10 Radical Functions and Equations

Crow Feeding Habits *(p. 573)*

Trapeze Artist *(p. 565)*

Asian Elephant *(p. 554)*

SEE the Big Idea
Tsunami *(p. 547)*

Firefighting *(p. 549)*

Maintaining Mathematical Proficiency

Evaluating Expressions Involving Square Roots

Example 1 Evaluate $-4(\sqrt{121} - 16)$.

$$-4(\sqrt{121} - 16) = -4(11 - 16) \qquad \text{Evaluate the square root.}$$
$$= -4(-5) \qquad \text{Subtract.}$$
$$= 20 \qquad \text{Multiply.}$$

Evaluate the expression.

1. $7\sqrt{25} + 10$ **2.** $-8 - \sqrt{\dfrac{64}{16}}$ **3.** $5\left(\dfrac{\sqrt{81}}{3} - 7\right)$ **4.** $-2(3\sqrt{9} + 13)$

Transforming Linear Functions

Example 2 Graph $f(x) = x$ and $g(x) = -3x - 4$. Describe the transformations from the graph of f to the graph of g.
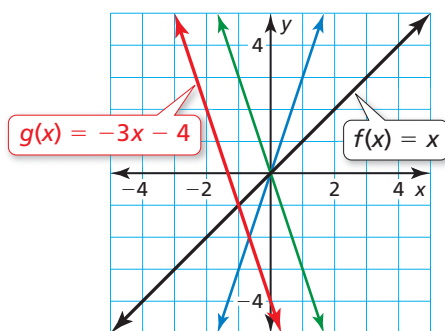
Note that you can rewrite g as $g(x) = -3f(x) - 4$.

Step 1 There is no horizontal translation from the graph of f to the graph of g.

Step 2 Stretch the graph of f vertically by a factor of 3 to get the graph of $h(x) = 3x$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -3x$.

Step 4 Translate the graph of r vertically 4 units down to get the graph of $g(x) = -3x - 4$.

Graph f and g. Describe the transformations from the graph of f to the graph of g.

5. $f(x) = x;\ g(x) = 2x - 2$ **6.** $f(x) = x;\ g(x) = \dfrac{1}{3}x + 5$ **7.** $f(x) = x;\ g(x) = -x + 3$

8. ABSTRACT REASONING Let a and b represent constants, where $b \geq 0$. Describe the transformations from the graph of $m(x) = ax + b$ to the graph of $n(x) = -2ax - 4b$.

Mathematical Practices

Mathematically proficient students distinguish correct reasoning from flawed reasoning.

Logical Reasoning

Core Concept

Logical Reasoning and Proof by Contradiction

Mathematics is a logical system that is built from only a few assumptions and undefined terms. The assumptions are called *axioms* or *postulates*. After starting with a collection of axioms and undefined terms, the remainder of mathematics is logically built using careful definitions and theorems (or rules), which are based on the axioms or on previously proven theorems.

To write an indirect proof, or *proof by contradiction*, identify the statement you want to prove. Assume temporarily that this statement is false by assuming that its opposite is true. Then reason logically until you reach a contradiction. Point out that the original statement must be true because the contradiction proves the temporary assumption false.

EXAMPLE 1 **Understanding a Proof**

A number is *rational* when it can be written as the ratio a/b of two integers, where $b \neq 0$. Use proof by contradiction to prove that $\sqrt{2}$ is not a rational number.

SOLUTION

Assume that $\sqrt{2}$ can be written as the ratio of two integers (in simplest form) and show that this assumption leads to a contradiction.

$$\sqrt{2} = \frac{a}{b} \qquad \text{Assume } \sqrt{2} \text{ is rational.}$$

$$2 = \frac{a^2}{b^2} \qquad \text{Square each side.}$$

$$a^2 = 2b^2 \qquad \text{Multiply each side by } b^2 \text{ and interchange left and right sides.}$$

This implies that a^2 is even, which is only true when a is even (divisible by 2). This implies that a^2 is divisible by 2^2, or 4. So, $2b^2$ is also divisible by 4, meaning that b^2 is divisible by 2, b^2 is even, and b is even. Because both a and b are even, they have a common factor (of at least 2).

This contradicts the assumption that the ratio a/b is written in simplest form. So, the initial assumption that $\sqrt{2}$ is rational must be false. Therefore, $\sqrt{2}$ is not a rational number.

Monitoring Progress

1. Which of the following square roots are rational numbers? Explain your reasoning.

$$\sqrt{0}, \sqrt{1}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}$$

2. The sequence of steps shown appears to prove that $1 = 0$. What is wrong with this argument?

$$x = 1 \qquad \text{Let } x = 1.$$
$$x - 1 = 0 \qquad \text{Subtract 1 from each side.}$$
$$x(x - 1) = 0 \qquad \text{Multiply each side by } x.$$
$$x = 0 \qquad \text{Divide each side by } (x - 1).$$

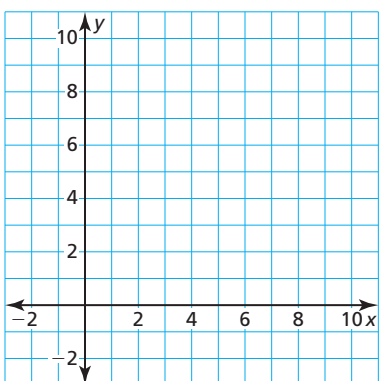# 10.1 Graphing Square Root Functions

Essential Question What are some of the characteristics of the graph of a square root function?
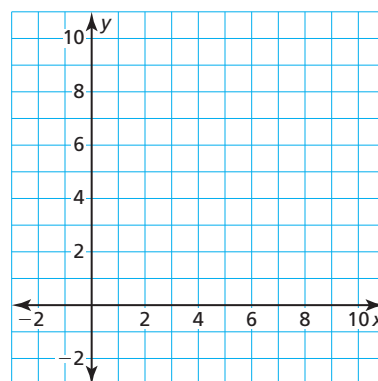
EXPLORATION 1 Graphing Square Root Functions

Work with a partner.

- Make a table of values for each function.
- Use the table to sketch the graph of each function.
- Describe the domain of each function.
- Describe the range of each function.

a. $y = \sqrt{x}$

b. $y = \sqrt{x} + 2$

LOOKING FOR A PATTERN

To be proficient in math, you need to look closely to discern a pattern or structure.

EXPLORATION 2 Writing Square Root Functions

Work with a partner. Write a square root function, $y = f(x)$, that has the given values. Then use the function to complete the table.

a.

x	f(x)
−4	0
−3	
−2	
−1	$\sqrt{3}$
0	2
1	

b.

x	f(x)
−4	1
−3	
−2	
−1	$1 + \sqrt{3}$
0	3
1	

Communicate Your Answer

3. What are some of the characteristics of the graph of a square root function?

4. Graph each function. Then compare the graph to the graph of $f(x) = \sqrt{x}$.

   a. $g(x) = \sqrt{x - 1}$ **b.** $g(x) = \sqrt{x} - 1$ **c.** $g(x) = 2\sqrt{x}$ **d.** $g(x) = -2\sqrt{x}$

Core Vocabulary

square root function, *p. 544*
radical function, *p. 545*

Previous
radicand
transformation
average rate of change

STUDY TIP

The graph of $f(x) = \sqrt{x}$ starts at (0, 0) and increases on the entire domain. So, the minimum value of f is 0.

What You Will Learn

▶ Graph square root functions.

▶ Compare square root functions using average rates of change.

▶ Solve real-life problems involving square root functions.

Graphing Square Root Functions

Core Concept

Square Root Functions

A **square root function** is a function that contains a square root with the independent variable in the radicand. The parent function for the family of square root functions is $f(x) = \sqrt{x}$. The domain of f is $x \geq 0$, and the range of f is $y \geq 0$.
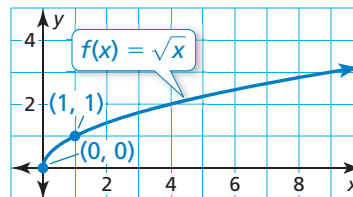
The value of the radicand in a square root function cannot be negative. So, the domain of a square root function includes x-values for which the radicand is greater than or equal to 0.

EXAMPLE 1 Describing the Domain of a Square Root Function

Describe the domain of $f(x) = 3\sqrt{x - 5}$.

SOLUTION

The radicand cannot be negative. So, $x - 5$ is greater than or equal to 0.

$x - 5 \geq 0$ Write an inequality for the domain.

$x \geq 5$ Add 5 to each side.

▶ The domain is the set of real numbers greater than or equal to 5.

EXAMPLE 2 Graphing a Square Root Function

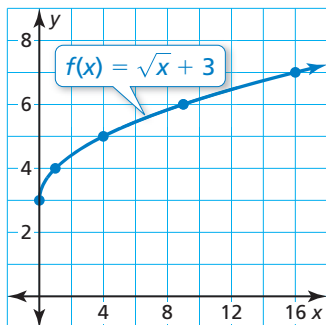Graph $f(x) = \sqrt{x} + 3$. Describe the range of the function.

SOLUTION

Step 1 Use the domain of f, $x \geq 0$, to make a table of values.

Step 2 Plot the ordered pairs.

x	0	1	4	9	16
f(x)	3	4	5	6	7

Step 3 Draw a smooth curve through the points, starting at (0, 3).

▶ From the graph, you can see that the range of f is $y \geq 3$.

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Describe the domain of the function.

1. $f(x) = 10\sqrt{x}$

2. $y = \sqrt{2x} + 7$

3. $h(x) = \sqrt{-x + 1}$

Graph the function. Describe the range.

4. $g(x) = \sqrt{x} - 4$

5. $y = \sqrt{x + 5}$

6. $n(x) = 5\sqrt{x}$

A **radical function** is a function that contains a radical expression with the independent variable in the radicand. A square root function is a type of radical function.

You can transform graphs of radical functions in the same way you transformed graphs of functions previously. In Example 2, notice that the graph of f is a vertical translation of the graph of the parent square root function.

Core Concept

Transformation	$f(x)$ Notation	Examples	
Horizontal Translation Graph shifts left or right.	$f(x - h)$	$g(x) = \sqrt{x - 2}$ $g(x) = \sqrt{x + 3}$	2 units right 3 units left
Vertical Translation Graph shifts up or down.	$f(x) + k$	$g(x) = \sqrt{x} + 7$ $g(x) = \sqrt{x} - 1$	7 units up 1 unit down
Reflection Graph flips over x- or y-axis.	$f(-x)$ $-f(x)$	$g(x) = \sqrt{-x}$ $g(x) = -\sqrt{x}$	in the y-axis in the x-axis
Horizontal Stretch or Shrink Graph stretches away from or shrinks toward y-axis.	$f(ax)$	$g(x) = \sqrt{3x}$ $g(x) = \sqrt{\frac{1}{2}x}$	shrink by a factor of $\frac{1}{3}$ stretch by a factor of 2
Vertical Stretch or Shrink Graph stretches away from or shrinks toward x-axis.	$a \cdot f(x)$	$g(x) = 4\sqrt{x}$ $g(x) = \frac{1}{5}\sqrt{x}$	stretch by a factor of 4 shrink by a factor of $\frac{1}{5}$

EXAMPLE 3 Comparing Graphs of Square Root Functions

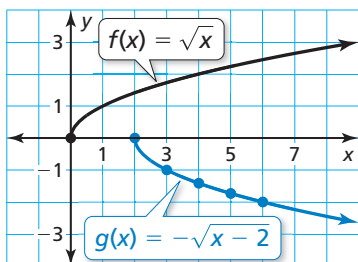Graph $g(x) = -\sqrt{x - 2}$. Compare the graph to the graph of $f(x) = \sqrt{x}$.

SOLUTION

Step 1 Use the domain of g, $x \geq 2$, to make a table of values.

x	2	3	4	5	6
g(x)	0	-1	-1.4	-1.7	-2

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points, starting at $(2, 0)$.

▶ The graph of g is a translation 2 units right and a reflection in the x-axis of the graph of f.

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Graph the function. Compare the graph to the graph of $f(x) = \sqrt{x}$.

7. $h(x) = \sqrt{\frac{1}{4}x}$ **8.** $g(x) = \sqrt{x} - 6$ **9.** $m(x) = -3\sqrt{x}$

To graph a square root function of the form $y = a\sqrt{x - h} + k$, where $a \neq 0$, start at (h, k).

EXAMPLE 4   **Graphing $y = a\sqrt{x - h} + k$**

Let $g(x) = -2\sqrt{x - 3} - 2$. (a) Describe the transformations from the graph of $f(x) = \sqrt{x}$ to the graph of g. (b) Graph g.

SOLUTION

a. **Step 1**   Translate the graph of f horizontally 3 units right to get the graph of $t(x) = \sqrt{x - 3}$.

    Step 2   Stretch the graph of t vertically by a factor of 2 to get the graph of $h(x) = 2\sqrt{x - 3}$.

    Step 3   Reflect the graph of h in the x-axis to get the graph of $r(x) = -2\sqrt{x - 3}$.

    Step 4   Translate the graph of r vertically 2 units down to get the graph of $g(x) = -2\sqrt{x - 3} - 2$.

> **REMEMBER**
>
> The graph of $y = a \cdot f(x - h) + k$ can be obtained from the graph of $y = f(x)$ using the steps you learned in Section 3.6.

b. **Step 1**   Use the domain, $x \geq 3$, to make a table of values.

    Step 2   Plot the ordered pairs.

    Step 3   Start at $(h, k) = (3, -2)$ and draw a smooth curve through the points.

x	3	4	7	12
g(x)	−2	−4	−6	−8
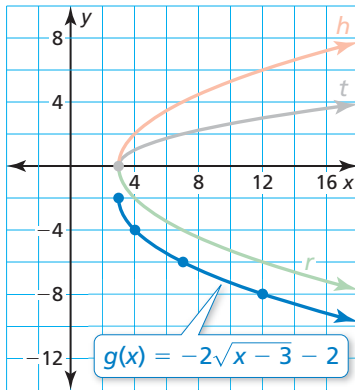
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10. Let $g(x) = \frac{1}{2}\sqrt{x + 4} + 1$. Describe the transformations from the graph of $f(x) = \sqrt{x}$ to the graph of g. Then graph g.

Comparing Average Rates of Change

EXAMPLE 5   **Comparing Square Root Functions**

The model $v(d) = \sqrt{2gd}$ represents the velocity v (in meters per second) of a free-falling object on the moon, where g is the constant 1.6 meters per second squared and d is the distance (in meters) the object has fallen. The velocity of a free-falling object on Earth is shown in the graph. Compare the velocities by finding and interpreting their average rates of change over the interval $d = 0$ to $d = 10$.

SOLUTION

To calculate the average rates of change, use points whose d-coordinates are 0 and 10.

