8 Similarity

Chapter Learning Target:
Understand similarity.

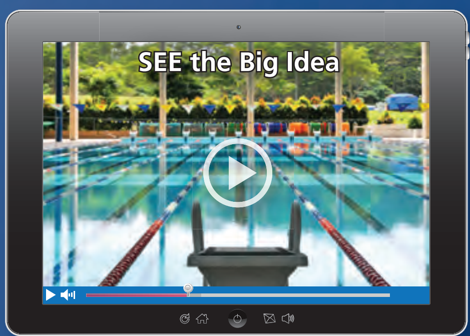
Chapter Success Criteria:
- I can identify corresponding parts of similar polygons.
- I can find and use the scale factor in similar polygons.
- I can prove triangles are similar.
- I can use proportionality theorems to solve problems.

Shuffleboard ***(p. 443)***

Ferris Wheel ***(p. 443)***

Flagpole ***(p. 430)***

Tennis Court ***(p. 425)***



Olympic-Size Swimming Pool ***(p. 420)***

Maintaining Mathematical Proficiency

Determining Whether Ratios Form a Proportion

Example 1 **Tell whether $\frac{2}{8}$ and $\frac{3}{12}$ form a proportion.**

Compare the ratios in simplest form.

$$\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$$

$$\frac{3}{12} = \frac{3 \div 3}{12 \div 3} = \frac{1}{4}$$

The ratios are equivalent.

▶ So, $\frac{2}{8}$ and $\frac{3}{12}$ form a proportion.

Tell whether the ratios form a proportion.

1. $\frac{5}{3}, \frac{35}{21}$

2. $\frac{9}{24}, \frac{24}{64}$

3. $\frac{8}{56}, \frac{6}{28}$

4. $\frac{18}{4}, \frac{27}{9}$

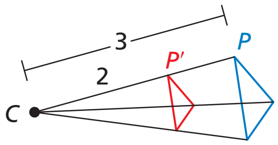
5. $\frac{15}{21}, \frac{55}{77}$

6. $\frac{26}{8}, \frac{39}{12}$

Finding a Scale Factor

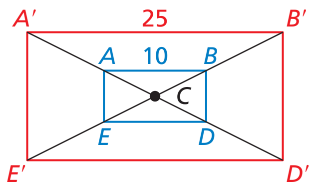
Example 2 **Find the scale factor of each dilation.**

a.



▶ Because $\frac{CP'}{CP} = \frac{2}{3}$, the scale factor is $k = \frac{2}{3}$.

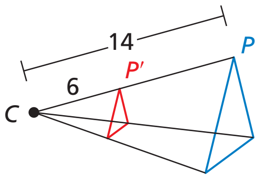
b.



▶ Because $\frac{A'B'}{AB} = \frac{25}{10}$, the scale factor is $k = \frac{25}{10} = \frac{5}{2}$.

Find the scale factor of the dilation.

7.



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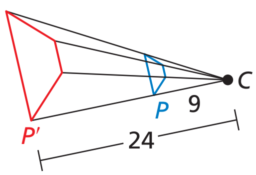

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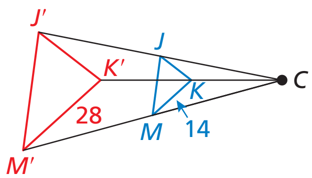


10. **ABSTRACT REASONING** If ratio X and ratio Y form a proportion and ratio Y and ratio Z form a proportion, do ratio X and ratio Z form a proportion? Explain your reasoning.

Mathematical Practices

Mathematically proficient students look for and make use of a pattern or structure.

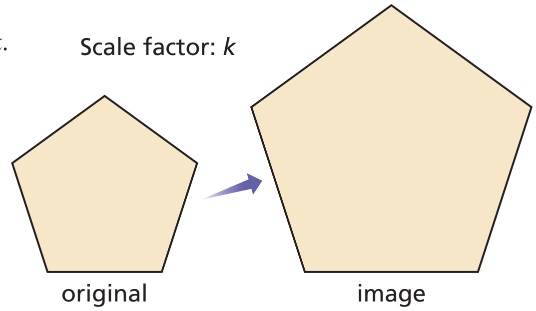
Discerning a Pattern or Structure

Core Concept

Dilations, Perimeter, Area, and Volume

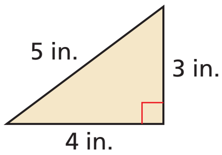
Consider a figure that is dilated by a scale factor of k.

