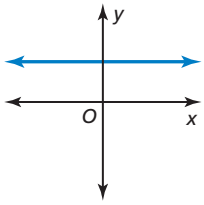
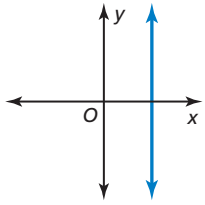
x	2	4	6	8
y	10	15	20	25

5.

x	5	5	5	5
y	−12	−9	−6	−3

Concept Summary

Slope

Positive slope	*Negative slope*	*Slope of 0*	*Undefined slope*
			
The line rises from left to right.	The line falls from left to right.	The line is horizontal.	The line is vertical.

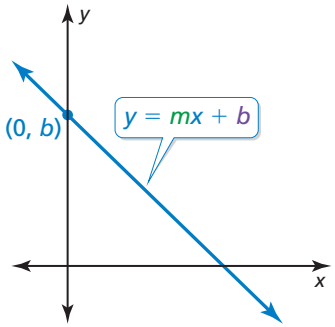
Using the Slope-Intercept Form of a Linear Equation

Core Concept

Slope-Intercept Form

Words A linear equation written in the form $y = mx + b$ is in **slope-intercept form**. The slope of the line is m, and the y-intercept of the line is b.

Algebra $y = mx + b$ (slope: m; y-intercept: b)



A linear equation written in the form $y = 0x + b$, or $y = b$, is a **constant function**. The graph of a constant function is a horizontal line.

EXAMPLE 3 Identifying Slopes and y-Intercepts

Find the slope and the y-intercept of the graph of each linear equation.

a. $y = 3x - 4$ **b.** $y = 6.5$ **c.** $-5x - y = -2$

SOLUTION

a. $y = mx + b$ Write the slope-intercept form.

(slope: m; y-intercept: b)

$y = 3x + (-4)$ Rewrite the original equation in slope-intercept form.

▶ The slope is 3, and the y-intercept is -4.

STUDY TIP

For a constant function, every input has the same output. For instance, in Example 3b, every input has an output of 6.5.

b. The equation represents a constant function. The equation can also be written as $y = 0x + 6.5$.

▶ The slope is 0, and the y-intercept is 6.5.

c. Rewrite the equation in slope-intercept form by solving for y.

$-5x - y = -2$ Write the original equation.

$+5x \quad +5x$ Add $5x$ to each side.

$-y = 5x - 2$ Simplify.

$\frac{-y}{-1} = \frac{5x - 2}{-1}$ Divide each side by –1.

$y = -5x + 2$ Simplify.

STUDY TIP

When you rewrite a linear equation in slope-intercept form, you are expressing y as a function of x.

▶ The slope is -5, and the y-intercept is 2.

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Find the slope and the y-intercept of the graph of the linear equation.

6. $y = -6x + 1$ **7.** $y = 8$ **8.** $x + 4y = -10$

EXAMPLE 4 Using Slope-Intercept Form to Graph

Graph $2x + y = 2$. Identify the x-intercept.

SOLUTION

Step 1 Rewrite the equation in slope-intercept form.

$$y = -2x + 2$$

Step 2 Find the slope and the y-intercept.

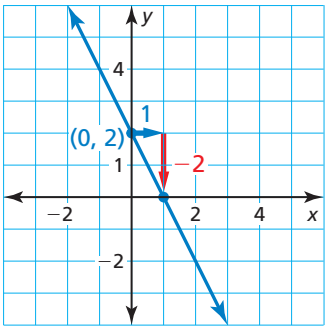
$$m = -2 \text{ and } b = 2$$

Step 3 The y-intercept is 2. So, plot (0, 2).

Step 4 Use the slope to find another point on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1}$$

Plot the point that is 1 unit right and 2 units down from (0, 2). Draw a line through the two points.



The line crosses the x-axis at (1, 0). So, the x-intercept is 1.

STUDY TIP

You can use the slope to find points on a line in either direction. In Example 4, note that the slope can be written as $\frac{2}{-1}$. So, you could move 1 unit left and 2 units up from (0, 2) to find the point (–1, 4).

REMEMBER

You can also find the x-intercept by substituting 0 for y in the equation $2x + y = 2$ and solving for x.

EXAMPLE 5 Graphing from a Verbal Description

A linear function g models a relationship in which the dependent variable increases 3 units for every 1 unit the independent variable increases. Graph g when $g(0) = 3$. Identify the slope, y-intercept, and x-intercept of the graph.

SOLUTION

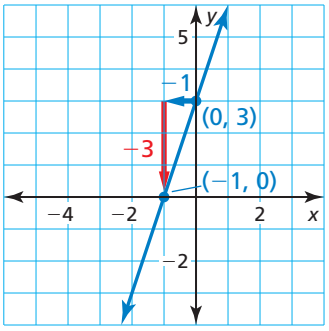
Because the function g is linear, it has a constant rate of change. Let x represent the independent variable and y represent the dependent variable.

Step 1 **Find the slope.** When the dependent variable increases by 3, the change in y is +3. When the independent variable increases by 1, the change in x is +1.

So, the slope is $\frac{3}{1}$, or 3.

Step 2 **Find the y-intercept.** The statement $g(0) = 3$ indicates that when $x = 0$, $y = 3$. So, the y-intercept is 3. Plot (0, 3).

Step 3 **Use the slope to find another point on the line.** A slope of 3 can be written as $\frac{-3}{-1}$. Plot the point that is 1 unit left and 3 units down from (0, 3). Draw a line through the two points. The line crosses the x-axis at $(-1, 0)$. So, the x-intercept is -1.



The slope is 3, the y-intercept is 3, and the x-intercept is -1.

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Graph the linear equation. Identify the x-intercept.

9. $y = 4x - 4$ **10.** $3x + y = -3$ **11.** $x + 2y = 6$

12. A linear function h models a relationship in which the dependent variable decreases 2 units for every 5 units the independent variable increases. Graph h when $h(0) = 4$. Identify the slope, y-intercept, and x-intercept of the graph.

Solving Real-Life Problems

In most real-life problems, slope is interpreted as a rate, such as miles per hour, dollars per hour, or people per year.

EXAMPLE 6 Modeling with Mathematics

A submersible that is exploring the ocean floor begins to ascend to the surface. The elevation h (in feet) of the submersible is modeled by the function $h(t) = 650t - 13{,}000$, where t is the time (in minutes) since the submersible began to ascend.

a. Graph the function and identify its domain and range.

b. Interpret the slope and the intercepts of the graph.

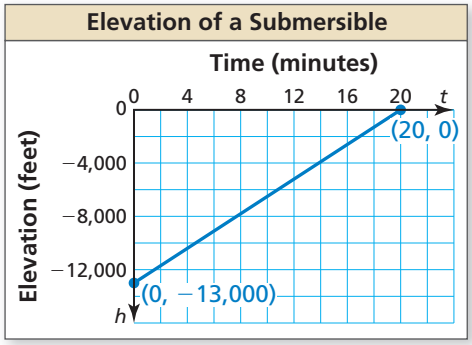
SOLUTION

1. Understand the Problem You know the function that models the elevation. You are asked to graph the function and identify its domain and range. Then you are asked to interpret the slope and intercepts of the graph.

2. Make a Plan Use the slope-intercept form of a linear equation to graph the function. Only graph values that make sense in the context of the problem. Examine the graph to interpret the slope and the intercepts.

3. Solve the Problem

a. The time t must be greater than or equal to 0. The elevation h is below sea level and must be less than or equal to 0. Use the slope of 650 and the h-intercept of $-13{,}000$ to graph the function in Quadrant IV.



▶ The domain is $0 \le t \le 20$, and the range is $-13{,}000 \le h \le 0$.

b. The slope is 650. So, the submersible ascends at a rate of 650 feet per minute. The h-intercept is $-13{,}000$. So, the elevation of the submersible after 0 minutes, or when the ascent begins, is $-13{,}000$ feet. The t-intercept is 20. So, the submersible takes 20 minutes to reach an elevation of 0 feet, or sea level.

4. Look Back You can check that your graph is correct by substituting the t-intercept for t in the function. If $h = 0$ when $t = 20$, the graph is correct.

$h = 650(20) - 13{,}000$ Substitute 20 for t in the original equation.

$h = 0$ ✓ Simplify.

STUDY TIP

Because t is the independent variable, the horizontal axis is the t-axis and the graph will have a "t-intercept." Similarly, the vertical axis is the h-axis and the graph will have an "h-intercept."

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13. WHAT IF? The elevation of the submersible is modeled by $h(t) = 500t - 10{,}000$. (a) Graph the function and identify its domain and range. (b) Interpret the slope and the intercepts of the graph.

3.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

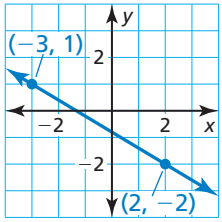
Vocabulary and Core Concept Check

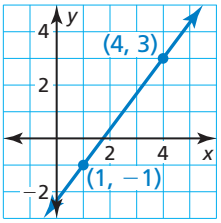
1. **COMPLETE THE SENTENCE** The _____ of a nonvertical line passing through two points is the ratio of the rise to the run.
2. **VOCABULARY** What is a constant function? What is the slope of a constant function?
3. **WRITING** What is the slope-intercept form of a linear equation? Explain why this form is called the slope-intercept form.
4. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

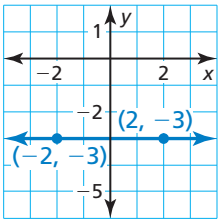
$y = -5x - 1$ $\quad 2x - y = 8$ $\quad y = x + 4$ $\quad y = -3x + 13$

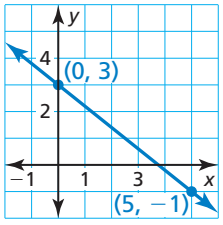
Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, describe the slope of the line. Then find the slope. *(See Example 1.)*

5. 

6. 

7. 

8. 

In Exercises 9–12, the points represented by the table lie on a line. Find the slope of the line. *(See Example 2.)*

9.

x	−9	−5	−1	3
y	−2	0	2	4

10.

x	−1	2	5	8
y	−6	−6	−6	−6

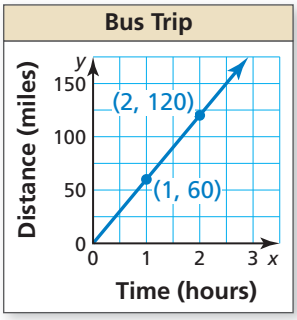
11.

x	0	0	0	0
y	−4	0	4	8

12.

x	−4	−3	−2	−1
y	2	−5	−12	−19

13. **ANALYZING A GRAPH** The graph shows the distance y (in miles) that a bus travels in x hours. Find and interpret the slope of the line.



14. **ANALYZING A TABLE** The table shows the amount x (in hours) of time you spend at a theme park and the admission fee y (in dollars) to the park. The points represented by the table lie on a line. Find and interpret the slope of the line.

Time (hours), x	Admission (dollars), y
6	54.99
7	54.99
8	54.99

In Exercises 15–22, find the slope and the *y*-intercept of the graph of the linear equation. *(See Example 3.)*

15. $y = -3x + 2$

16. $y = 4x - 7$

17. $y = 6x$

18. $y = -1$

19. $-2x + y = 4$

20. $x + y = -6$

21. $-5x = 8 - y$

22. $0 = 1 - 2y + 14x$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in finding the slope and the *y*-intercept of the graph of the equation.**

23.

$x = -4y$

The slope is −4, and the y-intercept is 0.

24.

$y = 3x - 6$

The slope is 3, and the y-intercept is 6.