    Earth: Use the graph to estimate. Use $(0, 0)$ and $(10, 14)$.

$$\frac{v(10) - v(0)}{10 - 0} \approx \frac{14 - 0}{10} = 1.4 \qquad \text{Average rate of change on Earth}$$

    Moon: Evaluate v when $d = 0$ and $d = 10$.

$$v(0) = \sqrt{2(1.6)(0)} = 0 \quad \text{and} \quad v(10) = \sqrt{2(1.6)(10)} = \sqrt{32} \approx 5.7$$

    Use $(0, 0)$ and $\left(10, \sqrt{32}\right)$.

$$\frac{v(10) - v(0)}{10 - 0} = \frac{\sqrt{32} - 0}{10} \approx 0.57 \qquad \text{Average rate of change on the moon}$$

▶ From 0 to 10 meters, the velocity of a free-falling object increases at an average rate of about 1.4 meters per second per meter on Earth and about 0.57 meter per second per meter on the moon.

Free-Falling Object on Earth

Distance fallen (meters)
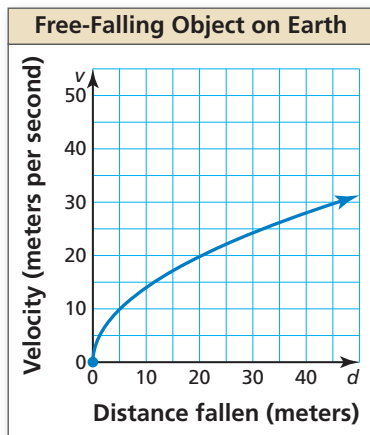
11. In Example 5, compare the velocities by finding and interpreting their average rates of change over the interval $d = 30$ to $d = 40$.

Solving Real-Life Problems

EXAMPLE 6 **Real-Life Application**

The velocity v (in meters per second) of a tsunami can be modeled by the function $v(x) = \sqrt{9.8x}$, where x is the water depth (in meters). (a) Use a graphing calculator to graph the function. At what depth does the velocity of the tsunami exceed 200 meters per second? (b) What happens to the average rate of change of the velocity as the water depth increases?

SOLUTION

1. **Understand the Problem** You know the function that models the velocity of a tsunami based on water depth. You are asked to graph the function using a calculator and find the water depth where the velocity exceeds 200 meters per second. Then you are asked to describe the average rate of change of the velocity as the water depth increases.

2. **Make a Plan** Graph the function using a calculator. Use the *trace* feature to find the value of x when $v(x)$ is about 200. Then calculate and compare average rates of change of the velocity over different intervals.

3. **Solve the Problem**

   a. **Step 1** Enter the function into your calculator and graph it.

      Step 2 Use the *trace* feature to find the value of x when $v(x) \approx 200$.

      ▶ The velocity exceeds 200 meters per second at a depth of about 4082 meters.

   b. Calculate the average rates of change over the intervals $x = 0$ to $x = 1000$, $x = 1000$ to $x = 2000$, and $x = 2000$ to $x = 3000$.

   $$\frac{v(1000) - v(0)}{1000 - 0} = \frac{\sqrt{9800} - 0}{1000} \approx 0.099 \qquad \text{0 to 1000 meters}$$

   $$\frac{v(2000) - v(1000)}{2000 - 1000} = \frac{\sqrt{19{,}600} - \sqrt{9800}}{1000} \approx 0.041 \qquad \text{1000 to 2000 meters}$$

   $$\frac{v(3000) - v(2000)}{3000 - 2000} = \frac{\sqrt{29{,}400} - \sqrt{19{,}600}}{1000} \approx 0.031 \qquad \text{2000 to 3000 meters}$$

   ▶ The average rate of change of the velocity decreases as the water depth increases.

4. **Look Back** To check the answer in part (a), find $v(x)$ when $x = 4082$.

   $$v(4082) = \sqrt{9.8(4082)} \approx 200 \ ✔$$

   In part (b), the slopes of the line segments (shown at the left) that represent the average rates of change over the intervals are decreasing. So, the answer to part (b) is reasonable.
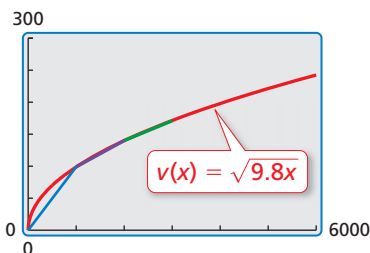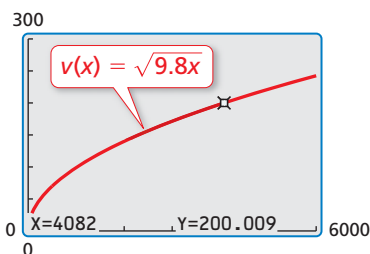
12. **WHAT IF?** At what depth does the velocity of the tsunami exceed 100 meters per second?

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A _____ is a function that contains a radical expression with the independent variable in the radicand.

2. **VOCABULARY** Is $y = 2x\sqrt{5}$ a square root function? Explain.

3. **WRITING** How do you describe the domain of a square root function?

4. **REASONING** Is the graph of $g(x) = 1.25\sqrt{x}$ a vertical stretch or a vertical shrink of the graph of $f(x) = \sqrt{x}$? Explain.
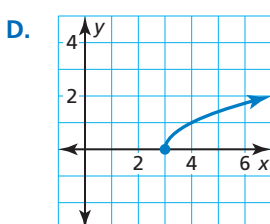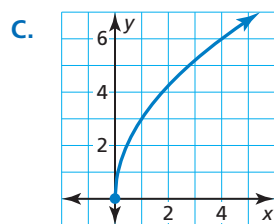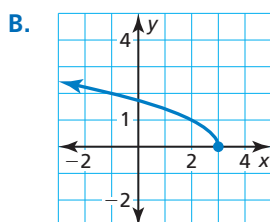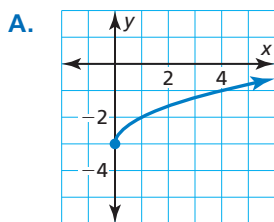
Monitoring Progress and Modeling with Mathematics

In Exercises 5–14, describe the domain of the function. *(See Example 1.)*

5. $y = 8\sqrt{x}$

6. $y = \sqrt{4x}$

7. $y = 4 + \sqrt{-x}$

8. $y = \sqrt{-\frac{1}{2}x} + 1$

9. $h(x) = \sqrt{x - 4}$

10. $p(x) = \sqrt{x + 7}$

11. $f(x) = \sqrt{-x + 8}$

12. $g(x) = \sqrt{-x - 1}$

13. $m(x) = 2\sqrt{x + 4}$

14. $n(x) = \frac{1}{2}\sqrt{-x - 2}$

In Exercises 15–18, match the function with its graph. Describe the range.

15. $y = \sqrt{x - 3}$

16. $y = 3\sqrt{x}$

17. $y = \sqrt{x} - 3$

18. $y = \sqrt{-x + 3}$

A.

B.

C.

D.

In Exercises 19–26, graph the function. Describe the range. *(See Example 2.)*

19. $y = \sqrt{3x}$

20. $y = 4\sqrt{-x}$

21. $y = \sqrt{x} + 5$

22. $y = -2 + \sqrt{x}$

23. $f(x) = -\sqrt{x} - 3$

24. $g(x) = \sqrt{x + 4}$

25. $h(x) = \sqrt{x + 2} - 2$

26. $f(x) = -\sqrt{x - 1} + 3$

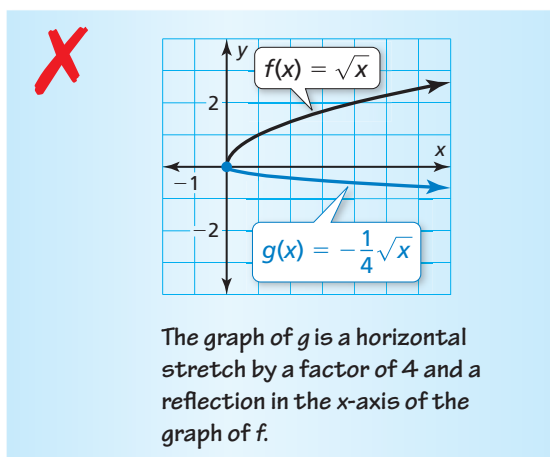**In Exercises 27–34, graph the function. Compare the graph to the graph of $f(x) = \sqrt{x}$.** *(See Example 3.)*

27. $g(x) = \frac{1}{4}\sqrt{x}$

28. $r(x) = \sqrt{2x}$

29. $h(x) = \sqrt{x + 3}$

30. $q(x) = \sqrt{x} + 8$

31. $p(x) = \sqrt{-\frac{1}{3}x}$

32. $g(x) = -5\sqrt{x}$

33. $m(x) = -\sqrt{x} - 6$

34. $n(x) = -\sqrt{x - 4}$

35. **ERROR ANALYSIS** Describe and correct the error in graphing the function $y = \sqrt{x + 1}$.
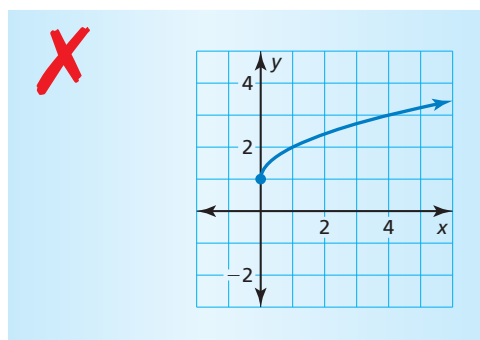
36. ERROR ANALYSIS Describe and correct the error in comparing the graph of $g(x) = -\frac{1}{4}\sqrt{x}$ to the graph of $f(x) = \sqrt{x}$.


The graph of g is a horizontal stretch by a factor of 4 and a reflection in the x-axis of the graph of f.

In Exercises 37–44, describe the transformations from the graph of $f(x) = \sqrt{x}$ to the graph of h. Then graph h. *(See Example 4.)*

37. $h(x) = 4\sqrt{x+2} - 1$ **38.** $h(x) = \frac{1}{2}\sqrt{x-6} + 3$

39. $h(x) = 2\sqrt{-x} - 6$ **40.** $h(x) = -\sqrt{x-3} - 2$

41. $h(x) = \frac{1}{3}\sqrt{x+3} - 3$

42. $h(x) = 2\sqrt{x-1} + 4$

43. $h(x) = -2\sqrt{x-1} + 5$

44. $h(x) = -5\sqrt{x+2} - 1$

45. COMPARING FUNCTIONS The model $S(d) = \sqrt{30df}$ represents the speed S (in miles per hour) of a van before it skids to a stop, where f is the drag factor of the road surface and d is the length (in feet) of the skid marks. The drag factor of Road Surface A is 0.75. The graph shows the speed of the van on Road Surface B. Compare the speeds by finding and interpreting their average rates of change over the interval $d = 0$ to $d = 15$. *(See Example 5.)*

Road Surface B
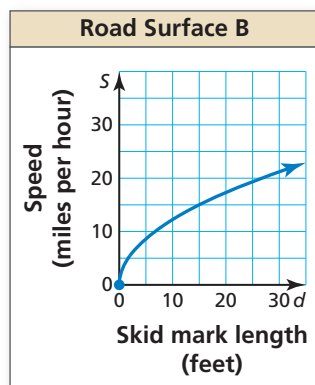
46. COMPARING FUNCTIONS The velocity v (in meters per second) of an object in motion is given by $v(E) = \sqrt{\dfrac{2E}{m}}$, where E is the kinetic energy of the object (in joules) and m is the mass of the object (in kilograms). The mass of Object A is 4 kilograms. The graph shows the velocity of Object B. Compare the velocities of the objects by finding and interpreting the average rates of change over the interval $E = 0$ to $E = 6$.

Object B
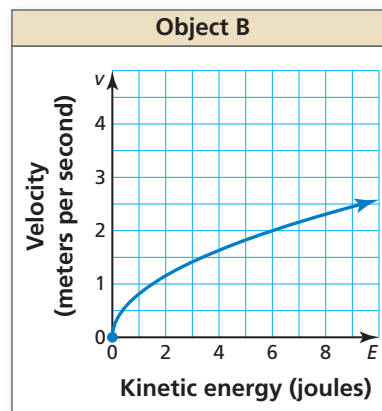
47. OPEN-ENDED Consider the graph of $y = \sqrt{x}$.

a. Write a function that is a vertical translation of the graph of $y = \sqrt{x}$.

b. Write a function that is a reflection of the graph of $y = \sqrt{x}$.

48. REASONING Can the domain of a square root function include negative numbers? Can the range include negative numbers? Explain your reasoning.

49. PROBLEM SOLVING The nozzle pressure of a fire hose allows firefighters to control the amount of water they spray on a fire. The flow rate f (in gallons per minute) can be modeled by the function $f = 120\sqrt{p}$, where p is the nozzle pressure (in pounds per square inch). *(See Example 6.)*

a. Use a graphing calculator to graph the function. At what pressure does the flow rate exceed 300 gallons per minute?

**b.** What happens to the average rate of change of the flow rate as the pressure increases?

50. PROBLEM SOLVING The speed s (in meters per second) of a long jumper before jumping can be modeled by the function $s = 10.9\sqrt{h}$, where h is the maximum height (in meters from the ground) of the jumper.

a. Use a graphing calculator to graph the function. A jumper is running 9.2 meters per second. Estimate the maximum height of the jumper.

b. Suppose the runway and pit are raised on a platform slightly higher than the ground. How would the graph of the function be transformed?

51. MATHEMATICAL CONNECTIONS The radius r of a circle is given by $r = \sqrt{\dfrac{A}{\pi}}$, where A is the area of the circle.

a. Describe the domain of the function. Use a graphing calculator to graph the function.

b. Use the *trace* feature to approximate the area of a circle with a radius of 5.4 inches.
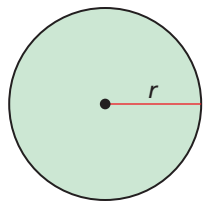
52. REASONING Consider the function $f(x) = 8a\sqrt{x}$.

a. For what value of a will the graph of f be identical to the graph of the parent square root function?

b. For what values of a will the graph of f be a vertical stretch of the graph of the parent square root function?

c. For what values of a will the graph of f be a vertical shrink and a reflection of the graph of the parent square root function?

53. REASONING The graph represents the function $f(x) = \sqrt{x}$.
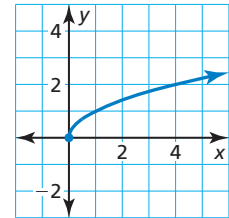
a. What is the minimum value of the function?

b. Does the function have a maximum value? Explain.

c. Write a square root function that has a maximum value. Does the function have a minimum value? Explain.

d. Write a square root function that has a minimum value of -4.