1. The perimeter of the image is k times the perimeter of the original figure.
2. The area of the image is k^2 times the area of the original figure.
3. If the original figure is three dimensional, then the volume of the image is k^3 times the volume of the original figure.



EXAMPLE 1 Finding Perimeter and Area after a Dilation

The triangle shown has side lengths of 3 inches, 4 inches, and 5 inches. Find the perimeter and area of the image when the triangle is dilated by a scale factor of (a) 2, (b) 3, and (c) 4.



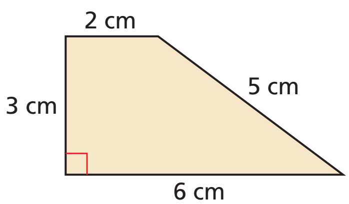
SOLUTION

Perimeter: $P = 5 + 3 + 4 = 12$ in. Area: $A = \frac{1}{2}(4)(3) = 6 \text{ in.}^2$

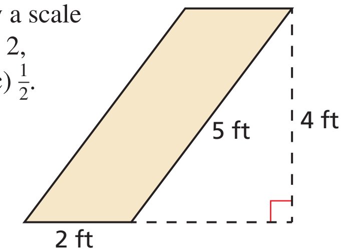
	Scale factor: k	**Perimeter: kP**	**Area: k^2A**
a.	2	$2(12) = 24$ in.	$(2^2)(6) = 24 \text{ in.}^2$
b.	3	$3(12) = 36$ in.	$(3^2)(6) = 54 \text{ in.}^2$
c.	4	$4(12) = 48$ in.	$(4^2)(6) = 96 \text{ in.}^2$

Monitoring Progress

1. Find the perimeter and area of the image when the trapezoid is dilated by a scale factor of (a) 2, (b) 3, and (c) 4.



2. Find the perimeter and area of the image when the parallelogram is dilated by a scale factor of (a) 2, (b) 3, and (c) $\frac{1}{2}$.



3. A rectangular prism is 3 inches wide, 4 inches long, and 5 inches tall. Find the surface area and volume of the image of the prism when it is dilated by a scale factor of (a) 2, (b) 3, and (c) 4.

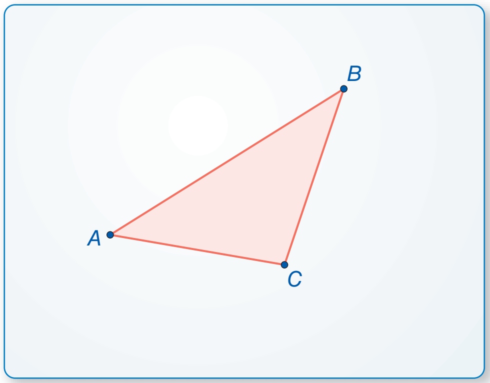
8.1 Similar Polygons

Essential Question

How are similar polygons related?

EXPLORATION 1 Comparing Triangles after a Dilation

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.



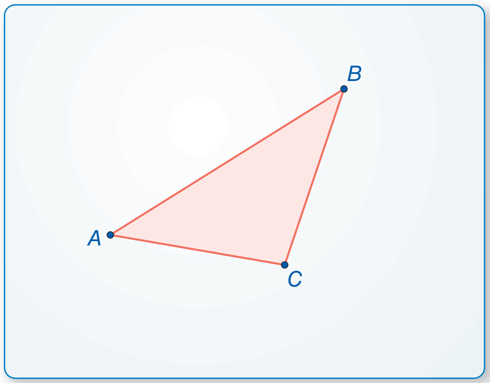
a. Compare the corresponding angles of $\triangle A'B'C'$ and $\triangle ABC$.

b. Find the ratios of the lengths of the sides of $\triangle A'B'C'$ to the lengths of the corresponding sides of $\triangle ABC$. What do you observe?

c. Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

EXPLORATION 2 Comparing Triangles after a Dilation

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$. Dilate $\triangle ABC$ to form a similar $\triangle A'B'C'$ using any scale factor k and any center of dilation.



a. Compare the perimeters of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?

b. Compare the areas of $\triangle A'B'C'$ and $\triangle ABC$. What do you observe?

c. Repeat parts (a) and (b) for several other triangles, scale factors, and centers of dilation. Do you obtain similar results?

LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

3. How are similar polygons related?

4. A $\triangle RST$ is dilated by a scale factor of 3 to form $\triangle R'S'T'$. The area of $\triangle RST$ is 1 square inch. What is the area of $\triangle R'S'T'$?

8.1 Lesson

Core Vocabulary

Previous
similar figures
similarity transformation
corresponding parts

What You Will Learn

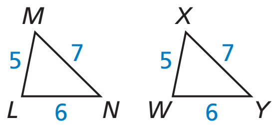
- Use similarity statements.
- Find corresponding lengths in similar polygons.
- Find perimeters and areas of similar polygons.
- Decide whether polygons are similar.

Using Similarity Statements

Recall from Section 4.6 that two geometric figures are similar figures if and only if there is a similarity transformation that maps one figure onto the other.

LOOKING FOR STRUCTURE

Notice that any two congruent figures are also similar. In $\triangle LMN$ and $\triangle WXY$ below, the scale factor is $\frac{5}{5} = \frac{6}{6} = \frac{7}{7} = 1$. So, you can write $\triangle LMN \sim \triangle WXY$ and $\triangle LMN \cong \triangle WXY$.



Core Concept

Corresponding Parts of Similar Polygons

In the diagram below, $\triangle ABC$ is similar to $\triangle DEF$. You can write "$\triangle ABC$ is similar to $\triangle DEF$" as $\triangle ABC \sim \triangle DEF$. A similarity transformation preserves angle measure. So, corresponding angles are congruent. A similarity transformation also enlarges or reduces side lengths by a scale factor k. So, corresponding side lengths are proportional.

Corresponding angles

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Ratios of corresponding side lengths

$$\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA} = k$$

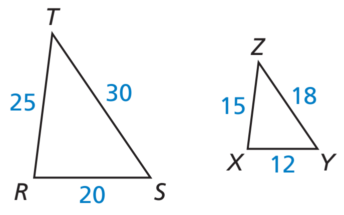
EXAMPLE 1 Using Similarity Statements

In the diagram, $\triangle RST \sim \triangle XYZ$.

a. Find the scale factor from $\triangle RST$ to $\triangle XYZ$.

b. List all pairs of congruent angles.

c. Write the ratios of the corresponding side lengths in a *statement of proportionality.*



READING

In a *statement of proportionality*, any pair of ratios forms a true proportion.

SOLUTION

a. $\frac{XY}{RS} = \frac{12}{20} = \frac{3}{5}$ $\qquad$ $\frac{YZ}{ST} = \frac{18}{30} = \frac{3}{5}$ $\qquad$ $\frac{ZX}{TR} = \frac{15}{25} = \frac{3}{5}$

So, the scale factor is $\frac{3}{5}$.

b. $\angle R \cong \angle X$, $\angle S \cong \angle Y$, and $\angle T \cong \angle Z$.

c. Because the ratios in part (a) are equal, $\frac{XY}{RS} = \frac{YZ}{ST} = \frac{ZX}{TR}$.

Monitoring Progress

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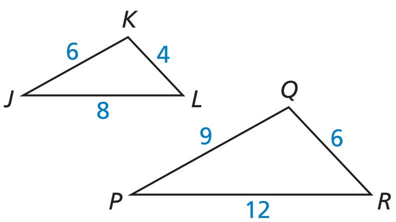
1. In the diagram, $\triangle JKL \sim \triangle PQR$. Find the scale factor from $\triangle JKL$ to $\triangle PQR$. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.



Finding Corresponding Lengths in Similar Polygons

> **READING**
>
> Corresponding lengths in similar triangles include side lengths, altitudes, medians, and midsegments.