In Exercises 25–32, graph the linear equation. Identify the *x*-intercept. *(See Example 4.)*

25. $y = -x + 7$

26. $y = \frac{1}{2}x + 3$

27. $y = 2x$

28. $y = -x$

29. $3x + y = -1$

30. $x + 4y = 8$

31. $-y + 5x = 0$

32. $2x - y + 6 = 0$

In Exercises 33 and 34, graph the function with the given description. Identify the slope, *y*-intercept, and *x*-intercept of the graph. *(See Example 5.)*

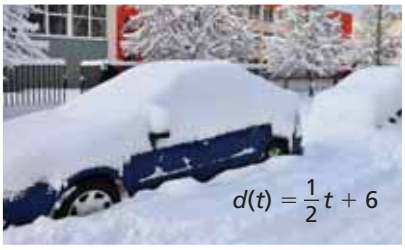
33. A linear function f models a relationship in which the dependent variable decreases 4 units for every 2 units the independent variable increases. The value of the function at 0 is −2.

34. A linear function h models a relationship in which the dependent variable increases 1 unit for every 5 units the independent variable decreases. The value of the function at 0 is 3.

35. **GRAPHING FROM A VERBAL DESCRIPTION** A linear function r models the growth of your right index fingernail. The length of the fingernail increases 0.7 millimeter every week. Graph r when $r(0) = 12$. Identify the slope and interpret the *y*-intercept of the graph.

36. **GRAPHING FROM A VERBAL DESCRIPTION** A linear function m models the amount of milk sold by a farm per month. The amount decreases 500 gallons for every $1 increase in price. Graph m when $m(0) = 3000$. Identify the slope and interpret the *x*- and *y*-intercepts of the graph.

37. **MODELING WITH MATHEMATICS** The function shown models the depth d (in inches) of snow on the ground during the first 9 hours of a snowstorm, where t is the time (in hours) after the snowstorm begins. *(See Example 6.)*



a. Graph the function and identify its domain and range.

b. Interpret the slope and the *d*-intercept of the graph.

38. **MODELING WITH MATHEMATICS** The function $c(x) = 0.5x + 70$ represents the cost c (in dollars) of renting a truck from a moving company, where x is the number of miles you drive the truck.

a. Graph the function and identify its domain and range.

b. Interpret the slope and the *c*-intercept of the graph.

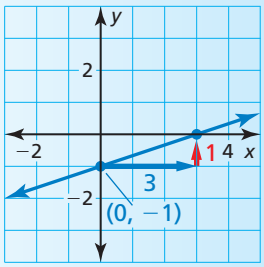
39. **COMPARING FUNCTIONS** A linear function models the cost of renting a truck from a moving company. The table shows the cost y (in dollars) when you drive the truck x miles. Graph the function and compare the slope and the *y*-intercept of the graph with the slope and the *c*-intercept of the graph in Exercise 38.

Miles, x	Cost (dollars), y
0	40
50	80
100	120

ERROR ANALYSIS **In Exercises 40 and 41, describe and correct the error in graphing the function.**

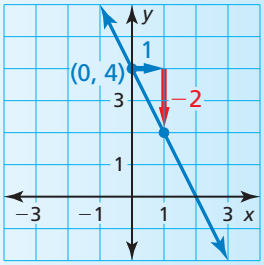
40.

$y + 1 = 3x$



41.

$-4x + y = -2$



42. **MATHEMATICAL CONNECTIONS** Graph the four equations in the same coordinate plane.

$$3y = -x - 3$$
$$2y - 14 = 4x$$
$$4x - 3 - y = 0$$
$$x - 12 = -3y$$

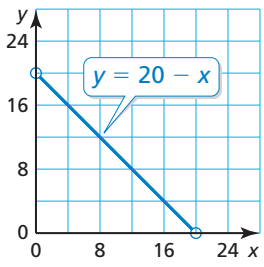
a. What enclosed shape do you think the lines form? Explain.

b. Write a conjecture about the equations of parallel lines.

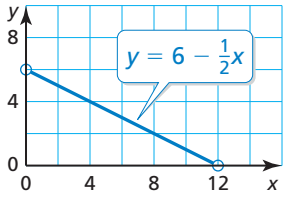
43. **MATHEMATICAL CONNECTIONS** The graph shows the relationship between the width y and the length x of a rectangle in inches. The perimeter of a second rectangle is 10 inches less than the perimeter of the first rectangle.

a. Graph the relationship between the width and length of the second rectangle.

b. How does the graph in part (a) compare to the the graph shown?



44. **MATHEMATICAL CONNECTIONS** The graph shows the relationship between the base length x and the side length (of the two equal sides) y of an isosceles triangle in meters. The perimeter of a second isosceles triangle is 8 meters more than the perimeter of the first triangle.



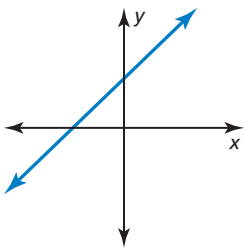
a. Graph the relationship between the base length and the side length of the second triangle.

b. How does the graph in part (a) compare to the graph shown?

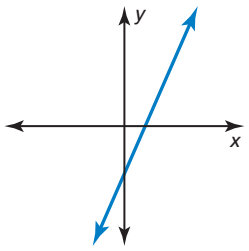
45. **ANALYZING EQUATIONS** Determine which of the equations could be represented by each graph.

$y = -3x + 8$	$y = -x - \frac{4}{3}$
$y = -7x$	$y = 2x - 4$
$y = \frac{7}{4}x - \frac{1}{4}$	$y = \frac{1}{3}x + 5$
$y = -4x - 9$	$y = 6$

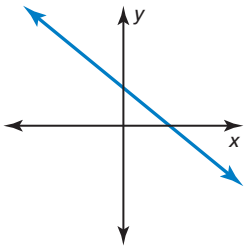
a.



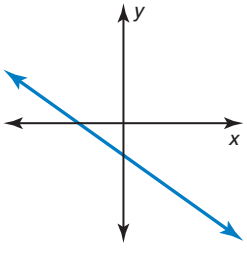
b.



c.



d.

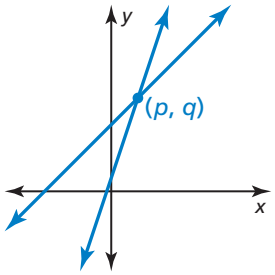


46. **MAKING AN ARGUMENT** Your friend says that you can write the equation of any line in slope-intercept form. Is your friend correct? Explain your reasoning.

47. WRITING Write the definition of the slope of a line in two different ways.

48. THOUGHT PROVOKING Your family goes on vacation to a beach 300 miles from your house. You reach your destination 6 hours after departing. Draw a graph that describes your trip. Explain what each part of your graph represents.

49. ANALYZING A GRAPH The graphs of the functions $g(x) = 6x + a$ and $h(x) = 2x + b$, where a and b are constants, are shown. They intersect at the point (p, q).

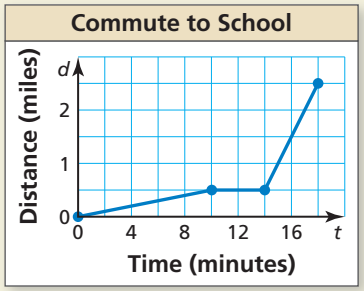


a. Label the graphs of g and h.

b. What do a and b represent?

c. Starting at the point (p, q), trace the graph of g until you get to the point with the x-coordinate $p + 2$. Mark this point C. Do the same with the graph of h. Mark this point D. How much greater is the y-coordinate of point C than the y-coordinate of point D?

50. HOW DO YOU SEE IT? You commute to school by walking and by riding a bus. The graph represents your commute.



a. Describe your commute in words.

b. Calculate and interpret the slopes of the different parts of the graph.

PROBLEM SOLVING **In Exercises 51 and 52, find the value of k so that the graph of the equation has the given slope or y-intercept.**

51. $y = 4kx - 5$; $m = \frac{1}{2}$

52. $y = -\frac{1}{3}x + \frac{5}{6}k$; $b = -10$

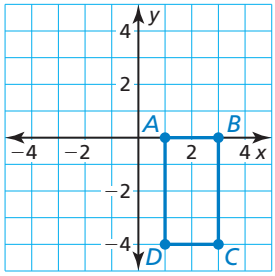
53. ABSTRACT REASONING To show that the slope of a line is constant, let (x_1, y_1) and (x_2, y_2) be any two points on the line $y = mx + b$. Use the equation of the line to express y_1 in terms of x_1 and y_2 in terms of x_2. Then use the slope formula to show that the slope between the points is m.

Maintaining Mathematical Proficiency

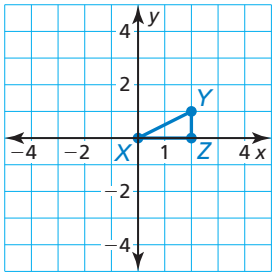
Reviewing what you learned in previous grades and lessons

Find the coordinates of the figure after the transformation. *(Skills Review Handbook)*

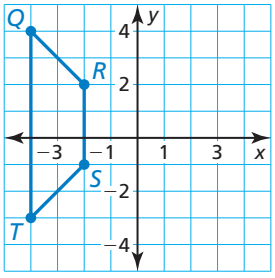
54. Translate the rectangle 4 units left.



55. Dilate the triangle with respect to the origin using a scale factor of 2.



56. Reflect the trapezoid in the y-axis.



Determine whether the equation represents a *linear* or *nonlinear* function. Explain. *(Section 3.2)*

57. $y - 9 = \frac{2}{x}$ **58.** $x = 3 + 15y$ **59.** $\frac{x}{4} + \frac{y}{12} = 1$ **60.** $y = 3x^4 - 6$

3.6 Transformations of Graphs of Linear Functions

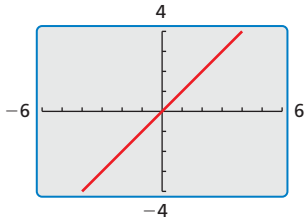
Essential Question How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

USING TOOLS STRATEGICALLY

To be proficient in math, you need to use the appropriate tools, including graphs, tables, and technology, to check your results.

EXPLORATION 1 Comparing Graphs of Functions

Work with a partner. The graph of $f(x) = x$ is shown. Sketch the graph of each function, along with f, on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?



a. $g(x) = x + 4$ **b.** $g(x) = x + 2$

c. $g(x) = x - 2$ **d.** $g(x) = x - 4$

EXPLORATION 2 Comparing Graphs of Functions

Work with a partner. Sketch the graph of each function, along with $f(x) = x$, on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?

a. $h(x) = \frac{1}{2}x$ **b.** $h(x) = 2x$ **c.** $h(x) = -\frac{1}{2}x$ **d.** $h(x) = -2x$

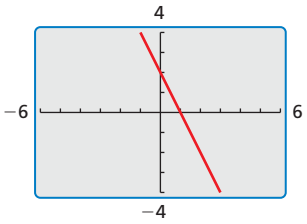
EXPLORATION 3 Matching Functions with Their Graphs

Work with a partner. Match each function with its graph. Use a graphing calculator to check your results. Then use the results of Explorations 1 and 2 to compare the graph of k to the graph of $f(x) = x$.

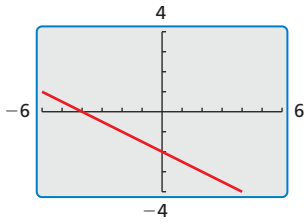
a. $k(x) = 2x - 4$ **b.** $k(x) = -2x + 2$

c. $k(x) = \frac{1}{2}x + 4$ **d.** $k(x) = -\frac{1}{2}x - 2$

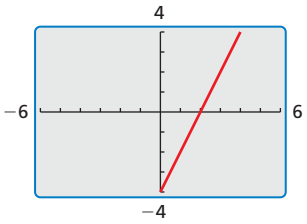
A.



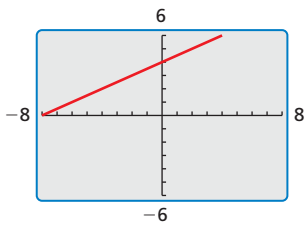
B.