54. HOW DO YOU SEE IT? Match each function with its graph. Explain your reasoning.

A. $f(x) = \sqrt{x} + 2$ **B.** $m(x) = f(x) - 4$

C. $n(x) = f(-x)$ **D.** $p(x) = f(3x)$
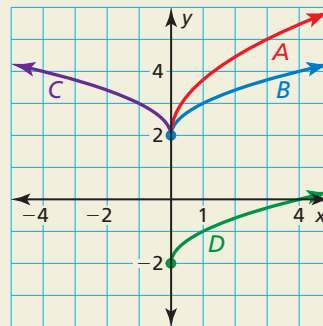
55. REASONING Without graphing, determine which function's graph rises more steeply, $f(x) = 5\sqrt{x}$ or $g(x) = \sqrt{5x}$. Explain your reasoning.

56. THOUGHT PROVOKING Use a graphical approach to find the solutions of $x - 1 = \sqrt{5x - 9}$. Show your work. Verify your solutions algebraically.

57. OPEN-ENDED Write a radical function that has a domain of all real numbers greater than or equal to -5 and a range of all real numbers less than or equal to 3.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression. *(Section 6.2)*

58. $\sqrt[3]{343}$ **59.** $\sqrt[3]{-64}$ **60.** $-\sqrt[3]{-\dfrac{1}{27}}$

Factor the polynomial. *(Section 7.5)*

61. $x^2 + 7x + 6$ **62.** $d^2 - 11d + 28$ **63.** $y^2 - 3y - 40$

10.2 Graphing Cube Root Functions

**Essential Question** What are some of the characteristics of the graph of a cube root function?

EXPLORATION 1   **Graphing Cube Root Functions**

Work with a partner.

• Make a table of values for each function. Use positive and negative values of x.

• Use the table to sketch the graph of each function.

• Describe the domain of each function.

• Describe the range of each function.

a. $y = \sqrt[3]{x}$

b. $y = \sqrt[3]{x + 3}$
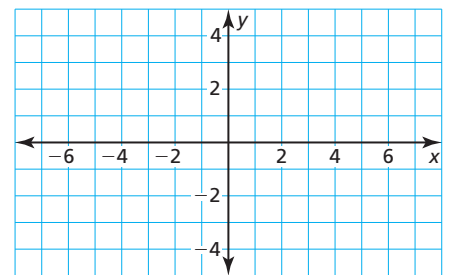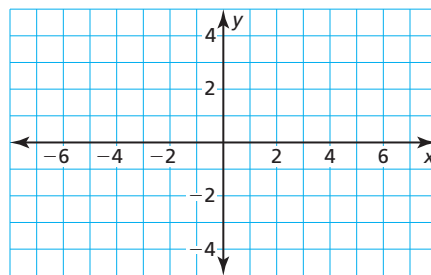
LOOKING FOR REGULARITY IN REPEATED REASONING

To be proficient in math, you need to notice whether calculations are repeated and look for both general methods and shortcuts.

EXPLORATION 2   **Writing Cube Root Functions**

Work with a partner. Write a cube root function, $y = f(x)$, that has the given values. Then use the function to complete the table.

a.

x	$f(x)$	x	$f(x)$
-4	0	1	
-3		2	
-2		3	
-1	$\sqrt[3]{3}$	4	2
0		5	

b.

x	$f(x)$	x	$f(x)$
-4	1	1	
-3		2	
-2		3	
-1	$1 + \sqrt[3]{3}$	4	3
0		5	

Communicate Your Answer

3. What are some of the characteristics of the graph of a cube root function?

4. Graph each function. Then compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

   a. $g(x) = \sqrt[3]{x - 1}$

   b. $g(x) = \sqrt[3]{x} - 1$

   c. $g(x) = 2\sqrt[3]{x}$

   d. $g(x) = -2\sqrt[3]{x}$

What You Will Learn

▶ Graph cube root functions.

▶ Compare cube root functions using average rates of change.

▶ Solve real-life problems involving cube root functions.

Graphing Cube Root Functions

Core Concept

Cube Root Functions

A **cube root function** is a radical function with an index of 3. The parent function for the family of cube root functions is $f(x) = \sqrt[3]{x}$. The domain and range of f are all real numbers.
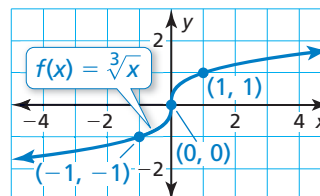
The graph of $f(x) = \sqrt[3]{x}$ increases on the entire domain.

You can transform graphs of cube root functions in the same way you transformed graphs of square root functions.

LOOKING FOR STRUCTURE

Use *x*-values so that the cube root of the radicand is an integer. This makes it easier to perform the calculations and plot the points.

EXAMPLE 1   **Comparing Graphs of Cube Root Functions**

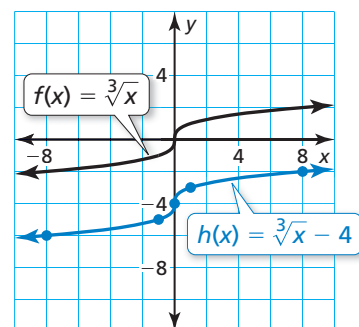Graph $h(x) = \sqrt[3]{x} - 4$. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

SOLUTION

Step 1   Make a table of values.

x	−8	−1	0	1	8
h(x)	−6	−5	−4	−3	−2

Step 2   Plot the ordered pairs.

Step 3   Draw a smooth curve through the points.

▶ The graph of h is a translation 4 units down of the graph of f.

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Graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

1. $h(x) = \sqrt[3]{x} + 3$

2. $m(x) = \sqrt[3]{x - 5}$

3. $g(x) = 4\sqrt[3]{x}$

EXAMPLE 2   **Comparing Graphs of Cube Root Functions**

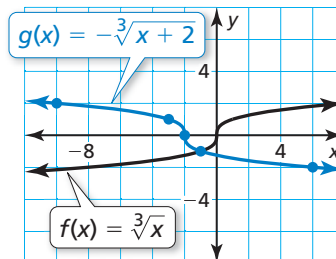Graph $g(x) = -\sqrt[3]{x + 2}$. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

SOLUTION

Step 1   Make a table of values.

x	−10	−3	−2	−1	6
g(x)	2	1	0	−1	−2

Step 2   Plot the ordered pairs.

Step 3   Draw a smooth curve through the points.

$g(x) = -\sqrt[3]{x + 2}$

$f(x) = \sqrt[3]{x}$

▶ The graph of g is a translation 2 units left and a reflection in the x-axis of the graph of f.

REMEMBER

The graph of $y = a \cdot f(x - h) + k$ can be obtained from the graph of $y = f(x)$ using the steps you learned in Section 3.6.

EXAMPLE 3   **Graphing $y = a\sqrt[3]{x - h} + k$**

Let $g(x) = 2\sqrt[3]{x - 3} + 4$. (a) Describe the transformations from the graph of $f(x) = \sqrt[3]{x}$ to the graph of g. (b) Graph g.

SOLUTION

a. Step 1   Translate the graph of f horizontally 3 units right to get the graph of $t(x) = \sqrt[3]{x - 3}$.

Step 2   Stretch the graph of t vertically by a factor of 2 to get the graph of $h(x) = 2\sqrt[3]{x - 3}$.

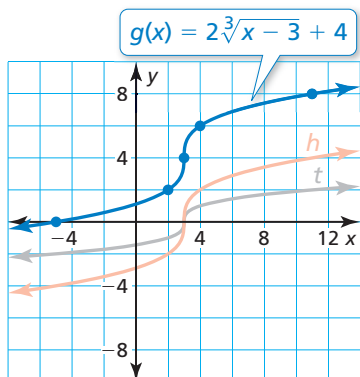**Step 3**   Because $a > 0$, there is no reflection.

Step 4   Translate the graph of h vertically 4 units up to get the graph of $g(x) = 2\sqrt[3]{x - 3} + 4$.

$g(x) = 2\sqrt[3]{x - 3} + 4$

b. Step 1   Make a table of values.

x	−5	2	3	4	11
g(x)	0	2	4	6	8

Step 2   Plot the ordered pairs.

Step 3   Draw a smooth curve through the points.

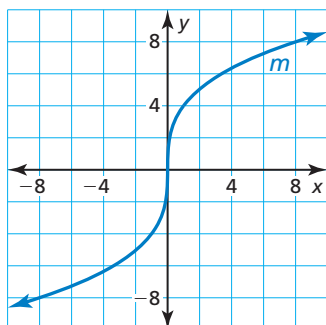## Monitoring Progress   🔊 Help in English and Spanish at *BigIdeasMath.com*

Graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

4. $g(x) = \sqrt[3]{0.5x} + 5$ **5.** $h(x) = 4\sqrt[3]{x - 1}$ **6.** $n(x) = \sqrt[3]{4 - x}$

7. Let $g(x) = -\frac{1}{2}\sqrt[3]{x + 2} - 4$. Describe the transformations from the graph of $f(x) = \sqrt[3]{x}$ to the graph of g. Then graph g.

Comparing Average Rates of Change

EXAMPLE 4   **Comparing Cube Root Functions**

The graph of cube root function m is shown. Compare the average rate of change of m to the average rate of change of $h(x) = \sqrt[3]{\dfrac{1}{4}x}$ over the interval $x = 0$ to $x = 8$.

SOLUTION

To calculate the average rates of change, use points whose x-coordinates are 0 and 8.

Function m: Use the graph to estimate. Use $(0, 0)$ and $(8, 8)$.

$$\frac{m(8) - m(0)}{8 - 0} \approx \frac{8 - 0}{8} = 1 \qquad \text{Average rate of change of } m$$

Function h: Evaluate h when $x = 0$ and $x = 8$.

$$h(0) = \sqrt[3]{\frac{1}{4}(0)} = 0 \qquad \text{and} \qquad h(8) = \sqrt[3]{\frac{1}{4}(8)} = \sqrt[3]{2} \approx 1.3$$

Use $(0, 0)$ and $\left(8, \sqrt[3]{2}\right)$.

$$\frac{h(8) - h(0)}{8 - 0} = \frac{\sqrt[3]{2} - 0}{8} \approx 0.16 \qquad \text{Average rate of change of } h$$

▶ The average rate of change of m is $1 \div \dfrac{\sqrt[3]{2}}{8} \approx 6.3$ times greater than the average rate of change of h over the interval $x = 0$ to $x = 8$.

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8. In Example 4, compare the average rates of change over the interval $x = 2$ to $x = 10$.

Solving Real-Life Problems

**EXAMPLE 5**   **Real-Life Application**

The shoulder height h (in centimeters) of a male Asian elephant can be modeled by the function $h = 62.5\sqrt[3]{t} + 75.8$, where t is the age (in years) of the elephant. Use a graphing calculator to graph the function. Estimate the age of an elephant whose shoulder height is 200 centimeters.

SOLUTION

Step 1   Enter $y_1 = 62.5\sqrt[3]{t} + 75.8$ and $y_2 = 200$ into your calculator and graph the equations. Choose a viewing window that shows the point where the graphs intersect.

Step 2   Use the *intersect* feature to find the x-coordinate of the intersection point.
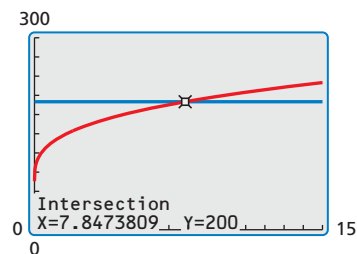
▶ The two graphs intersect at about $(8, 200)$. So, the elephant is about 8 years old.

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**9.** **WHAT IF?** Estimate the age of an elephant whose shoulder height is 175 centimeters.
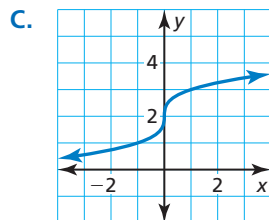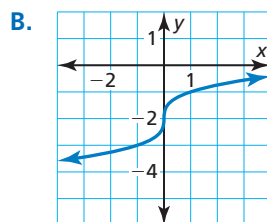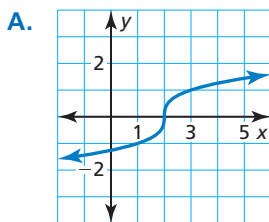
Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The _____ of the radical in a cube root function is 3.

2. **WRITING** Describe the domain and range of the function $f(x) = \sqrt[3]{x - 4} + 1$.

Monitoring Progress and Modeling with Mathematics
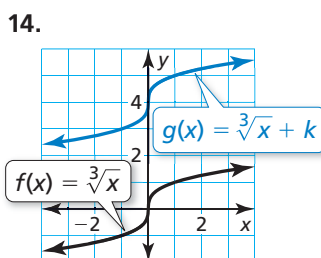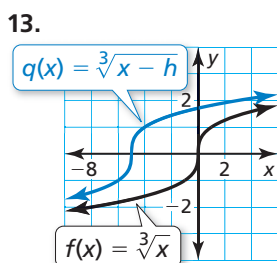
In Exercises 3–6, match the function with its graph.

3. $y = \sqrt[3]{x + 2}$

4. $y = \sqrt[3]{x - 2}$

5. $y = \sqrt[3]{x} + 2$

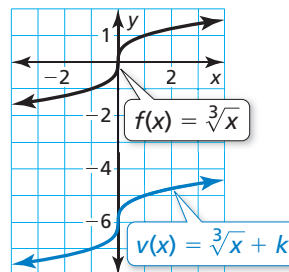6. $y = \sqrt[3]{x} - 2$

A.

B.

C.

D.
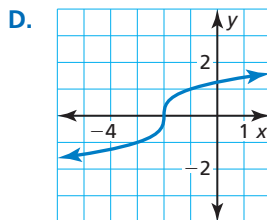
In Exercises 7–12, graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$. (See Example 1.)

7. $h(x) = \sqrt[3]{x} - 4$

8. $g(x) = \sqrt[3]{x + 1}$

9. $m(x) = \sqrt[3]{x} + 5$

10. $q(x) = \sqrt[3]{x} - 3$

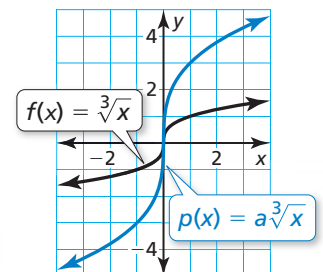11. $p(x) = 6\sqrt[3]{x}$

12. $j(x) = \sqrt[3]{\frac{1}{2}x}$

In Exercises 13–16, compare the graphs. Find the value of h, k, or a.

13.

14.

15.

16.

In Exercises 17–26, graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$. (See Example 2.)