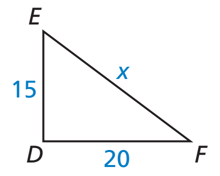
Core Concept

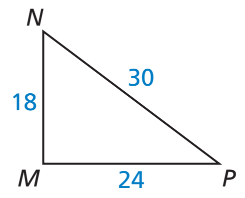
Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

EXAMPLE 2 Finding a Corresponding Length

In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of x.





SOLUTION

The triangles are similar, so the corresponding side lengths are proportional.

$\frac{MN}{DE} = \frac{NP}{EF}$ Write proportion.

$\frac{18}{15} = \frac{30}{x}$ Substitute.

$18x = 450$ Cross Products Property

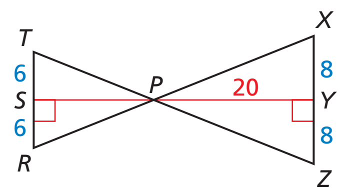
$x = 25$ Solve for x.

▶ The value of x is 25.

> **FINDING AN ENTRY POINT**
>
> There are several ways to write the proportion. For example, you could write $\frac{DF}{MP} = \frac{EF}{NP}$.

EXAMPLE 3 Finding a Corresponding Length

In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.



SOLUTION

First, find the scale factor from $\triangle XPZ$ to $\triangle TPR$.

$$\frac{TR}{XZ} = \frac{6 + 6}{8 + 8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{PS}{PY} = \frac{3}{4}$ Write proportion.

$\frac{PS}{20} = \frac{3}{4}$ Substitute 20 for PY.

$PS = 15$ Multiply each side by 20 and simplify.

▶ The length of the altitude $\overline{PS}$ is 15.

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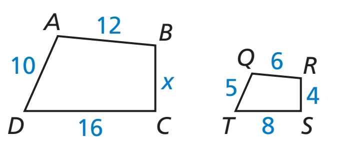
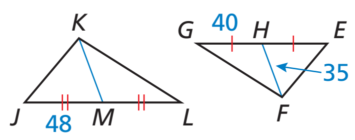
2. Find the value of x.



$ABCD \sim QRST$

3. Find KM.



$\triangle JKL \sim \triangle EFG$

Finding Perimeters and Areas of Similar Polygons

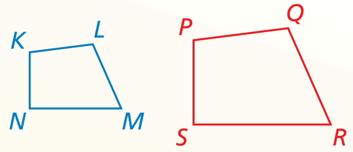
Theorem

ANALYZING RELATIONSHIPS

When two similar polygons have a scale factor of k, the ratio of their perimeters is equal to k.

Theorem 8.1 Perimeters of Similar Polygons

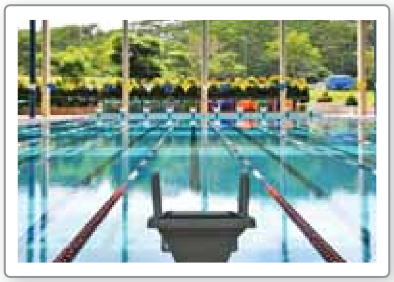
If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.



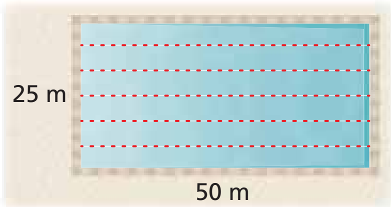
If $KLMN \sim PQRS$, then $\dfrac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \dfrac{PQ}{KL} = \dfrac{QR}{LM} = \dfrac{RS}{MN} = \dfrac{SP}{NK}$.

Proof Ex. 52, p. 426; *BigIdeasMath.com*

EXAMPLE 4 Modeling with Mathematics



A town plans to build a new swimming pool. An Olympic pool is rectangular with a length of 50 meters and a width of 25 meters. The new pool will be similar in shape to an Olympic pool but will have a length of 40 meters. Find the perimeters of an Olympic pool and the new pool.