C.



D.



Communicate Your Answer

4. How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

3.6 Lesson

What You Will Learn

- Translate and reflect graphs of linear functions.
- Stretch and shrink graphs of linear functions.
- Combine transformations of graphs of linear functions.

Core Vocabulary

family of functions, *p. 146*
parent function, *p. 146*
transformation, *p. 146*
translation, *p. 146*
reflection, *p. 147*
horizontal shrink, *p. 148*
horizontal stretch, *p. 148*
vertical stretch, *p. 148*
vertical shrink, *p. 148*

Previous
linear function

Translations and Reflections

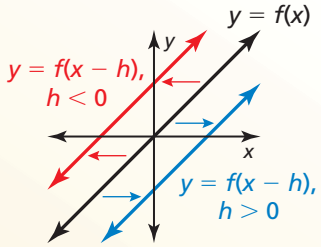
A **family of functions** is a group of functions with similar characteristics. The most basic function in a family of functions is the **parent function**. For nonconstant linear functions, the parent function is $f(x) = x$. The graphs of all other nonconstant linear functions are *transformations* of the graph of the parent function. A **transformation** changes the size, shape, position, or orientation of a graph.

Core Concept

A **translation** is a transformation that shifts a graph horizontally or vertically but does not change the size, shape, or orientation of the graph.

Horizontal Translations

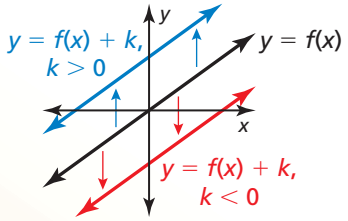
The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.



Subtracting h from the *inputs* before evaluating the function shifts the graph left when $h < 0$ and right when $h > 0$.

Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.



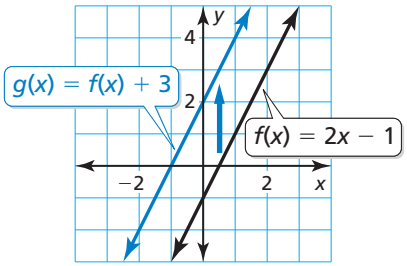
Adding k to the *outputs* shifts the graph down when $k < 0$ and up when $k > 0$.

EXAMPLE 1 Horizontal and Vertical Translations

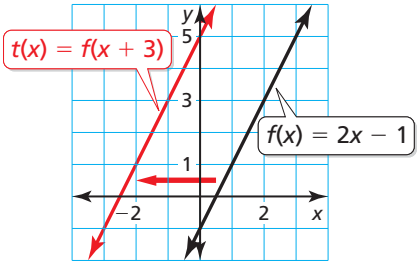
Let $f(x) = 2x - 1$. Graph (a) $g(x) = f(x) + 3$ and (b) $t(x) = f(x + 3)$. Describe the transformations from the graph of f to the graphs of g and t.

SOLUTION

a. The function g is of the form $y = f(x) + k$, where $k = 3$. So, the graph of g is a vertical translation 3 units up of the graph of f.



b. The function t is of the form $y = f(x - h)$, where $h = -3$. So, the graph of t is a horizontal translation 3 units left of the graph of f.



LOOKING FOR A PATTERN

In part (a), the output of g is equal to the output of f plus 3.

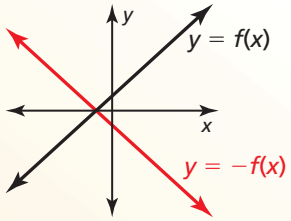
In part (b), the output of t is equal to the output of f when the input of f is 3 more than the input of t.

Core Concept

A **reflection** is a transformation that flips a graph over a line called the *line of reflection*.

Reflections in the *x*-axis

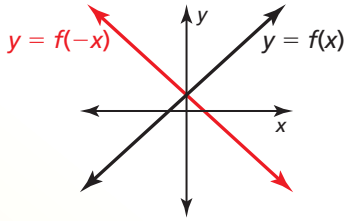
The graph of $y = -f(x)$ is a reflection in the x-axis of the graph of $y = f(x)$.



Multiplying the outputs by -1 changes their signs.

Reflections in the *y*-axis

The graph of $y = f(-x)$ is a reflection in the y-axis of the graph of $y = f(x)$.



Multiplying the inputs by -1 changes their signs.

STUDY TIP

A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

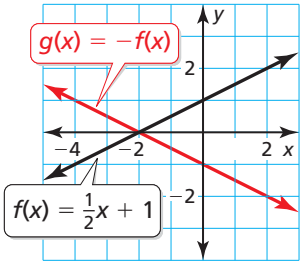
EXAMPLE 2 Reflections in the *x*-axis and the *y*-axis

Let $f(x) = \frac{1}{2}x + 1$. Graph (a) $g(x) = -f(x)$ and (b) $t(x) = f(-x)$. Describe the transformations from the graph of f to the graphs of g and t.

SOLUTION

a. To find the outputs of g, multiply the outputs of f by -1. The graph of g consists of the points $(x, -f(x))$.

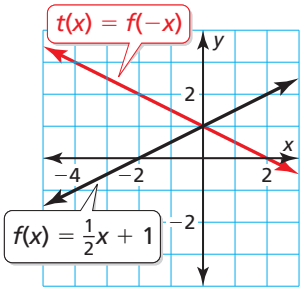
x	-4	-2	0
$f(x)$	-1	0	1
$-f(x)$	1	0	-1



The graph of g is a reflection in the x-axis of the graph of f.

b. To find the outputs of t, multiply the inputs by -1 and then evaluate f. The graph of t consists of the points $(x, f(-x))$.

x	-2	0	2
$-x$	2	0	-2
$f(-x)$	2	1	0



The graph of t is a reflection in the y-axis of the graph of f.

Monitoring Progress

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Using f, graph (a) g and (b) h. Describe the transformations from the graph of f to the graphs of g and h.

1. $f(x) = 3x + 1$; $g(x) = f(x) - 2$; $h(x) = f(x - 2)$
2. $f(x) = -4x - 2$; $g(x) = -f(x)$; $h(x) = f(-x)$

Stretches and Shrinks

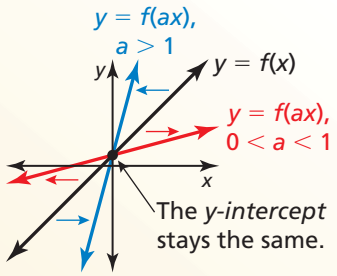
You can transform a function by multiplying all the x-coordinates (inputs) by the same factor a. When $a > 1$, the transformation is a **horizontal shrink** because the graph shrinks toward the y-axis. When $0 < a < 1$, the transformation is a **horizontal stretch** because the graph stretches away from the y-axis. In each case, the y-intercept stays the same.

You can also transform a function by multiplying all the y-coordinates (outputs) by the same factor a. When $a > 1$, the transformation is a **vertical stretch** because the graph stretches away from the x-axis. When $0 < a < 1$, the transformation is a **vertical shrink** because the graph shrinks toward the x-axis. In each case, the x-intercept stays the same.

Core Concept

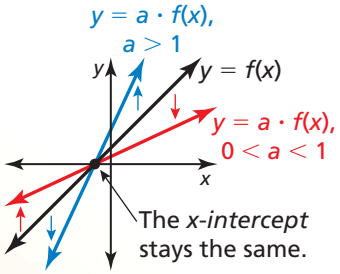
Horizontal Stretches and Shrinks

The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



Vertical Stretches and Shrinks

The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of a of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.



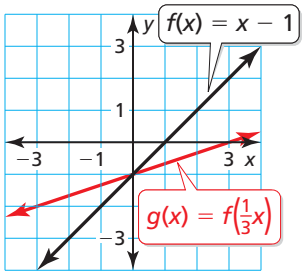
STUDY TIP

The graphs of $y = f(-ax)$ and $y = -a \cdot f(x)$ represent a stretch or shrink *and* a reflection in the x- or y-axis of the graph of $y = f(x)$.

EXAMPLE 3 Horizontal and Vertical Stretches

Let $f(x) = x - 1$. Graph (a) $g(x) = f\left(\frac{1}{3}x\right)$ and (b) $h(x) = 3f(x)$. Describe the transformations from the graph of f to the graphs of g and h.

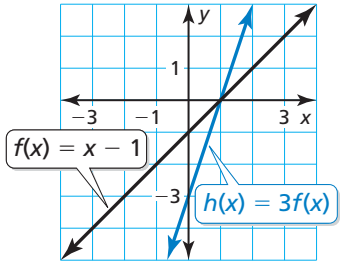
SOLUTION



a. To find the outputs of g, multiply the inputs by $\frac{1}{3}$. Then evaluate f. The graph of g consists of the points $\left(x, f\left(\frac{1}{3}x\right)\right)$.

x	-3	0	3
$\frac{1}{3}(x)$	-1	0	1
$f\left(\frac{1}{3}x\right)$	-2	-1	0

▶ The graph of g is a horizontal stretch of the graph of f by a factor of $1 \div \frac{1}{3} = 3$.



b. To find the outputs of h, multiply the outputs of f by 3. The graph of h consists of the points $(x, 3f(x))$.

x	0	1	2
$f(x)$	-1	0	1
$3f(x)$	-3	0	3

▶ The graph of h is a vertical stretch of the graph of f by a factor of 3.

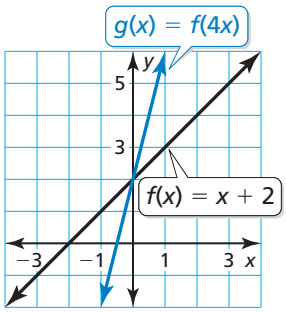
EXAMPLE 4 Horizontal and Vertical Shrinks

Let $f(x) = x + 2$. Graph (a) $g(x) = f(4x)$ and (b) $h(x) = \frac{1}{4}f(x)$. Describe the transformations from the graph of f to the graphs of g and h.

SOLUTION

a. To find the outputs of g, multiply the inputs by 4. Then evaluate f. The graph of g consists of the points $(x, f(4x))$.

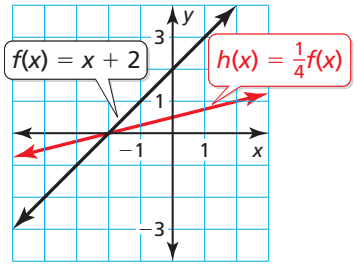
x	-1	0	1
$4x$	-4	0	4
$f(4x)$	-2	2	6



▶ The graph of g is a horizontal shrink of the graph of f by a factor of $\frac{1}{4}$.

b. To find the outputs of h, multiply the outputs of f by $\frac{1}{4}$. The graph of h consists of the points $\left(x, \frac{1}{4}f(x)\right)$.

x	-2	0	2
$f(x)$	0	2	4
$\frac{1}{4}f(x)$	0	$\frac{1}{2}$	1



▶ The graph of h is a vertical shrink of the graph of f by a factor of $\frac{1}{4}$.

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Using f, graph (a) g and (b) h. Describe the transformations from the graph of f to the graphs of g and h.

3. $f(x) = 4x - 2$; $g(x) = f\left(\frac{1}{2}x\right)$; $h(x) = 2f(x)$

4. $f(x) = -3x + 4$; $g(x) = f(2x)$; $h(x) = \frac{1}{2}f(x)$

STUDY TIP

You can perform transformations on the graph of *any* function f using these steps.

Combining Transformations

Core Concept

Transformations of Graphs

The graph of $y = a \cdot f(x - h) + k$ or the graph of $y = f(ax - h) + k$ can be obtained from the graph of $y = f(x)$ by performing these steps.

Step 1 Translate the graph of $y = f(x)$ horizontally h units.

Step 2 Use a to stretch or shrink the resulting graph from Step 1.

Step 3 Reflect the resulting graph from Step 2 when $a < 0$.

Step 4 Translate the resulting graph from Step 3 vertically k units.