17. $r(x) = -\sqrt[3]{x - 2}$

18. $h(x) = -\sqrt[3]{x} + 3$

19. $k(x) = 5\sqrt[3]{x + 1}$

20. $j(x) = 0.5\sqrt[3]{x - 4}$

21. $g(x) = 4\sqrt[3]{x} - 3$

22. $m(x) = 3\sqrt[3]{x} + 7$

23. $n(x) = \sqrt[3]{-8x} - 1$

24. $v(x) = \sqrt[3]{5x} + 2$

25. $q(x) = \sqrt[3]{2(x + 3)}$

26. $p(x) = \sqrt[3]{3(1 - x)}$

In Exercises 27–32, describe the transformations from the graph of $f(x) = \sqrt[3]{x}$ to the graph of the given function. Then graph the given function.
(See Example 3.)

27. $g(x) = \sqrt[3]{x - 4} + 2$

28. $n(x) = \sqrt[3]{x + 1} - 3$

29. $j(x) = -5\sqrt[3]{x + 3} + 2$

30. $k(x) = 6\sqrt[3]{x - 9} - 5$

31. $v(x) = \frac{1}{3}\sqrt[3]{x - 1} + 7$

32. $h(x) = -\frac{3}{2}\sqrt[3]{x + 4} - 3$
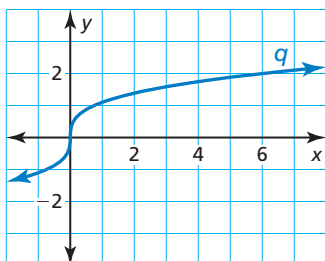
33. **ERROR ANALYSIS** Describe and correct the error in graphing the function $f(x) = \sqrt[3]{x} - 3$.
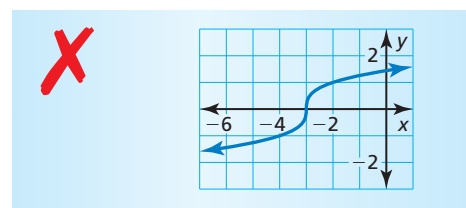
34. ERROR ANALYSIS Describe and correct the error in graphing the function $h(x) = \sqrt[3]{x} + 1$.
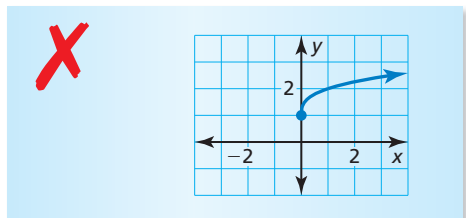
35. COMPARING FUNCTIONS The graph of cube root function q is shown. Compare the average rate of change of q to the average rate of change of $f(x) = 3\sqrt[3]{x}$ over the interval $x = 0$ to $x = 6$. *(See Example 4.)*

36. COMPARING FUNCTIONS The graphs of two cube root functions are shown. Compare the average rates of change of the two functions over the interval $x = -2$ to $x = 2$.
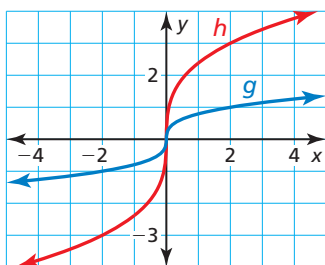

37. MODELING WITH MATHEMATICS For a drag race car that weighs 1600 kilograms, the velocity v (in kilometers per hour) reached by the end of a drag race can be modeled by the function $v = 23.8\sqrt[3]{p}$, where p is the car's power (in horsepower). Use a graphing calculator to graph the function. Estimate the power of a 1600-kilogram car that reaches a velocity of 220 kilometers per hour. *(See Example 5.)*

38. MODELING WITH MATHEMATICS The radius r of a sphere is given by the function $r = \sqrt[3]{\dfrac{3}{4\pi}V}$, where V is the volume of the sphere. Use a graphing calculator to graph the function. Estimate the volume of a spherical beach ball with a radius of 13 inches.

39. MAKING AN ARGUMENT Your friend says that all cube root functions are odd functions. Is your friend correct? Explain.

40. HOW DO YOU SEE IT? The graph represents the cube root function $f(x) = \sqrt[3]{x}$.
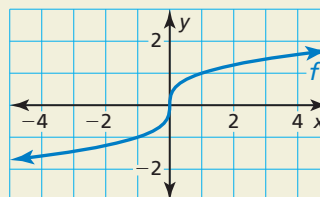
  a. On what interval is f negative? positive?

  b. On what interval, if any, is f decreasing? increasing?

  c. Does f have a maximum or minimum value? Explain.

  d. Find the average rate of change of f over the interval $x = -1$ to $x = 1$.

41. PROBLEM SOLVING Write a cube root function that passes through the point $(3, 4)$ and has an average rate of change of -1 over the interval $x = -5$ to $x = 2$.

42. THOUGHT PROVOKING Write the cube root function represented by the graph. Use a graphing calculator to check your answer.
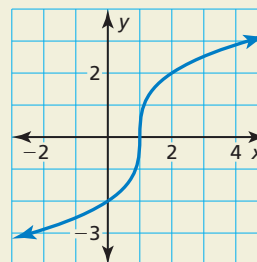
Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Factor the polynomial. *(Section 7.6)*

43. $3x^2 + 12x - 36$

44. $2x^2 - 11x + 9$

45. $4x^2 + 7x - 15$

Solve the equation using square roots. *(Section 9.3)*

46. $x^2 - 36 = 0$

47. $5x^2 + 20 = 0$

48. $(x + 4)^2 = 81$

49. $25(x - 2)^2 = 9$

556 Chapter 10 Radical Functions and Equations

Core Vocabulary

square root function, *p. 544*
radical function, *p. 545*
cube root function, *p. 552*

Core Concepts

Section 10.1

Square Root Functions, *p. 544*
Transformations of Square Root Functions, *p. 545*
Comparing Square Root Functions Using Average Rates of Change, *p. 546*

Section 10.2

Cube Root Functions, *p. 552*
Comparing Cube Root Functions Using Average Rates of Change, *p. 554*

Mathematical Practices

1. In Exercise 45 on page 549, what information are you given? What relationships are present? What is your goal?

2. What units of measure did you use in your answer to Exercise 38 on page 556? Explain your reasoning.

- - - - - - - - - - - - - - Study Skills - - - - - - - - - - - - - - -

Making Note Cards

Invest in three different colors of note cards. Use one color for each of the following: vocabulary words, rules, and calculator keystrokes.

- Using the first color of note cards, write a vocabulary word on one side of a card. On the other side, write the definition and an example. If possible, put the definition in your own words.

- Using the second color of note cards, write a rule on one side of a card. On the other side, write an explanation and an example.

- Using the third color of note cards, write a calculation on one side of a card. On the other side, write the keystrokes required to perform the calculation.

Use the note cards as references while completing your homework. Quiz yourself once a day.

Describe the domain of the function. *(Section 10.1)*

1. $y = \sqrt{x - 3}$

2. $f(x) = 15\sqrt{x}$

3. $y = \sqrt{3 - x}$

Graph the function. Describe the range. Compare the graph to the graph of $f(x) = \sqrt{x}$. *(Section 10.1)*

4. $g(x) = \sqrt{x} + 5$

5. $n(x) = \sqrt{x - 4}$
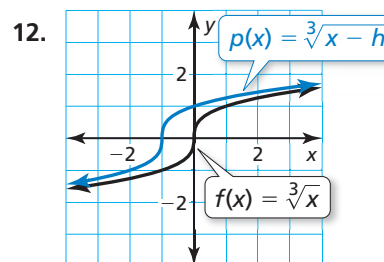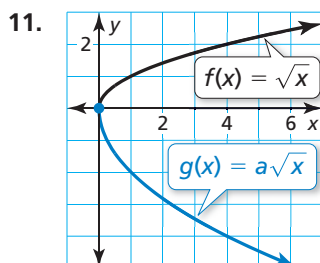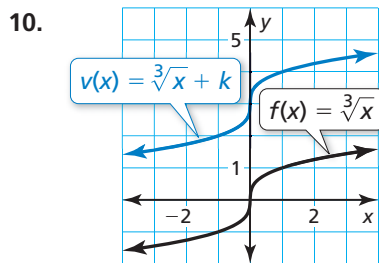
6. $r(x) = -\sqrt{x - 2} + 1$

Graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$. *(Section 10.2)*

7. $b(x) = \sqrt[3]{x + 2}$

8. $h(x) = -3\sqrt[3]{x} - 6$

9. $q(x) = \sqrt[3]{-4 - x}$

Compare the graphs. Find the value of h, k, or a. *(Section 10.1 and Section 10.2)*

10.

11.

12.

Describe the transformations from the graph of f to the graph of h. Then graph h. *(Section 10.1 and Section 10.2)*

13. $f(x) = \sqrt{x}; h(x) = -3\sqrt{x + 2} + 6$

14. $f(x) = \sqrt[3]{x}; h(x) = \frac{1}{2}\sqrt[3]{x} - 3$

15. The time t (in seconds) it takes a dropped object to fall h feet is given by $t = \frac{1}{4}\sqrt{h}$. *(Section 10.1)*

    a. Use a graphing calculator to graph the function. Describe the domain and range.

    b. It takes about 7.4 seconds for a stone dropped from the New River Gorge Bridge in West Virginia to reach the water below. About how high is the bridge above the New River?

16. The radius r of a sphere is given by the function $r = \sqrt[3]{\dfrac{3}{4\pi}V}$, where V is the volume of the sphere. Spaceship Earth is a spherical structure at Walt Disney World that has an inner radius of about 25 meters. Use a graphing calculator to graph the function. Estimate the volume of Spaceship Earth. *(Section 10.2)*

17. The graph of square root function g is shown. Compare the average rate of change of g to the average rate of change of $h(x) = \sqrt[3]{\dfrac{3}{2}x}$ over the interval $x = 0$ to $x = 3$.

    (Section 10.1 and Section 10.2)
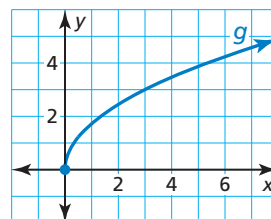
10.3 Solving Radical Equations

Essential Question How can you solve an equation that contains square roots?

Work with a partner. The table shows the time t (in seconds) that it takes a free-falling object (with no air resistance) to fall d feet.

a. Use the data in the table to sketch the graph of t as a function of d. Use the coordinate plane below.

b. Use your graph to estimate the time it takes the object to fall 240 feet.

c. The relationship between d and t is given by the function

$$t = \sqrt{\frac{d}{16}}.$$

Use this function to check your estimate in part (b).

d. It takes 5 seconds for the object to hit the ground. How far did it fall? Explain your reasoning.

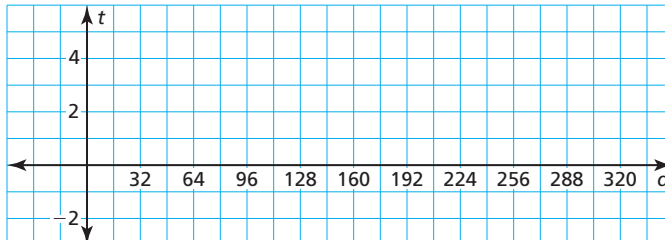| d (feet)	t (seconds)
0	0.00
32	1.41
64	2.00
96	2.45
128	2.83
160	3.16
192	3.46
224	3.74
256	4.00
288	4.24
320	4.47

MODELING WITH MATHEMATICS

To be proficient in math, you need to routinely interpret your mathematical results in the context of the situation and reflect on whether the results make sense.

EXPLORATION 2 **Solving a Square Root Equation**

Work with a partner. The speed s (in feet per second) of the free-falling object in Exploration 1 is given by the function

$$s = \sqrt{64d}.$$

Find the distance the object has fallen when it reaches each speed.

a. $s = 8$ ft/sec **b.** $s = 16$ ft/sec **c.** $s = 24$ ft/sec

Communicate Your Answer

3. How can you solve an equation that contains square roots?

4. Use your answer to Question 3 to solve each equation.

a. $5 = \sqrt{x + 20}$ **b.** $4 = \sqrt{x - 18}$

c. $\sqrt{x} + 2 = 3$ **d.** $-3 = -2\sqrt{x}$

What You Will Learn

▶ Solve radical equations.
▶ Identify extraneous solutions.
▶ Solve real-life problems involving radical equations.

Solving Radical Equations

A **radical equation** is an equation that contains a radical expression with a variable in the radicand. To solve a radical equation involving a square root, first use properties of equality to isolate the radical on one side of the equation. Then use the following property to eliminate the radical and solve for the variable.

Core Concept

Squaring Each Side of an Equation

Words   If two expressions are equal, then their squares are also equal.

Algebra   If $a = b$, then $a^2 = b^2$.

EXAMPLE 1   **Solving Radical Equations**

Solve each equation.

a. $\sqrt{x} + 5 = 13$

b. $3 - \sqrt{x} = 0$

SOLUTION

a.
$$\sqrt{x} + 5 = 13 \qquad \text{Write the equation.}$$
$$\sqrt{x} = 8 \qquad \text{Subtract 5 from each side.}$$
$$(\sqrt{x})^2 = 8^2 \qquad \text{Square each side of the equation.}$$
$$x = 64 \qquad \text{Simplify.}$$

▶ The solution is $x = 64$.

b.
$$3 - \sqrt{x} = 0 \qquad \text{Write the equation.}$$
$$3 = \sqrt{x} \qquad \text{Add } \sqrt{x} \text{ to each side.}$$
$$3^2 = (\sqrt{x})^2 \qquad \text{Square each side of the equation.}$$
$$9 = x \qquad \text{Simplify.}$$

▶ The solution is $x = 9$.

Check
$$\sqrt{x} + 5 = 13$$
$$\sqrt{64} + 5 \overset{?}{=} 13$$
$$8 + 5 \overset{?}{=} 13$$
$$13 = 13 ✓$$

Check
$$3 - \sqrt{x} = 0$$
$$3 - \sqrt{9} \overset{?}{=} 0$$
$$3 - 3 \overset{?}{=} 0$$
$$0 = 0 ✓$$

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Solve the equation. Check your solution.

1. $\sqrt{x} = 6$

2. $\sqrt{x} - 7 = 3$

3. $\sqrt{y} + 15 = 22$

4. $1 - \sqrt{c} = -2$

EXAMPLE 2   **Solving a Radical Equation**

$$4\sqrt{x+2}+3=19 \qquad \text{Original equation}$$
$$4\sqrt{x+2}=16 \qquad \text{Subtract 3 from each side.}$$
$$\sqrt{x+2}=4 \qquad \text{Divide each side by 4.}$$
$$\left(\sqrt{x+2}\right)^2=4^2 \qquad \text{Square each side of the equation.}$$
$$x+2=16 \qquad \text{Simplify.}$$
$$x=14 \qquad \text{Subtract 2 from each side.}$$

▶ The solution is $x=14$.