SOLUTION

1. **Understand the Problem** You are given the length and width of a rectangle and the length of a similar rectangle. You need to find the perimeters of both rectangles.
2. **Make a Plan** Find the scale factor of the similar rectangles and find the perimeter of an Olympic pool. Then use the Perimeters of Similar Polygons Theorem to write and solve a proportion to find the perimeter of the new pool.
3. **Solve the Problem** Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$. The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. Write and solve a proportion to find the perimeter x of the new pool.

$$\frac{x}{150} = \frac{4}{5} \qquad \text{Perimeters of Similar Polygons Theorem}$$

$$x = 120 \qquad \text{Multiply each side by 150 and simplify.}$$

So, the perimeter of an Olympic pool is 150 meters, and the perimeter of the new pool is 120 meters.

4. **Look Back** Check that the ratio of the perimeters is equal to the scale factor.

$$\frac{120}{150} = \frac{4}{5} \checkmark$$

STUDY TIP

You can also write the scale factor as a decimal. In Example 4, you can write the scale factor as 0.8 and multiply by 150 to get $x = 0.8(150) = 120$.

Monitoring Progress

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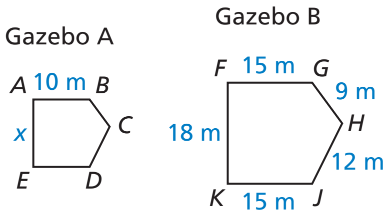
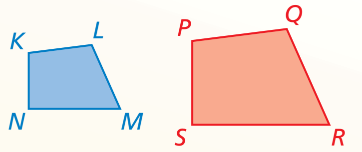
4. The two gazebos shown are similar pentagons. Find the perimeter of Gazebo A.



Theorem

Theorem 8.2 Areas of Similar Polygons

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.



If $KLMN \sim PQRS$, then $\dfrac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\dfrac{PQ}{KL}\right)^2 = \left(\dfrac{QR}{LM}\right)^2 = \left(\dfrac{RS}{MN}\right)^2 = \left(\dfrac{SP}{NK}\right)^2.$

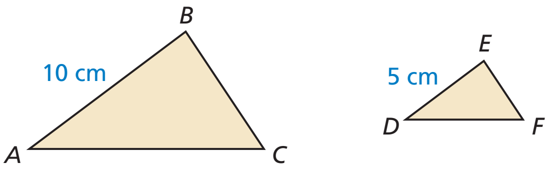
Proof Ex. 53, p. 426; *BigIdeasMath.com*

ANALYZING RELATIONSHIPS

When two similar polygons have a scale factor of k, the ratio of their areas is equal to k^2.

EXAMPLE 5 Finding Areas of Similar Polygons

In the diagram, $\triangle ABC \sim \triangle DEF$. Find the area of $\triangle DEF$.



Area of $\triangle ABC = 36$ cm^2

SOLUTION

Because the triangles are similar, the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$ is equal to the square of the ratio of AB to DE. Write and solve a proportion to find the area of $\triangle DEF$. Let A represent the area of $\triangle DEF$.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2 \qquad \text{Areas of Similar Polygons Theorem}$$

$$\frac{36}{A} = \left(\frac{10}{5}\right)^2 \qquad \text{Substitute.}$$

$$\frac{36}{A} = \frac{100}{25} \qquad \text{Square the right side of the equation.}$$

$$36 \cdot 25 = 100 \cdot A \qquad \text{Cross Products Property}$$

$$900 = 100A \qquad \text{Simplify.}$$

$$9 = A \qquad \text{Solve for } A.$$

The area of $\triangle DEF$ is 9 square centimeters.

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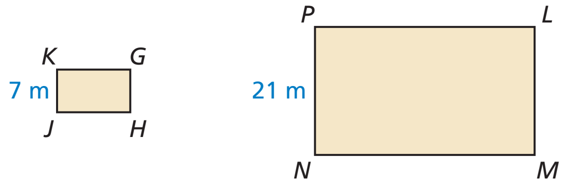
5. In the diagram, $GHJK \sim LMNP$. Find the area of $LMNP$.