EXAMPLE 5 Combining Transformations

Graph $f(x) = x$ and $g(x) = -2x + 3$. Describe the transformations from the graph of f to the graph of g.

SOLUTION

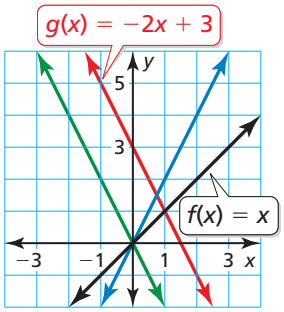
Note that you can rewrite g as $g(x) = -2f(x) + 3$.

Step 1 There is no horizontal translation from the graph of f to the graph of g.

Step 2 Stretch the graph of f vertically by a factor of 2 to get the graph of $h(x) = 2x$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -2x$.

Step 4 Translate the graph of r vertically 3 units up to get the graph of $g(x) = -2x + 3$.



ANOTHER WAY

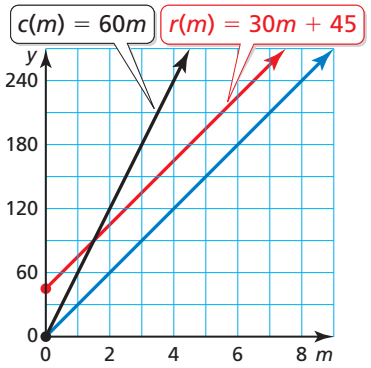
You could also rewrite g as $g(x) = f(-2x) + 3$. In this case, the transformations from the graph of f to the graph of g will be different from those in Example 5.

EXAMPLE 6 Solving a Real-Life Problem

A cable company charges customers \$60 per month for its service, with no installation fee. The cost to a customer is represented by $c(m) = 60m$, where m is the number of months of service. To attract new customers, the cable company reduces the monthly fee to \$30 but adds an installation fee of \$45. The cost to a new customer is represented by $r(m) = 30m + 45$, where m is the number of months of service. Describe the transformations from the graph of c to the graph of r.

SOLUTION

Note that you can rewrite r as $r(m) = \frac{1}{2}c(m) + 45$. In this form, you can use the order of operations to get the outputs of r from the outputs of c. First, multiply the outputs of c by $\frac{1}{2}$ to get $h(m) = 30m$. Then add 45 to the outputs of h to get $r(m) = 30m + 45$.



▶ The transformations are a vertical shrink by a factor of $\frac{1}{2}$ and then a vertical translation 45 units up.

Monitoring Progress

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5. Graph $f(x) = x$ and $h(x) = \frac{1}{4}x - 2$. Describe the transformations from the graph of f to the graph of h.

3.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

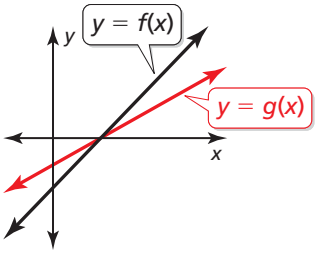
Vocabulary and Core Concept Check

1. **WRITING** Describe the relationship between $f(x) = x$ and all other nonconstant linear functions.

2. **VOCABULARY** Name four types of transformations. Give an example of each and describe how it affects the graph of a function.

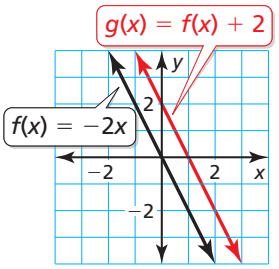
3. **WRITING** How does the value of a in the equation $y = f(ax)$ affect the graph of $y = f(x)$? How does the value of a in the equation $y = af(x)$ affect the graph of $y = f(x)$?

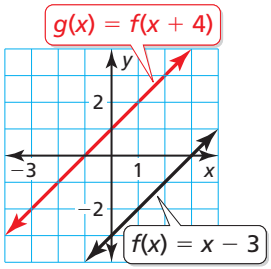
4. **REASONING** The functions f and g are linear functions. The graph of g is a vertical shrink of the graph of f. What can you say about the x-intercepts of the graphs of f and g? Is this always true? Explain.



Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, use the graphs of f and g to describe the transformation from the graph of f to the graph of g. *(See Example 1.)*

5. 

6. 

7. $f(x) = \frac{1}{3}x + 3$; $g(x) = f(x) - 3$

8. $f(x) = -3x + 4$; $g(x) = f(x) + 1$

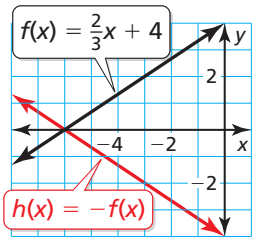
9. $f(x) = -x - 2$; $g(x) = f(x + 5)$

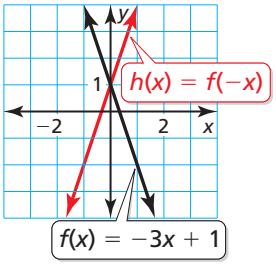
10. $f(x) = \frac{1}{2}x - 5$; $g(x) = f(x - 3)$

11. **MODELING WITH MATHEMATICS** You and a friend start biking from the same location. Your distance d (in miles) after t minutes is given by the function $d(t) = \frac{1}{5}t$. Your friend starts biking 5 minutes after you. Your friend's distance f is given by the function $f(t) = d(t - 5)$. Describe the transformation from the graph of d to the graph of f.

12. **MODELING WITH MATHEMATICS** The total cost C (in dollars) to cater an event with p people is given by the function $C(p) = 18p + 50$. The set-up fee increases by \$25. The new total cost T is given by the function $T(p) = C(p) + 25$. Describe the transformation from the graph of C to the graph of T.

In Exercises 13–16, use the graphs of f and h to describe the transformation from the graph of f to the graph of h. *(See Example 2.)*

13. 

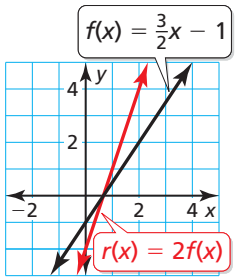
14. 

15. $f(x) = -5 - x$; $h(x) = f(-x)$

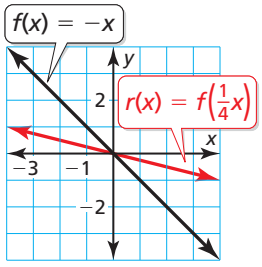
16. $f(x) = \frac{1}{4}x - 2$; $h(x) = -f(x)$

In Exercises 17–22, use the graphs of f and r to describe the transformation from the graph of f to the graph of r. *(See Example 3.)*

17.



18.



19. $f(x) = -2x - 4$; $r(x) = f\left(\frac{1}{2}x\right)$

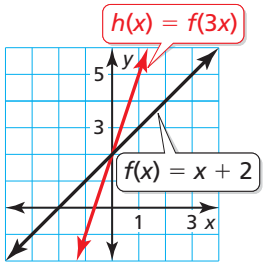
20. $f(x) = 3x + 5$; $r(x) = f\left(\frac{1}{3}x\right)$

21. $f(x) = \frac{2}{3}x + 1$; $r(x) = 3f(x)$

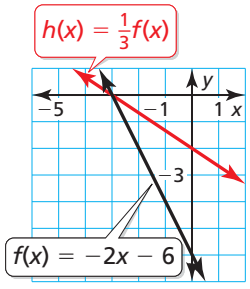
22. $f(x) = -\frac{1}{4}x - 2$; $r(x) = 4f(x)$

In Exercises 23–28, use the graphs of f and h to describe the transformation from the graph of f to the graph of h. *(See Example 4.)*

23.



24.



25. $f(x) = 3x - 12$; $h(x) = \frac{1}{6}f(x)$

26. $f(x) = -x + 1$; $h(x) = f(2x)$

27. $f(x) = -2x - 2$; $h(x) = f(5x)$

28. $f(x) = 4x + 8$; $h(x) = \frac{3}{4}f(x)$

In Exercises 29–34, use the graphs of f and g to describe the transformation from the graph of f to the graph of g.

29. $f(x) = x - 2$; $g(x) = \frac{1}{4}f(x)$

30. $f(x) = -4x + 8$; $g(x) = -f(x)$

31. $f(x) = -2x - 7$; $g(x) = f(x - 2)$

32. $f(x) = 3x + 8$; $g(x) = f\left(\frac{2}{3}x\right)$

33. $f(x) = x - 6$; $g(x) = 6f(x)$

34. $f(x) = -x$; $g(x) = f(x) - 3$

In Exercises 35–38, write a function g in terms of f so that the statement is true.

35. The graph of g is a horizontal translation 2 units right of the graph of f.

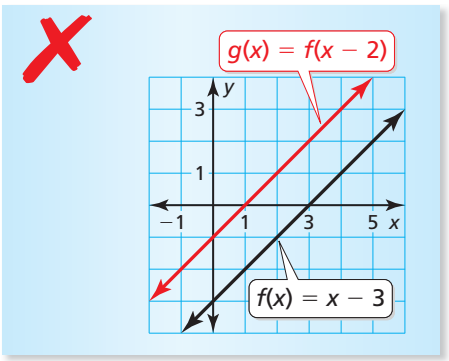
36. The graph of g is a reflection in the y-axis of the graph of f.

37. The graph of g is a vertical stretch by a factor of 4 of the graph of f.

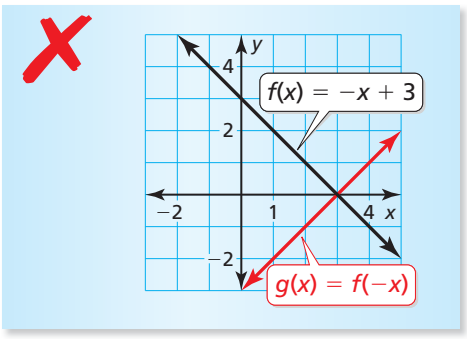
38. The graph of g is a horizontal shrink by a factor of $\frac{1}{5}$ of the graph of f.

ERROR ANALYSIS **In Exercises 39 and 40, describe and correct the error in graphing g.**

39.



40.



In Exercises 41–46, graph f and h. Describe the transformations from the graph of f to the graph of h. *(See Example 5.)*

41. $f(x) = x$; $h(x) = \frac{1}{3}x + 1$

42. $f(x) = x$; $h(x) = 4x - 2$

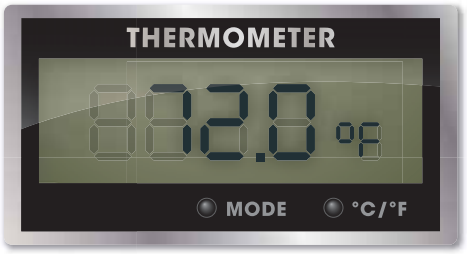
43. $f(x) = x$; $h(x) = -3x - 4$

44. $f(x) = x$; $h(x) = -\frac{1}{2}x + 3$

45. $f(x) = 2x$; $h(x) = 6x - 5$

46. $f(x) = 3x$; $h(x) = -3x - 7$

47. MODELING WITH MATHEMATICS The function $t(x) = -4x + 72$ represents the temperature from 5 P.M. to 11 P.M., where x is the number of hours after 5 P.M. The function $d(x) = 4x + 72$ represents the temperature from 10 A.M. to 4 P.M., where x is the number of hours after 10 A.M. Describe the transformation from the graph of t to the graph of d.