Check

$$4\sqrt{x+2}+3=19$$
$$4\sqrt{14+2}+3 \overset{?}{=} 19$$
$$4\sqrt{16}+3 \overset{?}{=} 19$$
$$4(4)+3 \overset{?}{=} 19$$
$$19=19 ✓$$

EXAMPLE 3   **Solving an Equation with Radicals on Both Sides**

Solve $\sqrt{2x-1}=\sqrt{x+4}$.
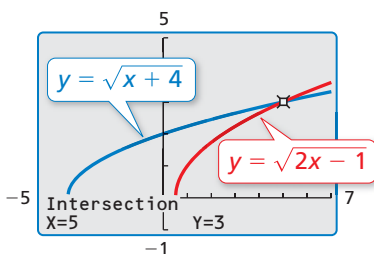
SOLUTION

Method 1
$$\sqrt{2x-1}=\sqrt{x+4} \qquad \text{Write the equation.}$$
$$\left(\sqrt{2x-1}\right)^2=\left(\sqrt{x+4}\right)^2 \qquad \text{Square each side of the equation.}$$
$$2x-1=x+4 \qquad \text{Simplify.}$$
$$x=5 \qquad \text{Solve for } x.$$

▶ The solution is $x=5$.

Method 2   Graph each side of the equation, as shown. Use the *intersect* feature to find the coordinates of the point of intersection. The x-value of the point of intersection is 5.

▶ So, the solution is $x=5$.

$y=\sqrt{x+4}$
$y=\sqrt{2x-1}$
Intersection X=5 Y=3

EXAMPLE 4   **Solving a Radical Equation Involving a Cube Root**

Solve $\sqrt[3]{5x-2}=12$.

SOLUTION

$$\sqrt[3]{5x-2}=12 \qquad \text{Write the equation.}$$
$$\left(\sqrt[3]{5x-2}\right)^3=12^3 \qquad \text{Cube each side of the equation.}$$
$$5x-2=1728 \qquad \text{Simplify.}$$
$$x=346 \qquad \text{Solve for } x.$$

▶ The solution is $x=346$.

LOOKING FOR STRUCTURE

You can extend the concept taught in Examples 1–3 to solve a radical equation involving a cube root. Instead of squaring each side of the equation, you *cube* each side to eliminate the radical.

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Solve the equation. Check your solution.

5. $\sqrt{x+4}+7=11$ **6.** $15=6+\sqrt{3w-9}$ **7.** $\sqrt{3x+1}=\sqrt{4x-7}$

8. $\sqrt{n}=\sqrt{5n-1}$ **9.** $\sqrt[3]{y}-4=1$ **10.** $\sqrt[3]{3c+7}=10$

Identifying Extraneous Solutions

Squaring each side of an equation can sometimes introduce an extraneous solution.

EXAMPLE 5 **Identifying an Extraneous Solution**

Solve $x = \sqrt{x + 6}$.

SOLUTION

$x = \sqrt{x + 6}$	Write the equation.
$x^2 = \left(\sqrt{x + 6}\right)^2$	Square each side of the equation.
$x^2 = x + 6$	Simplify.
$x^2 - x - 6 = 0$	Subtract x and 6 from each side.
$(x - 3)(x + 2) = 0$	Factor.
$x - 3 = 0 \quad \text{or} \quad x + 2 = 0$	Zero-Product Property
$x = 3 \quad \text{or} \quad x = -2$	Solve for x.

Check Check each solution in the original equation.

$3 \overset{?}{=} \sqrt{3 + 6}$ Substitute for x. $-2 \overset{?}{=} \sqrt{-2 + 6}$

$3 \overset{?}{=} \sqrt{9}$ Simplify. $-2 \overset{?}{=} \sqrt{4}$

$3 = 3$ ✔   Simplify. $-2 \neq 2$ ✗

▶ Because $x = -2$ does not satisfy the original equation, it is an extraneous solution. The only solution is $x = 3$.

EXAMPLE 6 **Identifying an Extraneous Solution**

Solve $13 + \sqrt{5n} = 3$.

SOLUTION

Check

$13 + \sqrt{5n} = 3$

$13 + \sqrt{5(20)} \overset{?}{=} 3$

$13 + \sqrt{100} \overset{?}{=} 3$

$23 \neq 3$ ✗

$13 + \sqrt{5n} = 3$	Write the equation.
$\sqrt{5n} = -10$	Subtract 13 from each side.
$\left(\sqrt{5n}\right)^2 = (-10)^2$	Square each side of the equation.
$5n = 100$	Simplify.
$n = 20$	Divide each side by 5.

▶ Because $n = 20$ does not satisfy the original equation, it is an extraneous solution. So, the equation has no solution.

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Solve the equation. Check your solution(s).

11. $\sqrt{4 - 3x} = x$ **12.** $\sqrt{3m} + 10 = 1$ **13.** $p + 1 = \sqrt{7p + 15}$

Solving Real-Life Problems

EXAMPLE 7 **Modeling with Mathematics**

The period P (in seconds) of a pendulum is given by the function $P = 2\pi\sqrt{\dfrac{L}{32}}$, where L is the pendulum length (in feet). A pendulum has a period of 4 seconds. Is this pendulum twice as long as a pendulum with a period of 2 seconds? Explain your reasoning.

SOLUTION

1. **Understand the Problem** You are given a function that represents the period P of a pendulum based on its length L. You need to find and compare the values of L for two values of P.

2. **Make a Plan** Substitute $P = 2$ and $P = 4$ into the function and solve for L. Then compare the values.

3. **Solve the Problem**

|		
$P = 2\pi\sqrt{\dfrac{L}{32}}$	Write the function.	$P = 2\pi\sqrt{\dfrac{L}{32}}$
$2 = 2\pi\sqrt{\dfrac{L}{32}}$	Substitute for P.	$4 = 2\pi\sqrt{\dfrac{L}{32}}$
$\dfrac{2}{2\pi} = \sqrt{\dfrac{L}{32}}$	Divide each side by 2π.	$\dfrac{4}{2\pi} = \sqrt{\dfrac{L}{32}}$
$\dfrac{1}{\pi} = \sqrt{\dfrac{L}{32}}$	Simplify.	$\dfrac{2}{\pi} = \sqrt{\dfrac{L}{32}}$
$\dfrac{1}{\pi^2} = \dfrac{L}{32}$	Square each side and simplify.	$\dfrac{4}{\pi^2} = \dfrac{L}{32}$
$\dfrac{32}{\pi^2} = L$	Multiply each side by 32.	$\dfrac{128}{\pi^2} = L$
$3.24 \approx L$	Use a calculator.	$12.97 \approx L$

▶ No, the length of the pendulum with a period of 4 seconds is $\dfrac{128}{\pi^2} \div \dfrac{32}{\pi^2} = 4$ times longer than the length of a pendulum with a period of 2 seconds.

4. **Look Back** Use the *trace* feature of a graphing calculator to check your solutions.
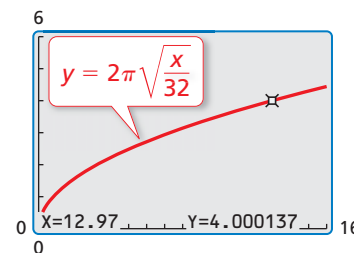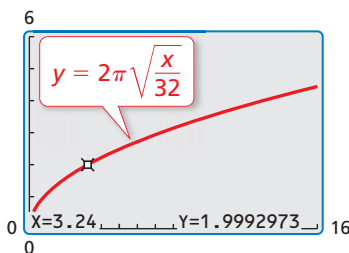
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14. What is the length of a pendulum that has a period of 2.5 seconds?

Vocabulary and Core Concept Check

1. **VOCABULARY** Why should you check every solution of a radical equation?

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$$\sqrt{x} + 6 = 10 \qquad 2\sqrt{x+3} = 32 \qquad x\sqrt{3} - 5 = 4 \qquad \sqrt{x-1} = 16$$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, solve the equation. Check your solution. *(See Example 1.)*

3. $\sqrt{x} = 9$

4. $\sqrt{y} = 4$

5. $7 = \sqrt{m} - 5$

6. $\sqrt{p} - 7 = -1$

7. $\sqrt{c} + 12 = 23$

8. $\sqrt{x} + 6 = 8$

9. $4 - \sqrt{a} = 2$

10. $-8 = 7 - \sqrt{r}$

11. $3\sqrt{y} - 18 = -3$

12. $2\sqrt{q} + 5 = 11$

In Exercises 13–20, solve the equation. Check your solution. *(See Example 2.)*

13. $\sqrt{a-3} + 5 = 9$

14. $\sqrt{b+7} - 5 = -2$

15. $2\sqrt{x+4} = 16$

16. $5\sqrt{y-2} = 10$

17. $-1 = \sqrt{5r+1} - 7$

18. $2 = \sqrt{4s-4} - 4$

19. $7 + 3\sqrt{3p-9} = 25$

20. $19 - 4\sqrt{3c-11} = 11$

21. **MODELING WITH MATHEMATICS**
The Cave of Swallows is a natural open-air pit cave in the state of San Luis Potosí, Mexico. The 1220-foot-deep cave was a popular destination for BASE jumpers. The function $t = \frac{1}{4}\sqrt{d}$ represents the time t (in seconds) that it takes a BASE jumper to fall d feet. How far does a BASE jumper fall in 3 seconds?
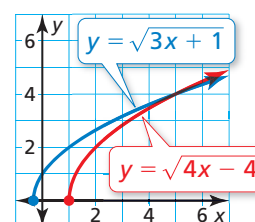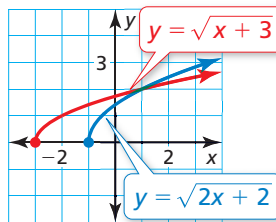
22. **MODELING WITH MATHEMATICS**
The edge length s of a cube with a surface area of A is given by $s = \sqrt{\dfrac{A}{6}}$. What is the surface area of a cube with an edge length of 4 inches?
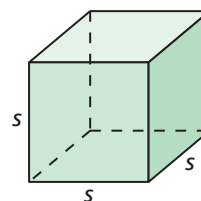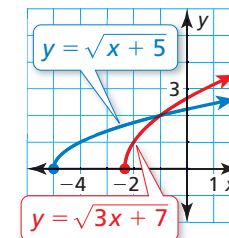

In Exercises 23–26, use the graph to solve the equation.

23. $\sqrt{2x+2} = \sqrt{x+3}$

24. $\sqrt{3x+1} = \sqrt{4x-4}$

$y = \sqrt{x+3}$
$y = \sqrt{2x+2}$

$y = \sqrt{3x+1}$
$y = \sqrt{4x-4}$

25. $\sqrt{x+2} - \sqrt{2x} = 0$

26. $\sqrt{x+5} - \sqrt{3x+7} = 0$

$y = \sqrt{x+2}$
$y = \sqrt{2x}$

$y = \sqrt{x+5}$
$y = \sqrt{3x+7}$
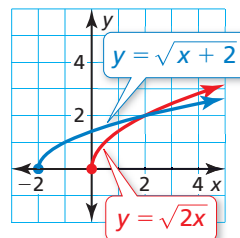
In Exercises 27–34, solve the equation. Check your solution. *(See Example 3.)*

27. $\sqrt{2x-9} = \sqrt{x}$

28. $\sqrt{y+1} = \sqrt{4y-8}$

29. $\sqrt{3g+1} = \sqrt{7g-19}$

30. $\sqrt{8h-7} = \sqrt{6h+7}$

31. $\sqrt{\dfrac{p}{2} - 2} = \sqrt{p-8}$

32. $\sqrt{2v-5} = \sqrt{\dfrac{v}{3} + 5}$

33. $\sqrt{2c+1} - \sqrt{4c} = 0$

34. $\sqrt{5r} - \sqrt{8r-2} = 0$

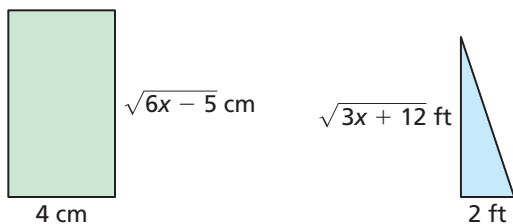**MATHEMATICAL CONNECTIONS** In Exercises 35 and 36, find the value of x.

35. Perimeter = 22 cm

36. Area = $\sqrt{5x - 4}$ ft²

$\sqrt{6x - 5}$ cm

4 cm

$\sqrt{3x + 12}$ ft

2 ft

In Exercises 37–44, solve the equation. Check your solution. *(See Example 4.)*

37. $\sqrt[3]{x} = 4$

38. $\sqrt[3]{y} = 2$

39. $6 = \sqrt[3]{8g}$

40. $\sqrt[3]{r + 19} = 3$

41. $\sqrt[3]{2s + 9} = -3$

42. $-5 = \sqrt[3]{10x + 15}$

43. $\sqrt[3]{y + 6} = \sqrt[3]{5y - 2}$

44. $\sqrt[3]{7j - 2} = \sqrt[3]{j + 4}$

In Exercises 45–48, determine which solution, if any, is an extraneous solution.

45. $\sqrt{6x - 5} = x; x = 5, x = 1$

46. $\sqrt{2y + 3} = y; y = -1, y = 3$

47. $\sqrt{12p + 16} = -2p; p = -1, p = 4$

48. $-3g = \sqrt{-18 - 27g}; g = -2, g = -1$

In Exercises 49–58, solve the equation. Check your solution(s). *(See Examples 5 and 6.)*

49. $y = \sqrt{5y - 4}$

50. $\sqrt{-14 - 9x} = x$

51. $\sqrt{1 - 3a} = 2a$

52. $2q = \sqrt{10q - 6}$

53. $9 + \sqrt{5p} = 4$

54. $\sqrt{3n} - 11 = -5$

55. $\sqrt{2m + 2} - 3 = 1$

56. $15 + \sqrt{4b - 8} = 13$

57. $r + 4 = \sqrt{-4r - 19}$

58. $\sqrt{3 - s} = s - 1$

ERROR ANALYSIS In Exercises 59 and 60, describe and correct the error in solving the equation.

59.

✗

$2 + 5\sqrt{x} = 12$

$5\sqrt{x} = 10$

$5x = 100$

$x = 20$

60.

✗

$x = \sqrt{12 - 4x}$

$x^2 = 12 - 4x$

$x^2 + 4x - 12 = 0$

$(x - 2)(x + 6) = 0$

$x = 2$ or $x = -6$

The solutions are $x = 2$ and $x = -6$.