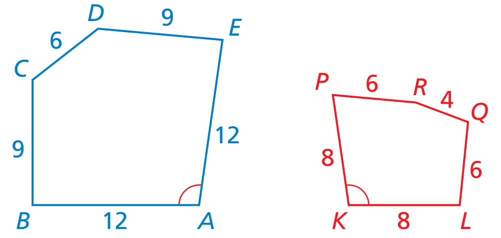


Area of $GHJK = 84$ m^2

Deciding Whether Polygons Are Similar

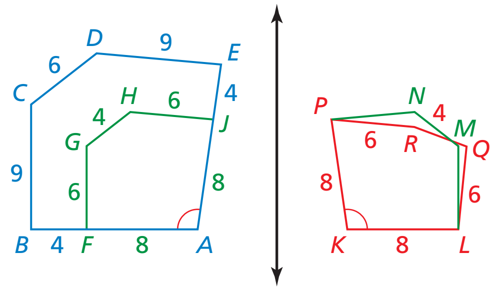
EXAMPLE 6 Deciding Whether Polygons Are Similar

Decide whether *ABCDE* and *KLQRP* are similar. Explain your reasoning.



SOLUTION

Corresponding sides of the pentagons are proportional with a scale factor of $\frac{2}{3}$. However, this does not necessarily mean the pentagons are similar. A dilation with center *A* and scale factor $\frac{2}{3}$ moves *ABCDE* onto *AFGHJ*. Then a reflection moves *AFGHJ* onto *KLMNP*.



KLMNP does not exactly coincide with *KLQRP*, because not all the corresponding angles are congruent. (Only $\angle A$ and $\angle K$ are congruent.)

▶ Because angle measure is not preserved, the two pentagons are not similar.

Monitoring Progress

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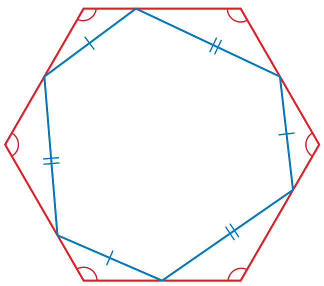
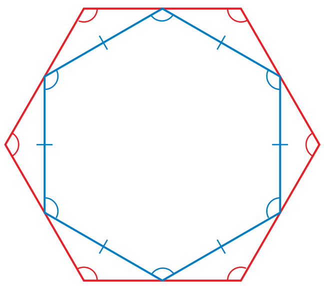
Refer to the floor tile designs below. In each design, the red shape is a regular hexagon.



Tile Design 1



Tile Design 2

6. Decide whether the hexagons in Tile Design 1 are similar. Explain.

7. Decide whether the hexagons in Tile Design 2 are similar. Explain.

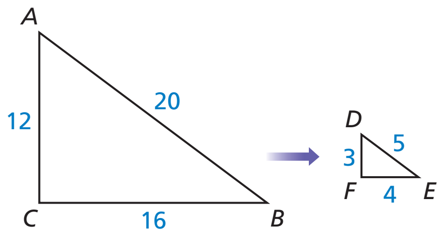
8.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. COMPLETE THE SENTENCE For two figures to be similar, the corresponding angles must be ____________, and the corresponding side lengths must be ________________.

2. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.



$\triangle ABC \sim \triangle DEF$

What is the scale factor?

What is the ratio of their areas?

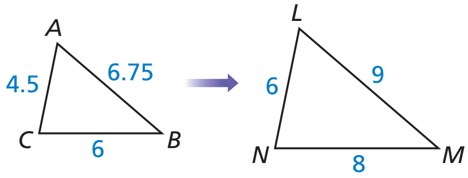
What is the ratio of their corresponding side lengths?

What is the ratio of their perimeters?

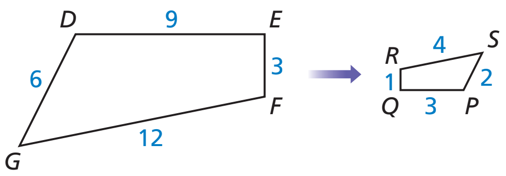
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, find the scale factor. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality. *(See Example 1.)*

3. $\triangle ABC \sim \triangle LMN$

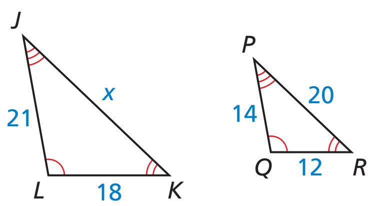


4. $DEFG \sim PQRS$

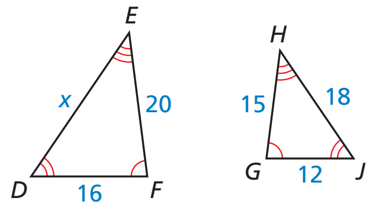


In Exercises 5–8, the polygons are similar. Find the value of x. *(See Example 2.)*

5.