48. MODELING WITH MATHEMATICS A school sells T-shirts to promote school spirit. The school's profit is given by the function $P(x) = 8x - 150$, where x is the number of T-shirts sold. During the play-offs, the school increases the price of the T-shirts. The school's profit during the play-offs is given by the function $Q(x) = 16x - 200$, where x is the number of T-shirts sold. Describe the transformations from the graph of P to the graph of Q. *(See Example 6.)*

49. USING STRUCTURE The graph of $g(x) = a \cdot f(x - b) + c$ is a transformation of the graph of the linear function f. Select the word or value that makes each statement true.

reflection	translation	-1
stretch	shrink	0
left	right	1
y-axis	x-axis	

a. The graph of g is a vertical _____ of the graph of f when $a = 4$, $b = 0$, and $c = 0$.

b. The graph of g is a horizontal translation _____ of the graph of f when $a = 1$, $b = 2$, and $c = 0$.

c. The graph of g is a vertical translation 1 unit up of the graph of f when $a = 1$, $b = 0$, and $c =$ ____.

50. USING STRUCTURE The graph of $h(x) = a \cdot f(bx - c) + d$ is a transformation of the graph of the linear function f. Select the word or value that makes each statement true.

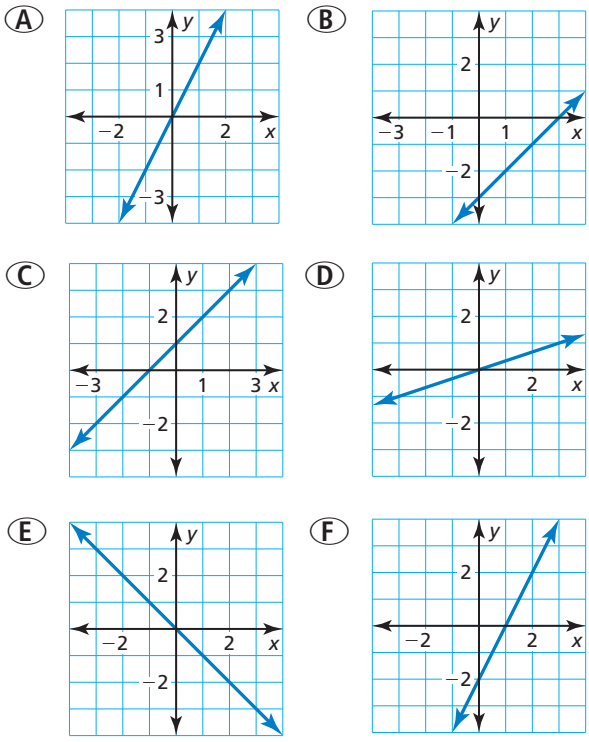
vertical	horizontal	0
stretch	shrink	$\frac{1}{5}$
y-axis	x-axis	5

a. The graph of h is a _____ shrink of the graph of f when $a = \frac{1}{3}$, $b = 1$, $c = 0$, and $d = 0$.

b. The graph of h is a reflection in the _____ of the graph of f when $a = 1$, $b = -1$, $c = 0$, and $d = 0$.

c. The graph of h is a horizontal stretch of the graph of f by a factor of 5 when $a = 1$, $b =$ _____, $c = 0$, and $d = 0$.

51. ANALYZING GRAPHS Which of the graphs are related by only a translation? Explain.



52. ANALYZING RELATIONSHIPS A swimming pool is filled with water by a hose at a rate of 1020 gallons per hour. The amount v (in gallons) of water in the pool after t hours is given by the function $v(t) = 1020t$. How does the graph of v change in each situation?

a. A larger hose is found. Then the pool is filled at a rate of 1360 gallons per hour.

b. Before filling up the pool with a hose, a water truck adds 2000 gallons of water to the pool.

53. ANALYZING RELATIONSHIPS You have \$50 to spend on fabric for a blanket. The amount m (in dollars) of money you have after buying y yards of fabric is given by the function $m(y) = -9.98y + 50$. How does the graph of m change in each situation?

a. You receive an additional \$10 to spend on the fabric.

b. The fabric goes on sale, and each yard now costs \$4.99.

54. THOUGHT PROVOKING Write a function g whose graph passes through the point (4, 2) and is a transformation of the graph of $f(x) = x$.

In Exercises 55–60, graph f and g. Write g in terms of f. Describe the transformation from the graph of f to the graph of g.

55. $f(x) = 2x - 5$; $g(x) = 2x - 8$

56. $f(x) = 4x + 1$; $g(x) = -4x - 1$

57. $f(x) = 3x + 9$; $g(x) = 3x + 15$

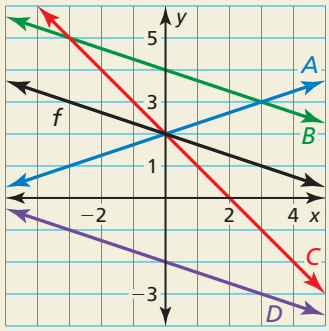
58. $f(x) = -x - 4$; $g(x) = x - 4$

59. $f(x) = x + 2$; $g(x) = \frac{2}{3}x + 2$

60. $f(x) = x - 1$; $g(x) = 3x - 3$

61. REASONING The graph of $f(x) = x + 5$ is a vertical translation 5 units up of the graph of $f(x) = x$. How can you obtain the graph of $f(x) = x + 5$ from the graph of $f(x) = x$ using a horizontal translation?

62. HOW DO YOU SEE IT? Match each function with its graph. Explain your reasoning.

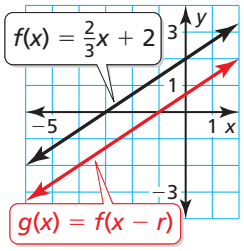


a. $a(x) = f(-x)$ **b.** $g(x) = f(x) - 4$

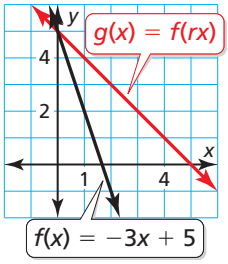
c. $h(x) = f(x) + 2$ **d.** $k(x) = f(3x)$

REASONING In Exercises 63–66, find the value of r.

63.



64.



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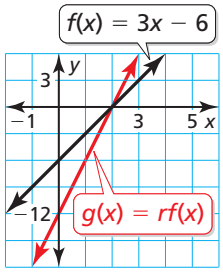
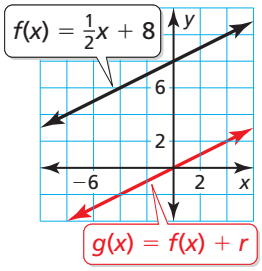


66.



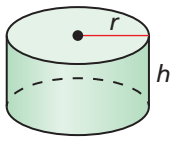
67. CRITICAL THINKING When is the graph of $y = f(x) + w$ the same as the graph of $y = f(x + w)$ for linear functions? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

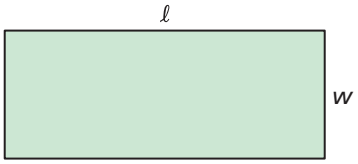
Solve the formula for the indicated variable. *(Section 1.5)*

68. Solve for h.



$V = \pi r^2 h$

69. Solve for w.



$P = 2\ell + 2w$

Solve the inequality. Graph the solution, if possible. *(Section 2.6)*

70. $|x - 3| \leq 14$ **71.** $|2x + 4| > 16$ **72.** $5|x + 7| < 25$ **73.** $-2|x + 1| \geq 18$

3.7 Graphing Absolute Value Functions

Essential Question How do the values of a, h, and k affect the graph of the absolute value function $g(x) = a|x - h| + k$?

The parent absolute value function is

$f(x) = |x|$. Parent absolute value function

The graph of f is V-shaped.

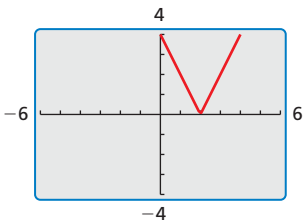
EXPLORATION 1 Identifying Graphs of Absolute Value Functions

Work with a partner. Match each absolute value function with its graph. Then use a graphing calculator to verify your answers.

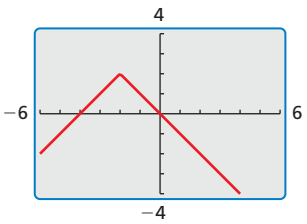
a. $g(x) = -|x - 2|$ **b.** $g(x) = |x - 2| + 2$ **c.** $g(x) = -|x + 2| - 2$

d. $g(x) = |x - 2| - 2$ **e.** $g(x) = 2|x - 2|$ **f.** $g(x) = -|x + 2| + 2$

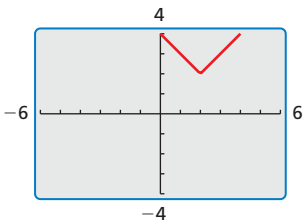
A.



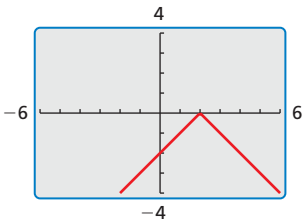
B.



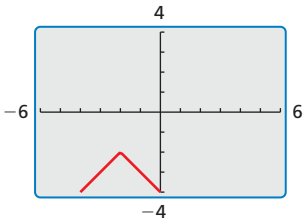
C.



D.



E.



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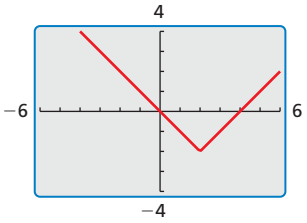


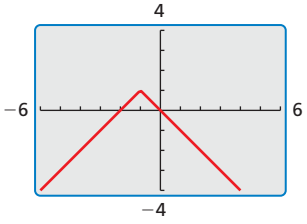
LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

2. How do the values of a, h, and k affect the graph of the absolute value function $g(x) = a|x - h| + k$?

3. Write the equation of the absolute value function whose graph is shown. Use a graphing calculator to verify your equation.



3.7 Lesson

What You Will Learn

- Translate graphs of absolute value functions.
- Stretch, shrink, and reflect graphs of absolute value functions.
- Combine transformations of graphs of absolute value functions.

Core Vocabulary

absolute value function, *p. 156*
vertex, *p. 156*
vertex form, *p. 158*

Previous
domain
range

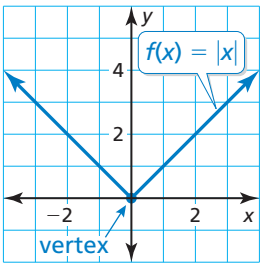
Translating Graphs of Absolute Value Functions

Core Concept

Absolute Value Function

An **absolute value function** is a function that contains an absolute value expression. The parent absolute value function is $f(x) = |x|$. The graph of $f(x) = |x|$ is V-shaped and symmetric about the y-axis. The **vertex** is the point where the graph changes direction. The vertex of the graph of $f(x) = |x|$ is $(0, 0)$.

The domain of $f(x) = |x|$ is all real numbers. The range is $y \geq 0$.



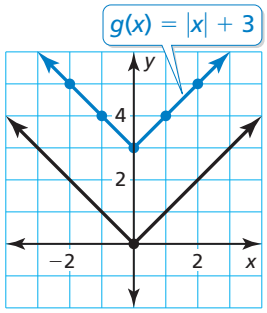
The graphs of all other absolute value functions are transformations of the graph of the parent function $f(x) = |x|$. The transformations presented in Section 3.6 also apply to absolute value functions.

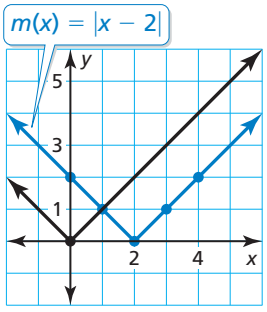
EXAMPLE 1 Graphing $g(x) = |x| + k$ and $g(x) = |x - h|$

Graph each function. Compare each graph to the graph of $f(x) = |x|$. Describe the domain and range.

a. $g(x) = |x| + 3$

b. $m(x) = |x - 2|$





SOLUTION

a. Step 1 Make a table of values.

x	-2	-1	0	1	2
$g(x)$	5	4	3	4	5

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.

The function g is of the form $y = f(x) + k$, where $k = 3$. So, the graph of g is a vertical translation 3 units up of the graph of f. The domain is all real numbers. The range is $y \geq 3$.

b. Step 1 Make a table of values.

x	0	1	2	3	4
$m(x)$	2	1	0	1	2

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.