61. REASONING Explain how to use mental math to solve $\sqrt{2x} + 5 = 1$.

62. WRITING Explain how you would solve $\sqrt[4]{m + 4} - \sqrt[4]{3m} = 0$.

63. MODELING WITH MATHEMATICS The formula $V = \sqrt{PR}$ relates the voltage V (in volts), power P (in watts), and resistance R (in ohms) of an electrical circuit. The hair dryer shown is on a 120-volt circuit. Is the resistance of the hair dryer half as much as the resistance of the same hair dryer on a 240-volt circuit? Explain your reasoning. *(See Example 7.)*

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64. MODELING WITH MATHEMATICS The time t (in seconds) it takes a trapeze artist to swing back and forth is represented by the function $t = 2\pi\sqrt{\dfrac{r}{32}}$, where r is the rope length (in feet). It takes the trapeze artist 6 seconds to swing back and forth. Is this rope $\frac{3}{2}$ as long as the rope used when it takes the trapeze artist 4 seconds to swing back and forth? Explain your reasoning.

REASONING In Exercises 65–68, determine whether the statement is *true* or *false*. If it is false, explain why.

65. If $\sqrt{a} = b$, then $(\sqrt{a})^2 = b^2$.

66. If $\sqrt{a} = \sqrt{b}$, then $a = b$.

67. If $a^2 = b^2$, then $a = b$.

68. If $a^2 = \sqrt{b}$, then $a^4 = (\sqrt{b})^2$.

69. COMPARING METHODS Consider the equation $x + 2 = \sqrt{2x - 3}$.

   a. Solve the equation by graphing. Describe the process.

   b. Solve the equation algebraically. Describe the process.

   c. Which method do you prefer? Explain your reasoning.

70. HOW DO YOU SEE IT? The graph shows two radical functions.
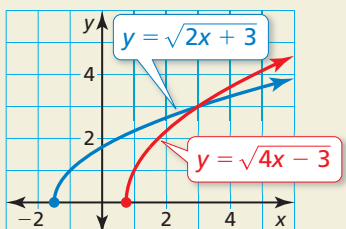
   a. Write an equation whose solution is the x-coordinate of the point of intersection of the graphs.

   b. Use the graph to solve the equation.

71. MATHEMATICAL CONNECTIONS The slant height s of a cone with a radius of r and a height of h is given by $s = \sqrt{r^2 + h^2}$. The slant heights of the two cones are equal. Find the radius of each cone.
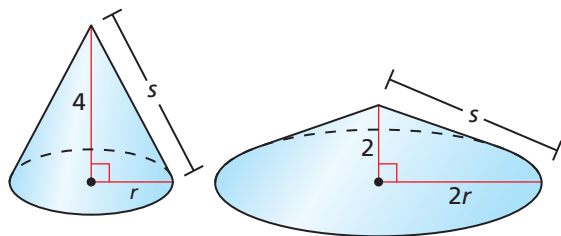
72. CRITICAL THINKING How is squaring $\sqrt{x + 2}$ different from squaring $\sqrt{x} + 2$?

USING STRUCTURE In Exercises 73–78, solve the equation. Check your solution.

73. $\sqrt{m + 15} = \sqrt{m} + \sqrt{5}$ **74.** $2 - \sqrt{x + 1} = \sqrt{x + 2}$

75. $\sqrt{5y + 9} + \sqrt{5y} = 9$

76. $\sqrt{2c - 8} - \sqrt{2c} - 4 = 0$

77. $2\sqrt{1 + 4h} - 4\sqrt{h} - 2 = 0$

78. $\sqrt{20 - 4z} + 2\sqrt{-z} = 10$

79. OPEN-ENDED Write a radical equation that has a solution of $x = 5$.

80. OPEN-ENDED Write a radical equation that has $x = 3$ and $x = 4$ as solutions.

81. MAKING AN ARGUMENT Your friend says the equation $\sqrt{(2x + 5)^2} = 2x + 5$ is always true, because after simplifying the left side of the equation, the result is an equation with infinitely many solutions. Is your friend correct? Explain.

82. THOUGHT PROVOKING Solve the equation $\sqrt[3]{x + 1} = \sqrt{x - 3}$. Show your work and explain your steps.

83. MODELING WITH MATHEMATICS The frequency f (in cycles per second) of a string of an electric guitar is given by the equation $f = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}}$, where ℓ is the length of the string (in meters), T is the string's tension (in newtons), and m is the string's mass per unit length (in kilograms per meter). The high E string of an electric guitar is 0.64 meter long with a mass per unit length of 0.000401 kilogram per meter.

   a. How much tension is required to produce a frequency of about 330 cycles per second?

   b. Would you need more or less tension to create the same frequency on a string with greater mass per unit length? Explain.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Find the product. *(Section 7.2)*

84. $(x + 8)(x - 2)$ **85.** $(3p - 1)(4p + 5)$ **86.** $(s + 2)(s^2 + 3s - 4)$

Graph the function. Compare the graph to the graph of $f(x) = x^2$. *(Section 8.1)*

87. $r(x) = 3x^2$ **88.** $g(x) = \frac{3}{4}x^2$ **89.** $h(x) = -5x^2$

10.4 Inverse of a Function

Essential Question How are a function and its inverse related?

Exploring Inverse Functions

Work with a partner. The functions *f* and *g* are *inverses* of each other. Compare the tables of values of the two functions. How are the functions related?

x	0	0.5	1	1.5	2	2.5	3	3.5
f(x)	0	0.25	1	2.25	4	6.25	9	12.25

x	0	0.25	1	2.25	4	6.25	9	12.25
g(x)	0	0.5	1	1.5	2	2.5	3	3.5

Exploring Inverse Functions

Work with a partner.

a. Plot the two sets of points represented by the tables in Exploration 1. Use the coordinate plane below.

b. Connect each set of points with a smooth curve.

c. Describe the relationship between the two graphs.

d. Write an equation for each function.
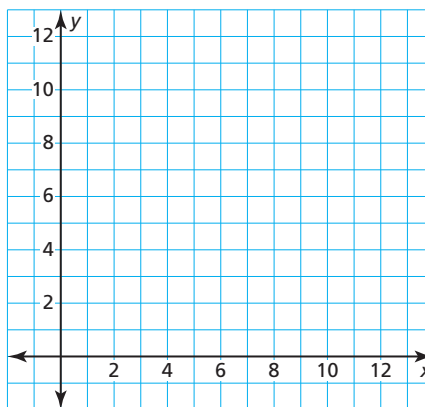
Communicate Your Answer

3. How are a function and its inverse related?

4. A table of values for a function *f* is given. Create a table of values for a function *g*, the inverse of *f*.

x	0	1	2	3	4	5	6	7
f(x)	1	2	3	4	5	6	7	8

5. Sketch the graphs of $f(x) = x + 4$ and its inverse in the same coordinate plane. Then write an equation of the inverse of *f*. Explain your reasoning.

ATTENDING TO PRECISION

To be proficient in math, you need to communicate precisely with others.

Core Vocabulary

inverse relation, *p. 568*
inverse function, *p. 569*

Previous
input
output
inverse operations
reflection
line of reflection

What You Will Learn

▶ Find inverses of relations.
▶ Explore inverses of functions.
▶ Find inverses of functions algebraically.
▶ Find inverses of nonlinear functions.

Finding Inverses of Relations

Recall that a relation pairs inputs with outputs. An **inverse relation** switches the input and output values of the original relation.
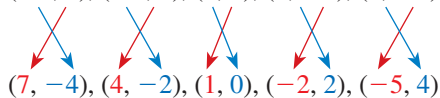
Core Concept

Inverse Relation

When a relation contains (a, b), the inverse relation contains (b, a).

EXAMPLE 1 Finding Inverses of Relations

Find the inverse of each relation.

a. $(-4, 7), (-2, 4), (0, 1), (2, -2), (4, -5)$ Switch the coordinates of each ordered pair.

$(7, -4), (4, -2), (1, 0), (-2, 2), (-5, 4)$ Inverse relation

b.

Input	−1	0	1	2	3	4
Output	5	10	15	20	25	30

Inverse relation:

Input	5	10	15	20	25	30
Output	−1	0	1	2	3	4

Switch the inputs and outputs.

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Find the inverse of the relation.

1. $(-3, -4), (-2, 0), (-1, 4), (0, 8), (1, 12), (2, 16), (3, 20)$

2.

Input	−2	−1	0	1	2
Output	4	1	0	1	4

Exploring Inverses of Functions

Throughout this book, you have used given inputs to find corresponding outputs of $y = f(x)$ for various types of functions. You have also used given outputs to find corresponding inputs. Now you will solve equations of the form $y = f(x)$ for x to obtain a formula for finding the input given a specific output of the function f.

EXAMPLE 2 **Writing a Formula for the Input of a Function**

Let $f(x) = 2x + 1$. Solve $y = f(x)$ for x. Then find the input when the output is -3.

SOLUTION

$$y = 2x + 1 \qquad \text{Set } y \text{ equal to } f(x).$$

$$y - 1 = 2x \qquad \text{Subtract 1 from each side.}$$

$$\frac{y - 1}{2} = x \qquad \text{Divide each side by 2.}$$

Find the input when $y = -3$.

$$x = \frac{-3 - 1}{2} \qquad \text{Substitute } -3 \text{ for } y.$$

$$= \frac{-4}{2} \qquad \text{Subtract.}$$

$$= -2 \qquad \text{Divide.}$$

> **Check**
>
> $f(-2) = 2(-2) + 1$
>
> $= -4 + 1$
>
> $= -3$ ✔

▶ So, the input is -2 when the output is -3.

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Solve $y = f(x)$ for x. Then find the input when the output is 4.

3. $f(x) = x - 6$ **4.** $f(x) = \frac{1}{2}x + 3$ **5.** $f(x) = 4x^2$

UNDERSTANDING MATHEMATICAL TERMS

The term *inverse functions* does not refer to a new type of function. Rather, it describes any pair of functions that are inverses.

In Example 2, notice the steps involved after substituting for x in $y = 2x + 1$ and after substituting for y in $x = \dfrac{y - 1}{2}$.

$$y = 2x + 1 \qquad\qquad\qquad x = \frac{y - 1}{2}$$

Step 1 Multiply by 2.   **Step 1** Subtract 1.

Step 2 Add 1.   *inverse operations in the reverse order*   **Step 2** Divide by 2.

Notice that these steps *undo* each other. **Inverse functions** are functions that undo each other. In Example 2, you can use the equation solved for x to write the inverse of f by switching the roles of x and y.

$$f(x) = 2x + 1 \quad \text{original function} \qquad\qquad g(x) = \frac{x - 1}{2} \quad \text{inverse function}$$

LOOKING FOR A PATTERN

Notice that the graph of the inverse function g is a reflection of the graph of the original function f. The line of reflection is $y = x$.
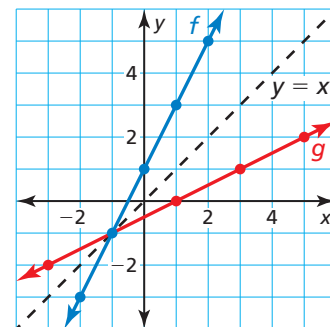
Because an inverse function interchanges the input and output values of the original function, the domain and range are also interchanged.

Original function: $f(x) = 2x + 1$

x	−2	−1	0	1	2
y	−3	−1	1	3	5

Inverse function: $g(x) = \dfrac{x - 1}{2}$

x	−3	−1	1	3	5
y	−2	−1	0	1	2

Finding Inverses of Functions Algebraically

Core Concept

Finding Inverses of Functions Algebraically

Step 1 Set y equal to $f(x)$.

Step 2 Switch x and y in the equation.

Step 3 Solve the equation for y.

STUDY TIP

On the previous page, you solved a function for x and switched the roles of x and y to find the inverse function. You can also find the inverse function by switching x and y first, and then solving for y.

EXAMPLE 3 Finding the Inverse of a Linear Function

Find the inverse of $f(x) = 4x - 9$.

SOLUTION

Method 1 Use the method above.

Step 1 $f(x) = 4x - 9$ Write the function.

$y = 4x - 9$ Set y equal to $f(x)$.

Step 2 $x = 4y - 9$ Switch x and y in the equation.

Step 3 $x + 9 = 4y$ Add 9 to each side.

$\dfrac{x + 9}{4} = y$ Divide each side by 4.

▶ The inverse of f is $g(x) = \dfrac{x + 9}{4}$, or $g(x) = \dfrac{1}{4}x + \dfrac{9}{4}$.

Method 2 Use inverse operations in the reverse order.

$f(x) = 4x - 9$ Multiply the input x by 4 and then subtract 9.

To find the inverse, apply inverse operations in the reverse order.

$g(x) = \dfrac{x + 9}{4}$ Add 9 to the input x and then divide by 4.

▶ The inverse of f is $g(x) = \dfrac{x + 9}{4}$, or $g(x) = \dfrac{1}{4}x + \dfrac{9}{4}$.

Check

The graph of g appears to be a reflection of the graph of f in the line $y = x$. ✔

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Find the inverse of the function. Then graph the function and its inverse.
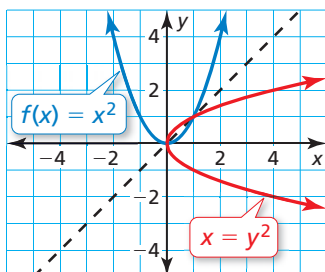
6. $f(x) = 6x$ **7.** $f(x) = -x + 5$ **8.** $f(x) = \frac{1}{4}x - 1$
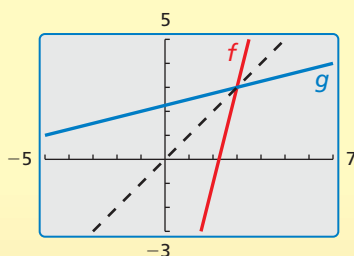
Finding Inverses of Nonlinear Functions

The inverse of the linear function in Example 3 is also a function. The inverse of a function, however, is *not* always a function. The graph of $f(x) = x^2$ is shown along with its reflection in the line $y = x$. Notice that the graph of the inverse of $f(x) = x^2$ does not pass the Vertical Line Test. So, the inverse is *not* a function.

When the domain of $f(x) = x^2$ is *restricted* to only nonnegative real numbers, the inverse of f is a function, as shown in the next example.
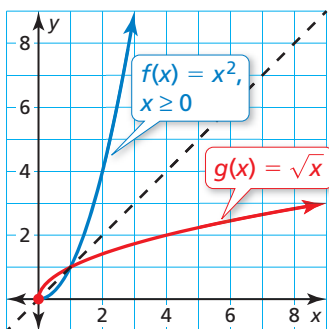
EXAMPLE 4 **Finding the Inverse of a Quadratic Function**

Find the inverse of $f(x) = x^2$, $x \geq 0$. Then graph the function and its inverse.

SOLUTION

$f(x) = x^2$ Write the function.

$y = x^2$ Set y equal to $f(x)$.

$x = y^2$ Switch x and y in the equation.

$\pm\sqrt{x} = y$ Take square root of each side.