6.



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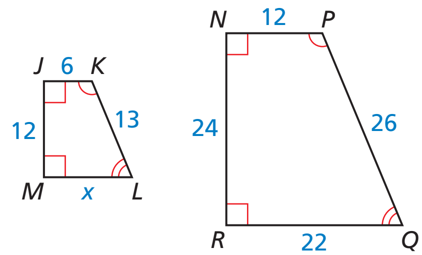

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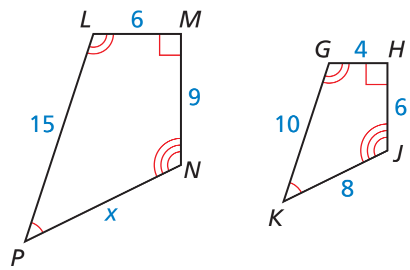
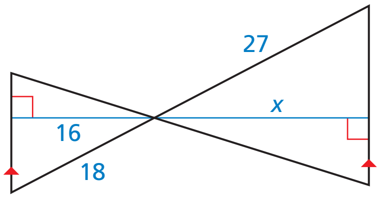
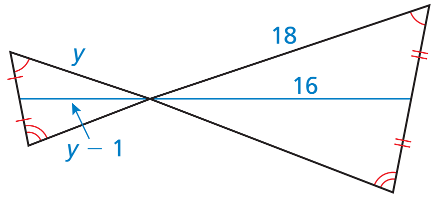


In Exercises 9 and 10, the black triangles are similar. Identify the type of segment shown in blue and find the value of the variable. *(See Example 3.)*

9.

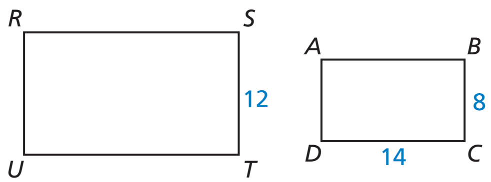


10.



In Exercises 11 and 12, *RSTU* ~ *ABCD*. Find the ratio of their perimeters.

11.



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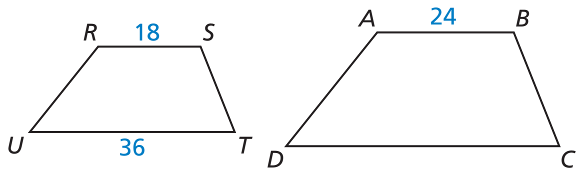


In Exercises 13–16, two polygons are similar. The perimeter of one polygon and the ratio of the corresponding side lengths are given. Find the perimeter of the other polygon.

13. perimeter of smaller polygon: 48 cm; ratio: $\frac{2}{3}$

14. perimeter of smaller polygon: 66 ft; ratio: $\frac{3}{4}$

15. perimeter of larger polygon: 120 yd; ratio: $\frac{1}{6}$

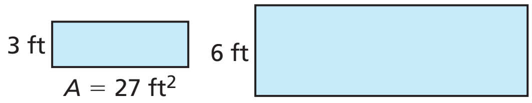
16. perimeter of larger polygon: 85 m; ratio: $\frac{2}{5}$

17. MODELING WITH MATHEMATICS A school gymnasium is being remodeled. The basketball court will be similar to an NCAA basketball court, which has a length of 94 feet and a width of 50 feet. The school plans to make the width of the new court 45 feet. Find the perimeters of an NCAA court and of the new court in the school. *(See Example 4.)*

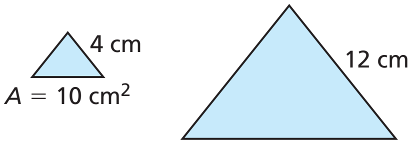
18. MODELING WITH MATHEMATICS Your family has decided to put a rectangular patio in your backyard, similar to the shape of your backyard. Your backyard has a length of 45 feet and a width of 20 feet. The length of your new patio is 18 feet. Find the perimeters of your backyard and of the patio.