The function m is of the form $y = f(x - h)$, where $h = 2$. So, the graph of m is a horizontal translation 2 units right of the graph of f. The domain is all real numbers. The range is $y \geq 0$.

Monitoring Progress

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Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range.

1. $h(x) = |x| - 1$

2. $n(x) = |x + 4|$

Stretching, Shrinking, and Reflecting

EXAMPLE 2 Graphing $g(x) = a|x|$

Graph each function. Compare each graph to the graph of $f(x) = |x|$. Describe the domain and range.

a. $q(x) = 2|x|$ **b.** $p(x) = -\frac{1}{2}|x|$

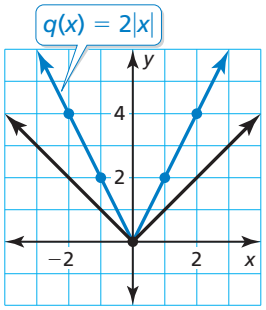
SOLUTION

a. Step 1 Make a table of values.

x	-2	-1	0	1	2
$q(x)$	4	2	0	2	4

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.



STUDY TIP

A vertical stretch of the graph of $f(x) = |x|$ is *narrower* than the graph of $f(x) = |x|$.

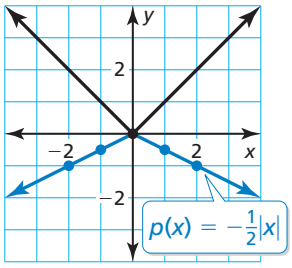
▶ The function q is of the form $y = a \cdot f(x)$, where $a = 2$. So, the graph of q is a vertical stretch of the graph of f by a factor of 2. The domain is all real numbers. The range is $y \geq 0$.

b. Step 1 Make a table of values.

x	-2	-1	0	1	2
$p(x)$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.



STUDY TIP

A vertical shrink of the graph of $f(x) = |x|$ is *wider* than the graph of $f(x) = |x|$.

▶ The function p is of the form $y = -a \cdot f(x)$, where $a = \frac{1}{2}$. So, the graph of p is a vertical shrink of the graph of f by a factor of $\frac{1}{2}$ and a reflection in the x-axis. The domain is all real numbers. The range is $y \leq 0$.

Monitoring Progress

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Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range.

3. $t(x) = -3|x|$ **4.** $v(x) = \frac{1}{4}|x|$

Core Concept

Vertex Form of an Absolute Value Function

An absolute value function written in the form $g(x) = a|x - h| + k$, where $a \neq 0$, is in **vertex form**. The vertex of the graph of g is (h, k).

Any absolute value function can be written in vertex form, and its graph is symmetric about the line $x = h$.

STUDY TIP

The function g is *not* in vertex form because the x variable does not have a coefficient of 1.

EXAMPLE 3 Graphing $f(x) = |x - h| + k$ and $g(x) = f(ax)$

Graph $f(x) = |x + 2| - 3$ and $g(x) = |2x + 2| - 3$. Compare the graph of g to the graph of f.

SOLUTION

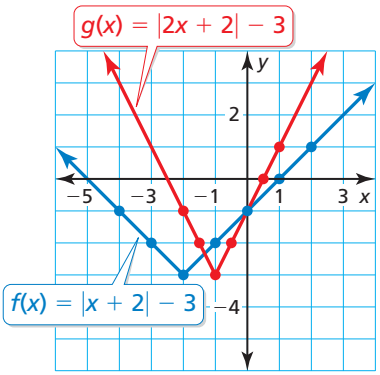
Step 1 Make a table of values for each function.

x	-4	-3	-2	-1	0	1	2
$f(x)$	-1	-2	-3	-2	-1	0	1

x	-2	-1.5	-1	-0.5	0	0.5	1
$g(x)$	-1	-2	-3	-2	-1	0	1

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph of each function. Notice that the vertex of the graph of f is $(-2, -3)$ and the graph is symmetric about $x = -2$.



Note that you can rewrite g as $g(x) = f(2x)$, which is of the form $y = f(ax)$, where $a = 2$. So, the graph of g is a horizontal shrink of the graph of f by a factor of $\frac{1}{2}$. The y-intercept is the same for both graphs. The points on the graph of f move halfway closer to the y-axis, resulting in the graph of g. When the input values of f are 2 times the input values of g, the output values of f and g are the same.

Monitoring Progress

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5. Graph $f(x) = |x - 1|$ and $g(x) = \left|\frac{1}{2}x - 1\right|$. Compare the graph of g to the graph of f.

6. Graph $f(x) = |x + 2| + 2$ and $g(x) = |-4x + 2| + 2$. Compare the graph of g to the graph of f.

Combining Transformations

EXAMPLE 4 **Graphing $g(x) = a|x - h| + k$**

Let $g(x) = -2|x - 1| + 3$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.

> **REMEMBER**
> You can obtain the graph of $y = a \cdot f(x - h) + k$ from the graph of $y = f(x)$ using the steps you learned in Section 3.6.

SOLUTION

a. Step 1 Translate the graph of f horizontally 1 unit right to get the graph of $t(x) = |x - 1|$.

Step 2 Stretch the graph of t vertically by a factor of 2 to get the graph of $h(x) = 2|x - 1|$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -2|x - 1|$.

Step 4 Translate the graph of r vertically 3 units up to get the graph of $g(x) = -2|x - 1| + 3$.

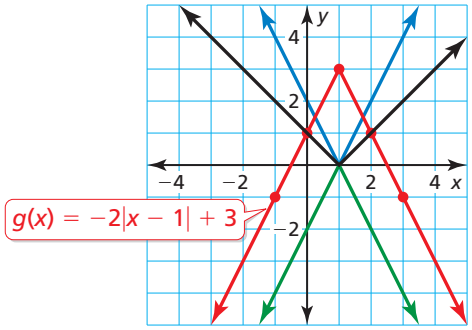
b. Method 1

Step 1 Make a table of values.

x	-1	0	1	2	3
$g(x)$	-1	1	3	1	-1

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.

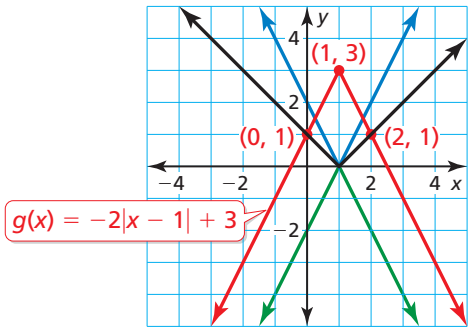


Method 2

Step 1 Identify and plot the vertex. $(h, k) = (1, 3)$

Step 2 Plot another point on the graph, such as $(2, 1)$. Because the graph is symmetric about the line $x = 1$, you can use symmetry to plot a third point, $(0, 1)$.

Step 3 Draw the V-shaped graph.



Monitoring Progress

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7. Let $g(x) = \left|-\frac{1}{2}x + 2\right| + 1$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.

3.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The point $(1, -4)$ is the _____ of the graph of $f(x) = -3|x - 1| - 4$.
2. **USING STRUCTURE** How do you know whether the graph of $f(x) = a|x - h| + k$ is a vertical stretch or a vertical shrink of the graph of $f(x) = |x|$?
3. **WRITING** Describe three different types of transformations of the graph of an absolute value function.
4. **REASONING** The graph of which function has the same y-intercept as the graph of $f(x) = |x - 2| + 5$? Explain.

$g(x) = |3x - 2| + 5$ $\quad$ $h(x) = 3|x - 2| + 5$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range. *(See Examples 1 and 2.)*

5. $d(x) = |x| - 4$
6. $r(x) = |x| + 5$
7. $m(x) = |x + 1|$
8. $v(x) = |x - 3|$
9. $p(x) = \frac{1}{3}|x|$
10. $j(x) = 3|x|$
11. $a(x) = -5|x|$
12. $q(x) = -\frac{3}{2}|x|$

In Exercises 13–16, graph the function. Compare the graph to the graph of $f(x) = |x - 6|$.

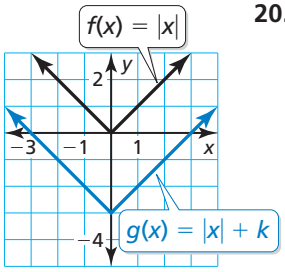
13. $h(x) = |x - 6| + 2$
14. $n(x) = \frac{1}{2}|x - 6|$
15. $k(x) = -3|x - 6|$
16. $g(x) = |x - 1|$

In Exercises 17 and 18, graph the function. Compare the graph to the graph of $f(x) = |x + 3| - 2$.

17. $y(x) = |x + 4| - 2$
18. $b(x) = |x + 3| + 3$

In Exercises 19–22, compare the graphs. Find the value of h, k, or a.

19.



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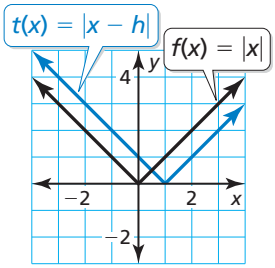

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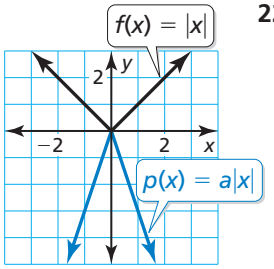

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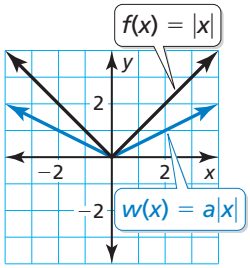


In Exercises 23–26, write an equation that represents the given transformation(s) of the graph of $g(x) = |x|$.

23. vertical translation 7 units down
24. horizontal translation 10 units left
25. vertical shrink by a factor of $\frac{1}{4}$
26. vertical stretch by a factor of 3 and a reflection in the x-axis

In Exercises 27–32, graph and compare the two functions. *(See Example 3.)*

27. $f(x) = |x - 4|$; $g(x) = |3x - 4|$
28. $h(x) = |x + 5|$; $t(x) = |2x + 5|$
29. $p(x) = |x + 1| - 2$; $q(x) = \left|\frac{1}{4}x + 1\right| - 2$
30. $w(x) = |x - 3| + 4$; $y(x) = |5x - 3| + 4$
31. $a(x) = |x + 2| + 3$; $b(x) = |-4x + 2| + 3$
32. $u(x) = |x - 1| + 2$; $v(x) = \left|-\frac{1}{2}x - 1\right| + 2$

In Exercises 33–40, describe the transformations from the graph of $f(x) = |x|$ to the graph of the given function. Then graph the given function. *(See Example 4.)*

33. $r(x) = |x + 2| - 6$

34. $c(x) = |x + 4| + 4$

35. $d(x) = -|x - 3| + 5$

36. $v(x) = -3|x + 1| + 4$

37. $m(x) = \frac{1}{2}|x + 4| - 1$

38. $s(x) = |2x - 2| - 3$

39. $j(x) = |-x + 1| - 5$

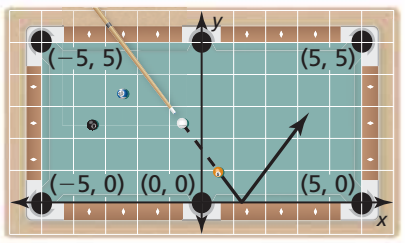
40. $n(x) = \left|-\frac{1}{3}x + 1\right| + 2$

41. MODELING WITH MATHEMATICS The number of pairs of shoes sold s (in thousands) increases and then decreases as described by the function $s(t) = -2|t - 15| + 50$, where t is the time (in weeks).

a. Graph the function.

b. What is the greatest number of pairs of shoes sold in 1 week?