▶ Because the domain of f is restricted to nonnegative values of x, the range of the inverse must also be restricted to nonnegative values. So, the inverse of f is $g(x) = \sqrt{x}$.

You can use the graph of a function f to determine whether the inverse of f is a function by applying the *Horizontal Line Test*.

Core Concept

Horizontal Line Test

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

EXAMPLE 5   **Finding the Inverse of a Radical Function**

Consider the function $f(x) = \sqrt{x + 2}$. Determine whether the inverse of f is a function. Then find the inverse.

SOLUTION

Graph the function f. Because no horizontal line intersects the graph more than once, the inverse of f is a function. Find the inverse.

$y = \sqrt{x + 2}$ Set y equal to $f(x)$.

$x = \sqrt{y + 2}$ Switch x and y in the equation.
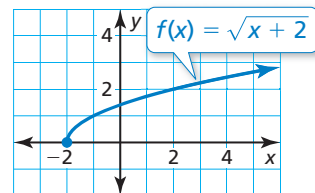
$x^2 = \left(\sqrt{y + 2}\right)^2$ Square each side.

$x^2 = y + 2$ Simplify.
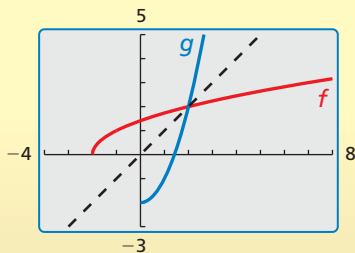
$x^2 - 2 = y$ Subtract 2 from each side.

▶ Because the range of f is $y \geq 0$, the domain of the inverse must be restricted to $x \geq 0$. So, the inverse of f is $g(x) = x^2 - 2$, where $x \geq 0$.

Check

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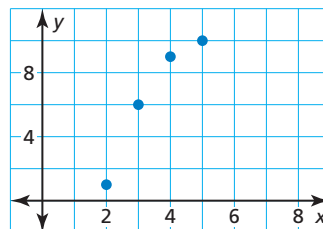**Find the inverse of the function. Then graph the function and its inverse.**

9. $f(x) = -x^2$, $x \leq 0$ **10.** $f(x) = 4x^2 + 3$, $x \geq 0$

11. Is the inverse of $f(x) = \sqrt{2x - 1}$ a function? Find the inverse.

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A relation contains the point $(-3, 10)$. The _____ contains the point $(10, -3)$.

2. **DIFFERENT WORDS, SAME QUESTION** Consider the function f represented by the graph. Which is different? Find "both" answers.

   Graph the inverse of the function.

   Reflect the graph of the function in the *x*-axis.

   Reflect the graph of the function in the line $y = x$.

   Switch the inputs and outputs of the function and graph the resulting function.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the inverse of the relation. *(See Example 1.)*

3. $(1, 0), (3, -8), (4, -3), (7, -5), (9, -1)$
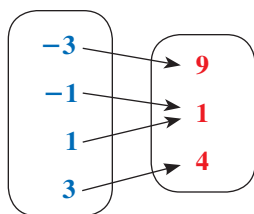
4. $(2, 1), (4, -3), (6, 7), (8, 1), (10, -4)$

5.

Input	−5	−5	0	5	10
Output	8	6	0	6	8

6.

Input	−12	−8	−5	−3	−2
Output	2	5	−1	10	−2

7.
Input	Output
−3	9
−1	1
1	4
3	

8.
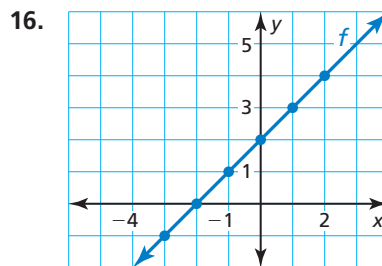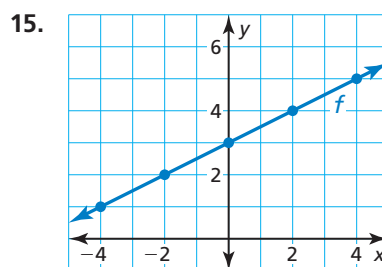| Input	Output
0	4
2	−5
7	0
15	−9
	11
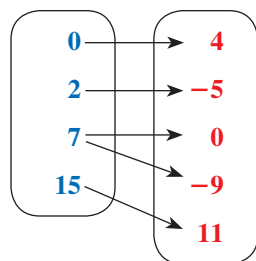
In Exercises 9–14, solve $y = f(x)$ for x. Then find the input when the output is 2. *(See Example 2.)*

9. $f(x) = x + 5$

10. $f(x) = 2x - 3$

11. $f(x) = \frac{1}{4}x - 1$

12. $f(x) = \frac{2}{3}x + 4$

13. $f(x) = 9x^2$

14. $f(x) = \frac{1}{2}x^2 - 7$

In Exercises 15 and 16, graph the inverse of the function by reflecting the graph in the line $y = x$. Describe the domain and range of the inverse.

15.

16.

In Exercises 17–22, find the inverse of the function. Then graph the function and its inverse. *(See Example 3.)*
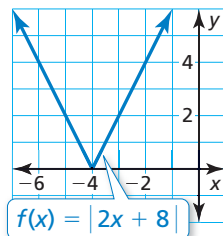
17. $f(x) = 4x - 1$

18. $f(x) = -2x + 5$

19. $f(x) = -3x - 2$

20. $f(x) = 2x + 3$

21. $f(x) = \frac{1}{3}x + 8$

22. $f(x) = -\frac{3}{2}x + \frac{7}{2}$

In Exercises 23–28, find the inverse of the function. Then graph the function and its inverse. *(See Example 4.)*
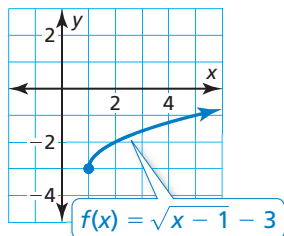
23. $f(x) = 4x^2, x \geq 0$

24. $f(x) = -\frac{1}{25}x^2, x \leq 0$

25. $f(x) = -x^2 + 10, x \leq 0$

26. $f(x) = 2x^2 + 6, x \geq 0$

27. $f(x) = \frac{1}{9}x^2 + 2, x \geq 0$ **28.** $f(x) = -4x^2 - 8, x \leq 0$

In Exercises 29–32, use the Horizontal Line Test to determine whether the inverse of f is a function.
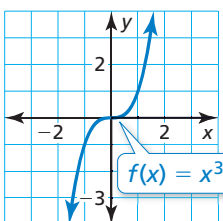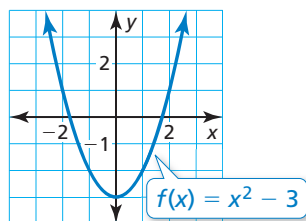
29.

$f(x) = |2x + 8|$

30.

$f(x) = \sqrt{x - 1} - 3$

31.

$f(x) = x^3$

32.

$f(x) = x^2 - 3$

In Exercises 33–42, determine whether the inverse of f is a function. Then find the inverse. *(See Example 5.)*

33. $f(x) = \sqrt{x + 3}$

34. $f(x) = \sqrt{x - 5}$

35. $f(x) = \sqrt{2x - 6}$

36. $f(x) = \sqrt{4x + 1}$

37. $f(x) = 3\sqrt{x - 8}$

38. $f(x) = -\frac{1}{4}\sqrt{5x + 2}$

39. $f(x) = -\sqrt{3x + 5} - 2$

40. $f(x) = 2\sqrt{x - 7} + 6$

41. $f(x) = 2x^2$

42. $f(x) = |x|$

43. ERROR ANALYSIS Describe and correct the error in finding the inverse of the function $f(x) = 3x + 5$.

$y = 3x + 5$

$y - 5 = 3x$

$\dfrac{y - 5}{3} = x$

The inverse of f is $g(x) = \dfrac{y - 5}{3}$, or $g(x) = \dfrac{y}{3} - \dfrac{5}{3}$.

44. ERROR ANALYSIS Describe and correct the error in finding and graphing the inverse of the function $f(x) = \sqrt{x - 3}$.

$y = \sqrt{x - 3}$

$x = \sqrt{y - 3}$

$x^2 = y - 3$

$x^2 + 3 = y$

The inverse of f is

$g(x) = x^2 + 3$.

45. MODELING WITH MATHEMATICS The euro is the unit of currency for the European Union. On a certain day, the number E of euros that could be obtained for D U.S. dollars was represented by the formula shown.

$$E = 0.74683D$$

Solve the formula for D. Then find the number of U.S. dollars that could be obtained for 250 euros on that day.

46. MODELING WITH MATHEMATICS A crow is flying at a height of 50 feet when it drops a walnut to break it open. The height h (in feet) of the walnut above ground can be modeled by $h = -16t^2 + 50$, where t is the time (in seconds) since the crow dropped the walnut. Solve the equation for t. After how many seconds will the walnut be 15 feet above the ground?

50 ft
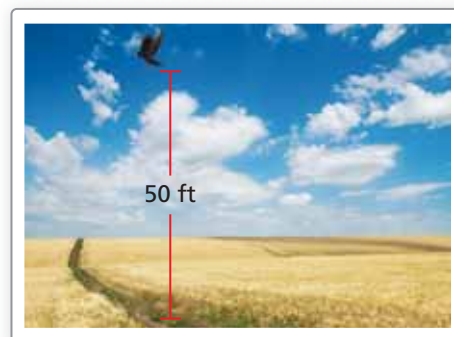
MATHEMATICAL CONNECTIONS In Exercises 47 and 48, s is the side length of an equilateral triangle. Solve the formula for s. Then evaluate the new formula for the given value.
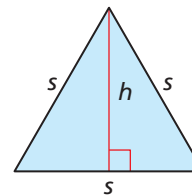
47. Height: $h = \dfrac{\sqrt{3}s}{2}$; $h = 16$ in.

48. Area: $A = \dfrac{\sqrt{3}s^2}{4}$; $A = 11$ ft²

In Exercises 49–54, find the inverse of the function. Then graph the function and its inverse.

49. $f(x) = 2x^3$
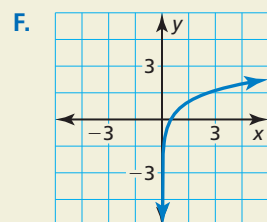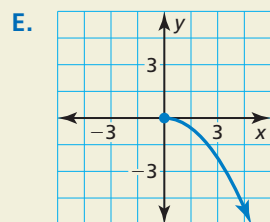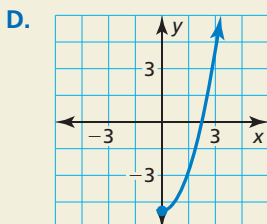
50. $f(x) = x^3 - 4$
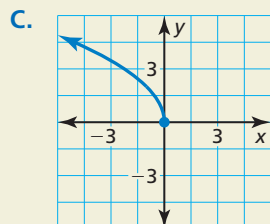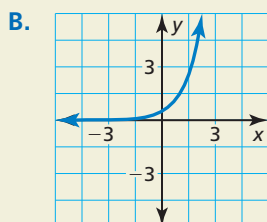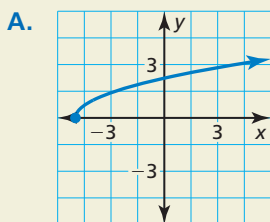
51. $f(x) = (x - 5)^3$

52. $f(x) = 8(x + 2)^3$

53. $f(x) = 4\sqrt[3]{x}$

54. $f(x) = -\sqrt[3]{x - 1}$

55. MAKING AN ARGUMENT Your friend says that the inverse of the function $f(x) = 3$ is a function because all linear functions pass the Horizontal Line Test. Is your friend correct? Explain.

56. HOW DO YOU SEE IT? Pair the graph of each function with the graph of its inverse.

A.

B.

C.

D.

E.

F.

57. WRITING Describe changes you could make to the function $f(x) = x^2 - 5$ so that its inverse is a function. Describe the domain and range of the new function and its inverse.

58. CRITICAL THINKING Can an even function with at least two values in its domain have an inverse that is a function? Explain.

59. OPEN-ENDED Write a function such that the graph of its inverse is a line with a slope of 4.

**60. CRITICAL THINKING** Consider the function $g(x) = -x$.

a. Graph $g(x) = -x$ and explain why it is its own inverse.

b. Graph other linear functions that are their own inverses. Write equations of the lines you graph.

c. Use your results from part (b) to write a general equation that describes the family of linear functions that are their own inverses.

61. REASONING Show that the inverse of any linear function $f(x) = mx + b$, where $m \neq 0$, is also a linear function. Write the slope and y-intercept of the graph of the inverse in terms of m and b.

62. THOUGHT PROVOKING The graphs of $f(x) = x^3 - 3x$ and its inverse are shown. Find the greatest interval $-a \leq x \leq a$ for which the inverse of f is a function. Write an equation of the inverse function.

$f(x) = x^3 - 3x$
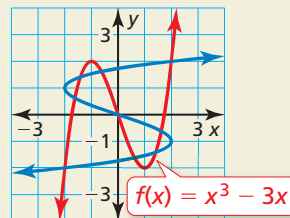
63. REASONING Is the inverse of $f(x) = 2|x + 1|$ a function? Are there any values of a, h, and k for which the inverse of $f(x) = a|x - h| + k$ is a function? Explain your reasoning.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Find the sum or difference. *(Section 7.1)*

64. $(2x - 9) - (6x + 5)$

65. $(8y + 1) + (-y - 12)$

66. $(t^2 - 4t - 4) + (7t^2 + 12t + 3)$

67. $(-3d^2 + 10d - 8) - (7d^2 - d - 6)$

Graph the function. Compare the graph to the graph of $f(x) = x^2$. *(Section 8.2)*

68. $g(x) = x^2 + 6$

69. $h(x) = -x^2 - 2$

70. $p(x) = -4x^2 + 5$

71. $q(x) = \frac{1}{3}x^2 - 1$

Core Vocabulary

radical equation, *p. 560*
inverse relation, *p. 568*
inverse function, *p. 569*

Core Concepts

Section 10.3

Squaring Each Side of an Equation, *p. 560*
Identifying Extraneous Solutions, *p. 562*

Section 10.4

Inverse Relation, *p. 568*
Finding Inverses of Functions Algebraically, *p. 570*
Finding Inverses of Nonlinear Functions, *p. 570*
Horizontal Line Test, *p. 571*

Mathematical Practices

1. Could you also solve Exercises 37–44 on page 565 by graphing? Explain.

2. What external resources could you use to check the reasonableness of your answer in Exercise 45 on page 573?

Performance Task

Medication and the Mosteller Formula

When taking medication, it is critical to take the correct dosage. For children in particular, body surface area (BSA) is a key component in calculating that dosage. The Mosteller Formula is commonly used to approximate body surface area. How will you use this formula to calculate BSA for the optimum dosage?

To explore the answers to this question and more, go to *BigIdeasMath.com*.

10.1 Graphing Square Root Functions *(pp. 543–550)*

a. Describe the domain of $f(x) = 4\sqrt{x + 2}$.

The radicand cannot be negative. So, $x + 2$ is greater than or equal to 0.

$x + 2 \geq 0$ Write an inequality for the domain.

$x \geq -2$ Subtract 2 from each side.

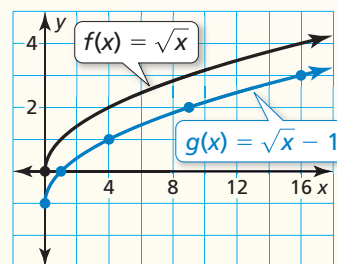▶ The domain is the set of real numbers greater than or equal to -2.

b. Graph $g(x) = \sqrt{x} - 1$. Describe the range. Compare the graph to the graph of $f(x) = \sqrt{x}$.

Step 1 Use the domain of g, $x \geq 0$, to make a table of values.

x	0	1	4	9	16
g(x)	−1	0	1	2	3

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points, starting at $(0, -1)$.

▶ The range of g is $y \geq -1$. The graph of g is a translation 1 unit down of the graph of f.

Graph the function. Describe the domain and range. Compare the graph to the graph of $f(x) = \sqrt{x}$.

1. $g(x) = \sqrt{x} + 7$

2. $h(x) = \sqrt{x - 6}$

3. $r(x) = -\sqrt{x + 3} - 1$

4. Let $g(x) = \frac{1}{4}\sqrt{x - 6} + 2$. Describe the transformations from the graph of $f(x) = \sqrt{x}$ to the graph of g. Then graph g.

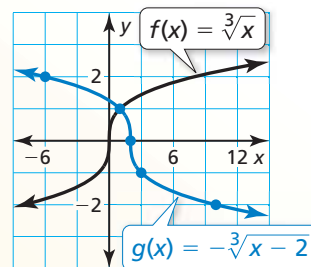## 10.2 Graphing Cube Root Functions *(pp. 551–556)*

Graph $g(x) = -\sqrt[3]{x} - 2$. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.

Step 1 Make a table of values.

x	−6	1	2	3	10
g(x)	2	1	0	−1	−2

Step 2 Plot the ordered pairs.

Step 3 Draw a smooth curve through the points.

▶ The graph of g is a translation 2 units right and a reflection in the x-axis of the graph of f.

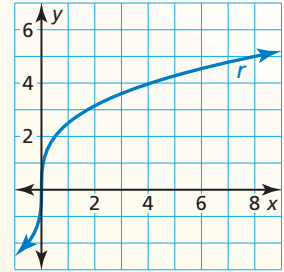**Graph the function. Compare the graph to the graph of $f(x) = \sqrt[3]{x}$.**

5. $g(x) = \sqrt[3]{x} + 4$ **6.** $h(x) = -8\sqrt[3]{x}$ **7.** $s(x) = \sqrt[3]{-2(x - 3)}$

8. Let $g(x) = -3\sqrt[3]{x + 2} - 1$. Describe the transformations from the graph of $f(x) = \sqrt[3]{x}$ to the graph of g. Then graph g.

9. The graph of cube root function r is shown. Compare the average rate of change of r to the average rate of change of $p(x) = \sqrt[3]{\frac{1}{2}x}$ over the interval $x = 0$ to $x = 8$.

10.3   **Solving Radical Equations**   *(pp. 559–566)*

Solve $\sqrt{10 - 3x} = x$.

$\sqrt{10 - 3x} = x$ Write the equation.

$\left(\sqrt{10 - 3x}\right)^2 = x^2$ Square each side of the equation.

$10 - 3x = x^2$ Simplify.

$0 = x^2 + 3x - 10$ Subtract 10 and add $3x$ to each side.

$0 = (x - 2)(x + 5)$ Factor.

$x - 2 = 0$ or $x + 5 = 0$ Zero-Product Property

$x = 2$ or $x = -5$ Solve for x.

> **Check** Check each solution in the original equation.
>
> $\sqrt{10 - 3(2)} \overset{?}{=} 2$ Substitute for x. $\sqrt{10 - 3(-5)} \overset{?}{=} -5$
>
> $\sqrt{4} \overset{?}{=} 2$ Simplify. $\sqrt{25} \overset{?}{=} -5$
>
> $2 = 2$ ✔   Simplify. $5 \neq -5$ ✘

▶   Because $x = -5$ does not satisfy the original equation, it is an extraneous solution. The only solution is $x = 2$.

Solve the equation. Check your solution(s).

10. $8 + \sqrt{x} = 18$ **11.** $\sqrt[3]{x - 1} = 3$ **12.** $\sqrt{5x - 9} = \sqrt{4x}$

13. $x = \sqrt{3x + 4}$ **14.** $8\sqrt{x - 5} + 34 = 58$ **15.** $\sqrt{5x + 6} = 5$

16. The radius r of a cylinder is represented by the function $r = \sqrt{\dfrac{V}{\pi h}}$, where V is the volume and h is the height of the cylinder. What is the volume of the cylindrical can?

2 in.

4 in.

a. Find the inverse of the relation.

Input	−4	−2	0	2	4	6
Output	−3	0	3	6	9	12

Inverse relation: Switch the inputs
                                                          and outputs.

Input	−3	0	3	6	9	12
Output	−4	−2	0	2	4	6

b. Find the inverse of $f(x) = \sqrt{x} - 4$. Then graph the function and its inverse.

$$y = \sqrt{x} - 4 \qquad \text{Set } y \text{ equal to } f(x).$$

$$x = \sqrt{y} - 4 \qquad \text{Switch } x \text{ and } y \text{ in the equation.}$$

$$x^2 = \left(\sqrt{y} - 4\right)^2 \qquad \text{Square each side.}$$
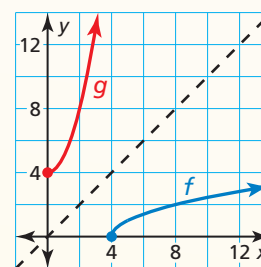
$$x^2 = y - 4 \qquad \text{Simplify.}$$

$$x^2 + 4 = y \qquad \text{Add 4 to each side.}$$

Because the range of f is $y \geq 0$, the domain of the inverse must be restricted to $x \geq 0$.

▶ So, the inverse of f is $g(x) = x^2 + 4$, where $x \geq 0$.

Find the inverse of the relation.

17. $(1, -10), (3, -4), (5, 4), (7, 14), (9, 26)$

18.

Input	−4	−2	0	2	4
Output	6	3	0	−3	−6

Find the inverse of the function. Then graph the function and its inverse.

19. $f(x) = -5x + 10$

20. $f(x) = 3x^2 - 1, \, x \geq 0$

21. $f(x) = \frac{1}{2}\sqrt{2x + 6}$

22. Consider the function $f(x) = x^2 + 4$. Use the Horizontal Line Test to determine whether the inverse of f is a function.

23. In bowling, a handicap is an adjustment to a bowler's score to even out differences in ability levels. In a particular league, you can find a bowler's handicap h by using the formula $h = 0.8(210 - a)$, where a is the bowler's average. Solve the formula for a. Then find a bowler's average when the bowler's handicap is 28.

10 Chapter Test

Find the inverse of the function.

1. $f(x) = 5x - 8$

2. $f(x) = 2\sqrt{x + 3} - 1$

3. $f(x) = -\frac{1}{3}x^2 + 4, x \geq 0$

Graph the function f. Describe the domain and range. Compare the graph of f to the graph of g.

4. $f(x) = -\sqrt{x + 6}; g(x) = \sqrt{x}$

5. $f(x) = \sqrt{x - 3} + 2; g(x) = \sqrt{x}$

6. $f(x) = \sqrt[3]{x} - 5; g(x) = \sqrt[3]{x}$

7. $f(x) = -2\sqrt[3]{x + 1}; g(x) = \sqrt[3]{x}$

Solve the equation. Check your solution(s).

8. $9 - \sqrt{x} = 3$

9. $\sqrt{2x - 7} - 3 = 6$

10. $\sqrt{8x - 21} = \sqrt{18 - 5x}$

11. $x + 5 = \sqrt{7x + 53}$

12. When solving the equation $x - 5 = \sqrt{ax + b}$, you obtain $x = 2$ and $x = 8$. Explain why at least one of these solutions must be extraneous.

Describe the transformations from the graph of $f(x) = \sqrt[3]{x}$ to the graph of the given function. Then graph the given function.

13. $h(x) = 4\sqrt[3]{x - 1} + 5$

14. $w(x) = -\sqrt[3]{x + 7} - 2$

15. The velocity v (in meters per second) of a roller coaster at the bottom of a hill is given by $v = \sqrt{19.6h}$, where h is the height (in meters) of the hill. (a) Use a graphing calculator to graph the function. Describe the domain and range. (b) How tall must the hill be for the velocity of the roller coaster at the bottom of the hill to be at least 28 meters per second? (c) What happens to the average rate of change of the velocity as the height of the hill increases?

16. The speed s (in meters per second) of sound through air is given by $s = 20\sqrt{T + 273}$, where T is the temperature (in degrees Celsius).

   a. What is the temperature when the speed of sound through air is 340 meters per second?

   **b.** How long does it take you to hear the wolf howl when the temperature is $-17°C$?

1760 m you

wolf
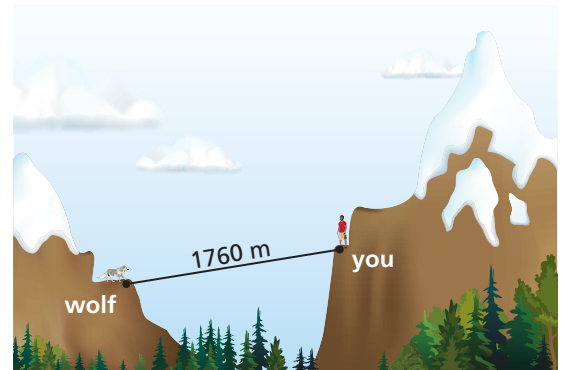
17. How can you restrict the domain of the function $f(x) = (x - 3)^2$ so that the inverse of f is a function?

18. Write a radical function that has a domain of all real numbers less than or equal to 0 and a range of all real numbers greater than or equal to 9.

1. Fill in the function so that it is represented by the graph.

$f(x) = \boxed{} \sqrt{x - \boxed{}} + \boxed{}$
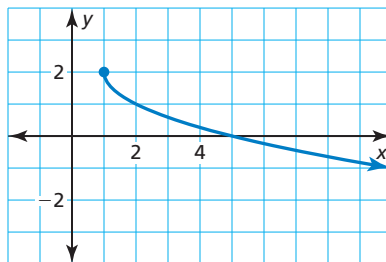
2. Consider the equation $y = mx + b$. Fill in values for m and b so that each statement is true.

   a. When $m = $ _____ and $b = $ _____, the graph of the equation passes through the point $(-1, 4)$.

   b. When $m = $ _____ and $b = $ _____, the graph of the equation has a positive slope and passes through the point $(-2, -5)$.

   c. When $m = $ _____ and $b = $ _____, the graph of the equation is perpendicular to the graph of $y = 4x - 3$ and passes through the point $(1, 6)$.

3. Which graph represents the inverse of the function $f(x) = 2x + 4$?

   Ⓐ

   Ⓑ

   Ⓒ

   Ⓓ
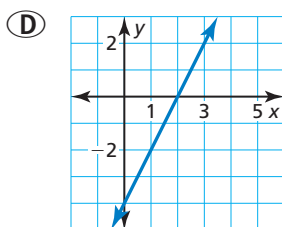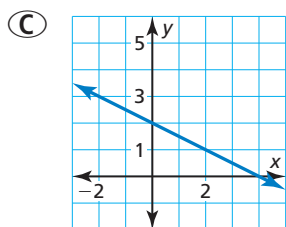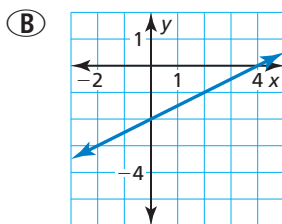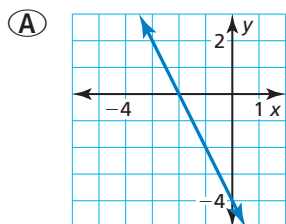
4. Consider the equation $x = \sqrt{ax + b}$. Student A claims this equation has one real solution. Student B claims this equation has two real solutions. Use the numbers to answer parts (a)–(c).

   | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |

   a. Choose values for a and b to create an equation that supports Student A's claim.

   b. Choose values for a and b to create an equation that supports Student B's claim.

   c. Choose values for a and b to create an equation that does not support either student's claim.

5. Which equation represents the nth term of the sequence 3, 12, 48, 192, . . .?

    Ⓐ $a_n = 3(4)^{n-1}$ **Ⓑ** $a_n = 3(9)^{n-1}$

    Ⓒ $a_n = 9n - 6$ **Ⓓ** $a_n = 9n + 3$

6. Consider the function $f(x) = \frac{1}{2}\sqrt[3]{x} + 3$. The graph represents function g. Select all the statements that are true.
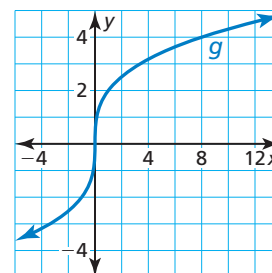
> The x-intercept of the graph of f is greater than the x-intercept of the graph of g.

> The graph of g is always increasing.

> The average rate of change of g decreases as x increases.

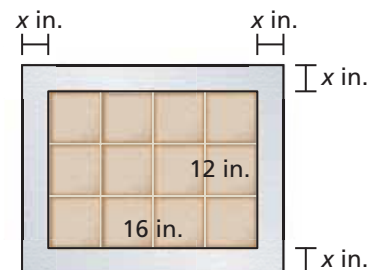> The average rate of change of f increases as x increases.

> The average rate of change of g is greater than the average rate of change of f over the interval $x = 0$ to $x = 8$.

7. Place each function into one of the three categories.

No zeros	One zero	Two zeros
$f(x) = 3x^2 + 4x + 2$ $f(x) = -x^2 + 2x$	$f(x) = 4x^2 - 8x + 4$ $f(x) = x^2 - 3x - 21$	$f(x) = 7x^2$ $f(x) = -6x^2 - 5$

8. You are making a tabletop with a tiled center and a uniform mosaic border.

    a. Write the polynomial in standard form that represents the perimeter of the tabletop.

    b. Write the polynomial in standard form that represents the area of the tabletop.

    c. The perimeter of the tabletop is less than 80 inches, and the area of tabletop is at least 252 square inches. Select all the possible values of x.

> 0.5   1   1.5   2   2.5   3   3.5   4