42. MODELING WITH MATHEMATICS On the pool table shown, you bank the five ball off the side represented by the x-axis. The path of the ball is described by the function $p(x) = \frac{4}{3}\left|x - \frac{5}{4}\right|$.



a. At what point does the five ball bank off the side?

b. Do you make the shot? Explain your reasoning.

43. USING TRANSFORMATIONS The points $A\left(-\frac{1}{2}, 3\right)$, $B(1, 0)$, and $C(-4, -2)$ lie on the graph of the absolute value function f. Find the coordinates of the points corresponding to A, B, and C on the graph of each function.

a. $g(x) = f(x) - 5$

b. $h(x) = f(x - 3)$

c. $j(x) = -f(x)$

d. $k(x) = 4f(x)$

44. USING STRUCTURE Explain how the graph of each function compares to the graph of $y = |x|$ for positive and negative values of k, h, and a.

a. $y = |x| + k$

b. $y = |x - h|$

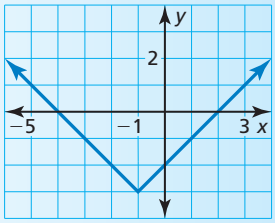
c. $y = a|x|$

d. $y = |ax|$

ERROR ANALYSIS **In Exercises 45 and 46, describe and correct the error in graphing the function.**

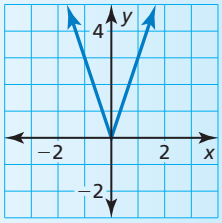
45.

$y = |x - 1| - 3$



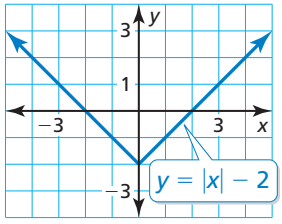
46.

$y = -3|x|$

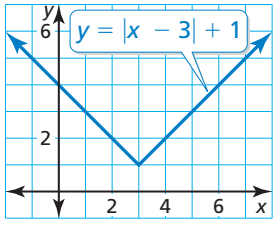


MATHEMATICAL CONNECTIONS **In Exercises 47 and 48, write an absolute value function whose graph forms a square with the given graph.**

47.

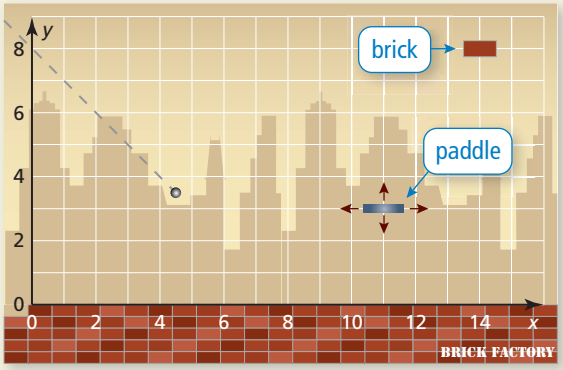


48.



49. WRITING Compare the graphs of $p(x) = |x - 6|$ and $q(x) = |x| - 6$.

50. HOW DO YOU SEE IT? The object of a computer game is to break bricks by deflecting a ball toward them using a paddle. The graph shows the current path of the ball and the location of the last brick.



a. You can move the paddle up, down, left, and right. At what coordinates should you place the paddle to break the last brick? Assume the ball deflects at a right angle.

b. You move the paddle to the coordinates in part (a), and the ball is deflected. How can you write an absolute value function that describes the path of the ball?

In Exercises 51–54, graph the function. Then rewrite the absolute value function as two linear functions, one that has the domain $x < 0$ and one that has the domain $x \geq 0$.

51. $y = |x|$

52. $y = |x| - 3$

53. $y = -|x| + 9$

54. $y = -4|x|$

In Exercises 55–58, graph and compare the two functions.

55. $f(x) = |x - 1| + 2$; $g(x) = 4|x - 1| + 8$

56. $s(x) = |2x - 5| - 6$; $t(x) = \frac{1}{2}|2x - 5| - 3$

57. $v(x) = -2|3x + 1| + 4$; $w(x) = 3|3x + 1| - 6$

58. $c(x) = 4|x + 3| - 1$; $d(x) = -\frac{4}{3}|x + 3| + \frac{1}{3}$

59. REASONING Describe the transformations from the graph of $g(x) = -2|x + 1| + 4$ to the graph of $h(x) = |x|$. Explain your reasoning.

60. THOUGHT PROVOKING Graph an absolute value function f that represents the route a wide receiver runs in a football game. Let the x-axis represent distance (in yards) across the field horizontally. Let the y-axis represent distance (in yards) down the field. Be sure to limit the domain so the route is realistic.

61. SOLVING BY GRAPHING Graph $y = 2|x + 2| - 6$ and $y = -2$ in the same coordinate plane. Use the graph to solve the equation $2|x + 2| - 6 = -2$. Check your solutions.

62. MAKING AN ARGUMENT Let p be a positive constant. Your friend says that because the graph of $y = |x| + p$ is a *positive* vertical translation of the graph of $y = |x|$, the graph of $y = |x + p|$ is a *positive* horizontal translation of the graph of $y = |x|$. Is your friend correct? Explain.

63. ABSTRACT REASONING Write the vertex of the absolute value function $f(x) = |ax - h| + k$ in terms of a, h, and k.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. *(Section 2.4)*

64. $8a - 7 \leq 2(3a - 1)$

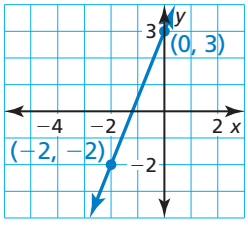
65. $-3(2p + 4) > -6p - 5$

66. $4(3h + 1.5) \geq 6(2h - 2)$

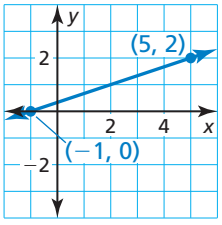
67. $-4(x + 6) < 2(2x - 9)$

Find the slope of the line. *(Section 3.5)*

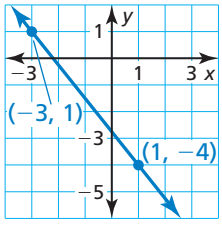
68.



69.



70.



3.4–3.7 What Did You Learn?

Core Vocabulary

standard form, *p. 130*
x-intercept, *p. 131*
y-intercept, *p. 131*
slope, *p. 136*
rise, *p. 136*
run, *p. 136*
slope-intercept form, *p. 138*
constant function, *p. 138*
family of functions, *p. 146*
parent function, *p. 146*
transformation, *p. 146*
translation, *p. 146*
reflection, *p. 147*
horizontal shrink, *p. 148*
horizontal stretch, *p. 148*
vertical stretch, *p. 148*
vertical shrink, *p. 148*
absolute value function, *p. 156*
vertex, *p. 156*
vertex form, *p. 158*

Core Concepts

Section 3.4

Horizontal and Vertical Lines, *p. 130*
Using Intercepts to Graph Equations, *p. 131*

Section 3.5

Slope, *p. 136*
Slope-Intercept Form, *p. 138*

Section 3.6

Horizontal Translations, *p. 146*
Vertical Translations, *p. 146*
Reflections in the *x*-axis, *p. 147*
Reflections in the *y*-axis, *p. 147*
Horizontal Stretches and Shrinks, *p. 148*
Vertical Stretches and Shrinks, *p. 148*
Transformations of Graphs, *p. 149*

Section 3.7

Absolute Value Function, *p. 156*
Vertex Form of an Absolute Value Function, *p. 158*

Mathematical Practices

1. Explain how you determined what units of measure to use for the horizontal and vertical axes in Exercise 37 on page 142.
2. Explain your plan for solving Exercise 48 on page 153.

Performance Task

The Cost of a T-Shirt

You receive bids for making T-shirts for your class fundraiser from four companies. To present the pricing information, one company uses a table, one company uses a written description, one company uses an equation, and one company uses a graph. How will you compare the different representations and make the final choice?

To explore the answers to this question and more, go to ***BigIdeasMath.com***.

3 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

3.1 Functions *(pp. 103–110)*

Determine whether the relation is a function. Explain.

Input, x	2	5	7	9	14
Output, y	5	11	19	12	3

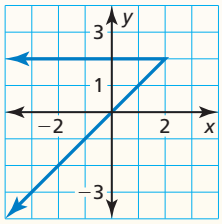
Every input has exactly one output.

▶ So, the relation is a function.

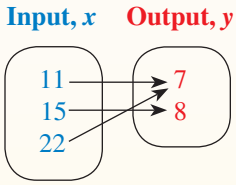
Determine whether the relation is a function. Explain.

1. (0, 1), (5, 6), (7, 9)

2.



3.



4. The function $y = 10x + 100$ represents the amount y (in dollars) of money in your bank account after you babysit for x hours.

a. Identify the independent and dependent variables.

b. You babysit for 4 hours. Find the domain and range of the function.

3.2 Linear Functions *(pp. 111–120)*

Does the table or equation represent a *linear* or *nonlinear* function? Explain.

a.

		+4	+4	+4
x	6	10	14	18
y	5	9	14	20
		+4	+5	+6

As x increases by 4, y increases by different amounts. The rate of change is *not* constant.

▶ So, the function is nonlinear.

b. $y = 3x - 4$

The equation is in the form $y = mx + b$.

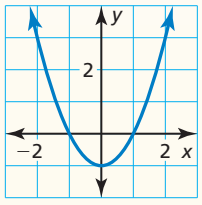
▶ So, the equation represents a linear function.

Does the table or graph represent a *linear* or *nonlinear* function? Explain.

5.

x	2	7	12	17
y	2	−1	−4	−7

6.



7. The function $y = 60 - 8x$ represents the amount y (in dollars) of money you have after buying x movie tickets. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain.

3.3 Function Notation *(pp. 121–126)*

a. Evaluate $f(x) = 3x - 9$ when $x = 2$.

$f(x) = 3x - 9$	Write the function.
$f(2) = 3(2) - 9$	Substitute 2 for x.
$= 6 - 9$	Multiply.
$= -3$	Subtract.

▶ When $x = 2$, $f(x) = -3$.

b. For $f(x) = 4x$, find the value of x for which $f(x) = 12$.

$f(x) = 4x$	Write the function.
$12 = 4x$	Substitute 12 for $f(x)$.
$3 = x$	Divide each side by 4.

▶ When $x = 3$, $f(x) = 12$.

Evaluate the function when $x = -3$, 0, and 5.

8. $f(x) = x + 8$ **9.** $g(x) = 4 - 3x$

Find the value of x so that the function has the given value.

10. $k(x) = 7x$; $k(x) = 49$ **11.** $r(x) = -5x - 1$; $r(x) = 19$

Graph the linear function.

12. $g(x) = -2x - 3$ **13.** $h(x) = \frac{2}{3}x + 4$

3.4 Graphing Linear Equations in Standard Form *(pp. 129–134)*

Use intercepts to graph the equation $2x + 3y = 6$.

Step 1 Find the intercepts.

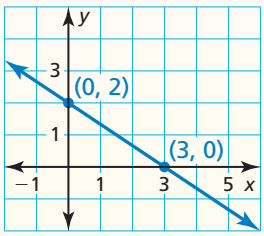
To find the x-intercept, substitute 0 for y and solve for x.

$$2x + 3y = 6$$
$$2x + 3(0) = 6$$
$$x = 3$$

To find the y-intercept, substitute 0 for x and solve for y.

$$2x + 3y = 6$$
$$2(0) + 3y = 6$$
$$y = 2$$

Step 2 Plot the points and draw the line.
The x-intercept is 3, so plot the point (3, 0).
The y-intercept is 2, so plot the point (0, 2).
Draw a line through the points.



Graph the linear equation.

14. $8x - 4y = 16$ **15.** $-12x - 3y = 36$ **16.** $y = -5$ **17.** $x = 6$

3.5 Graphing Linear Equations in Slope-Intercept Form *(pp. 135–144)*

a. The points represented by the table lie on a line. How can you find the slope of the line from the table? What is the slope of the line?

Choose any two points from the table and use the slope formula. Use the points $(x_1, y_1) = (1, -7)$ and $(x_2, y_2) = (4, 2)$.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-7)}{4 - 1} = \frac{9}{3}, \text{ or } 3$$

x	y
1	−7
4	2
7	11
10	20

▶ The slope is 3.

b. Graph $-\frac{1}{2}x + y = 1$. Identify the x-intercept.

Step 1 Rewrite the equation in slope-intercept form.

$$y = \frac{1}{2}x + 1$$

Step 2 Find the slope and the y-intercept.

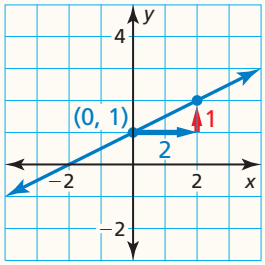
$$m = \frac{1}{2} \text{ and } b = 1$$

Step 3 The y-intercept is 1. So, plot (0, 1).

Step 4 Use the slope to find another point on the line.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$$

Plot the point that is 2 units right and 1 unit up from (0, 1). Draw a line through the two points.



▶ The line crosses the x-axis at $(-2, 0)$. So, the x-intercept is -2.

The points represented by the table lie on a line. Find the slope of the line.

18.

x	y
6	9
11	15
16	21
21	27

19.

x	y
3	−5
3	−2
3	5
3	8

20.

x	y
−4	−1
−3	−1
1	−1
9	−1

Graph the linear equation. Identify the x-intercept.

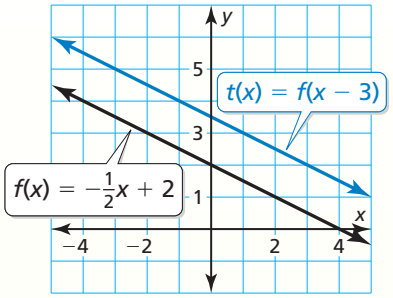
21. $y = 2x + 4$ **22.** $-5x + y = -10$ **23.** $x + 3y = 9$

24. A linear function h models a relationship in which the dependent variable decreases 2 units for every 3 units the independent variable increases. Graph h when $h(0) = 2$. Identify the slope, y-intercept, and x-intercept of the graph.

3.6 Transformations of Graphs of Linear Functions *(pp. 145–154)*

a. Let $f(x) = -\frac{1}{2}x + 2$. Graph $t(x) = f(x - 3)$. Describe the transformation from the graph of f to the graph of t.

The function t is of the form $y = f(x - h)$, where $h = 3$. So, the graph of t is a horizontal translation 3 units right of the graph of f.



b. Graph $f(x) = x$ and $g(x) = -3x - 2$. Describe the transformations from the graph of f to the graph of g.

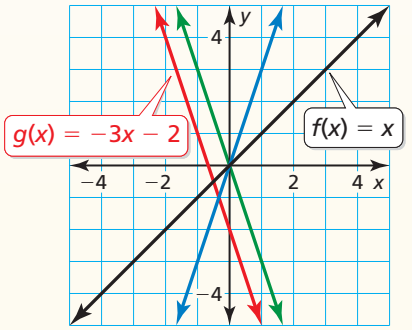
Note that you can rewrite g as $g(x) = -3f(x) - 2$.

Step 1 There is no horizontal translation from the graph of f to the graph of g.

Step 2 Stretch the graph of f vertically by a factor of 3 to get the graph of $h(x) = 3x$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -3x$.

Step 4 Translate the graph of r vertically 2 units down to get the graph of $g(x) = -3x - 2$.



Let $f(x) = 3x + 4$. Graph f and h. Describe the transformation from the graph of f to the graph of h.

25. $h(x) = f(x + 3)$

26. $h(x) = f(x) + 1$

27. $h(x) = f(-x)$

28. $h(x) = -f(x)$

29. $h(x) = 3f(x)$

30. $h(x) = f(6x)$

31. Graph $f(x) = x$ and $g(x) = 5x + 1$. Describe the transformations from the graph of f to the graph of g.

3.7 Graphing Absolute Value Functions *(pp. 155–162)*

Let $g(x) = -3|x + 1| + 2$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.

a. Step 1 Translate the graph of f horizontally 1 unit left to get the graph of $t(x) = |x + 1|$.

Step 2 Stretch the graph of t vertically by a factor of 3 to get the graph of $h(x) = 3|x + 1|$.

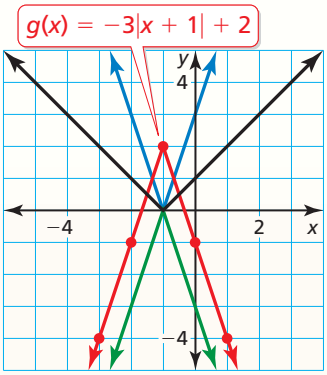
Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -3|x + 1|$.

Step 4 Translate the graph of r vertically 2 units up to get the graph of $g(x) = -3|x + 1| + 2$.

b. Method 1

Step 1 Make a table of values.

x	-3	-2	-1	0	1
$g(x)$	-4	-1	2	-1	-4



Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.

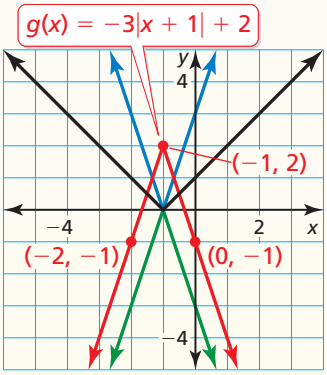
Method 2

Step 1 Identify and plot the vertex.

$(h, k) = (-1, 2)$

Step 2 Plot another point on the graph such as $(0, -1)$. Because the graph is symmetric about the line $x = -1$, you can use symmetry to plot a third point, $(-2, -1)$.

Step 3 Draw the V-shaped graph.



Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range.

32. $m(x) = |x| + 6$ **33.** $p(x) = |x - 4|$ **34.** $q(x) = 4|x|$ **35.** $r(x) = -\frac{1}{4}|x|$

36. Graph $f(x) = |x - 2| + 4$ and $g(x) = |3x - 2| + 4$. Compare the graph of g to the graph of f.

37. Let $g(x) = \frac{1}{3}|x - 1| - 2$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.

3 Chapter Test

Determine whether the relation is a function. If the relation is a function, determine whether the function is *linear* or *nonlinear*. Explain.

1.

x	-1	0	1	2
y	6	5	9	14

2. $y = -2x + 3$

3. $x = -2$

Graph the equation and identify the intercept(s). If the equation is linear, find the slope of the line.

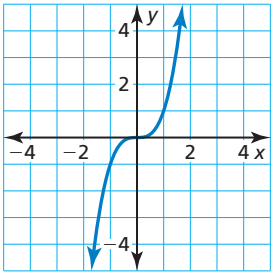
4. $2x - 3y = 6$

5. $y = 4.5$

6. $y = |x - 1| - 2$

Find the domain and range of the function represented by the graph. Determine whether the domain is *discrete* or *continuous*. Explain.

7.



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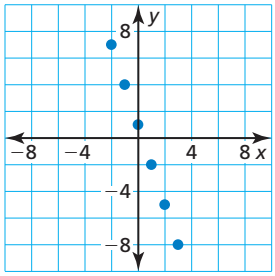


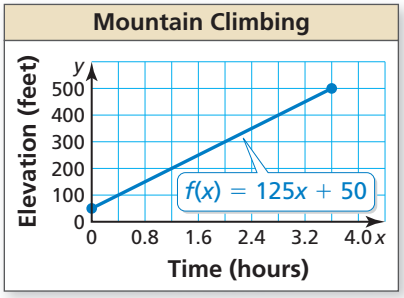
Graph f and g. Describe the transformations from the graph of f to the graph of g.

9. $f(x) = x$; $g(x) = -x + 3$

10. $f(x) = |x|$; $g(x) = |2x + 4|$

11. Function A represents the amount of money in a jar based on the number of quarters in the jar. Function B represents your distance from home over time. Compare the domains.

12. A mountain climber is scaling a 500-foot cliff. The graph shows the elevation of the climber over time.



a. Find and interpret the slope and the y-intercept of the graph.

b. Explain two ways to find $f(3)$. Then find $f(3)$ and interpret its meaning.

c. How long does it take the climber to reach the top of the cliff? Justify your answer.

13. Without graphing, compare the slopes and the intercepts of the graphs of the functions $f(x) = x + 1$ and $g(x) = f(2x)$.

14. A rock band releases a new single. Weekly sales s (in thousands of dollars) increase and then decrease as described by the function $s(t) = -2|t - 20| + 40$, where t is the time (in weeks).

a. Identify the independent and dependent variables.

b. Graph s. Describe the transformations from the graph of $f(x) = |x|$ to the graph of s.

3 Cumulative Assessment

1. You claim you can create a table of values that represents a linear function. Your friend claims he can create a table of values that represents a nonlinear function. Using the given numbers, what values can you use for x (the input) and y (the output) to support your claim? What values can your friend use?

	Your claim			
x				
y				

	Friend's claim			
x				
y				

-4	-3	-2	-1	0
1	2	3	4	5

2. A car rental company charges an initial fee of \$42 and a daily fee of \$12.

a. Use the numbers and symbols to write a function that represents this situation.

x	$f(x)$	42	12	138
$+$	$-$	$\times$	$\div$	$=$

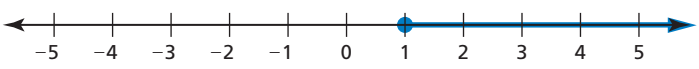
b. The bill is \$138. How many days did you rent the car?

3. Fill in values for a and b so that each statement is true for the inequality $ax - b > 0$.

a. When $a =$ _____ and $b =$ _____, $x > \frac{b}{a}$.

b. When $a =$ _____ and $b =$ _____, $x < \frac{b}{a}$.

4. Fill in the inequality with $<$, $\leq$, $>$, or $\geq$ so that the solution of the inequality is represented by the graph.



$-3(x + 7)$ ____ -24

5. Use the numbers to fill in the coefficients of $ax + by = 40$ so that when you graph the function, the x-intercept is -10 and the y-intercept is 8.

-10	-8	-5	-4	4	5	8	10

6. Solve each equation. Then classify each equation based on the solution. Explain your reasoning.

a. $2x - 9 = 5x - 33$

b. $5x - 6 = 10x + 10$

c. $2(8x - 3) = 4(4x + 7)$

d. $-7x + 5 = 2(x - 10.1)$

e. $6(2x + 4) = 4(4x + 10)$

f. $8(3x + 4) = 2(12x + 16)$

7. The table shows the cost of bologna at a deli. Plot the points represented by the table in a coordinate plane. Decide whether you should connect the points with a line. Explain your reasoning.

Pounds, *x*	0.5	1	1.5	2
Cost, *y*	\$3	\$6	\$9	\$12

8. The graph of g is a horizontal translation right, then a vertical stretch, then a vertical translation down of the graph of $f(x) = x$. Use the numbers and symbols to create g.

-3	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	3

x	$g(x)$	$+$	$-$	$\times$	$\div$	$=$

9. What is the sum of the integer solutions of the compound inequality $2|x - 5| < 16$?

Ⓐ 72 Ⓑ 75 Ⓒ 85 Ⓓ 88

10. Your bank offers a text alert service that notifies you when your checking account balance drops below a specific amount. You set it up so you are notified when your balance drops below \$700. The balance is currently \$3000. You only use your account for paying your rent (no other deposits or deductions occur). Your rent each month is \$625.

a. Write an inequality that represents the number of months m you can pay your rent without receiving a text alert.

b. What is the maximum number of months you can pay your rent without receiving a text alert?

c. Suppose you start paying rent in June. Select all the months you can pay your rent without making a deposit.

June	July	August	September	October