In Exercises 19–22, the polygons are similar. The area of one polygon is given. Find the area of the other polygon. *(See Example 5.)*

19.



20.



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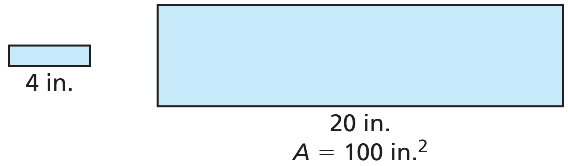
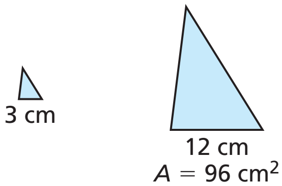
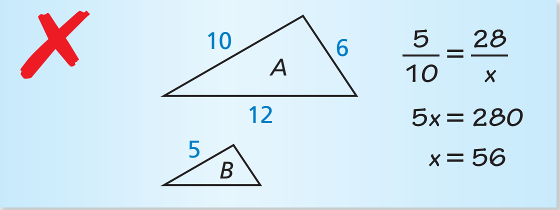


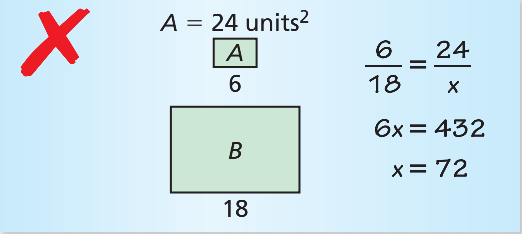
22.



23. ERROR ANALYSIS Describe and correct the error in finding the perimeter of triangle B. The triangles are similar.

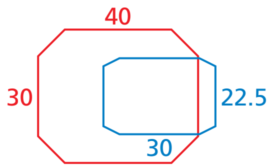


24. ERROR ANALYSIS Describe and correct the error in finding the area of rectangle B. The rectangles are similar.

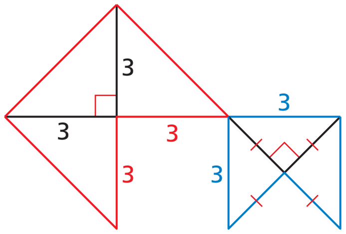


In Exercises 25 and 26, decide whether the red and blue polygons are similar. *(See Example 6.)*

25.



26.

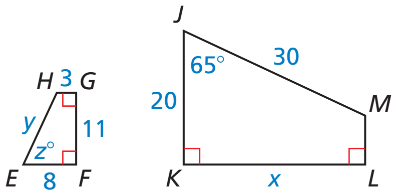


27. REASONING Triangles ABC and DEF are similar. Which statement is correct? Select all that apply.

Ⓐ $\frac{BC}{EF} = \frac{AC}{DF}$ Ⓑ $\frac{AB}{DE} = \frac{CA}{FE}$

Ⓒ $\frac{AB}{EF} = \frac{BC}{DE}$ Ⓓ $\frac{CA}{FD} = \frac{BC}{EF}$

ANALYZING RELATIONSHIPS In Exercises 28–34, *JKLM* ~ *EFGH*.



28. Find the scale factor of $JKLM$ to $EFGH$.

29. Find the scale factor of $EFGH$ to $JKLM$.

30. Find the values of x, y, and z.

31. Find the perimeter of each polygon.

32. Find the ratio of the perimeters of $JKLM$ to $EFGH$.

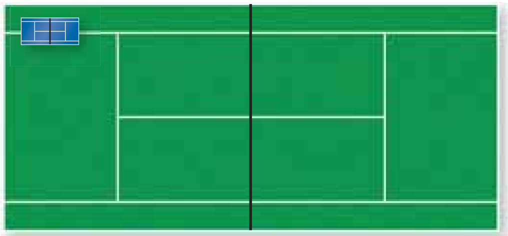
33. Find the area of each polygon.

34. Find the ratio of the areas of $JKLM$ to $EFGH$.

35. USING STRUCTURE Rectangle A is similar to rectangle B. Rectangle A has side lengths of 6 and 12. Rectangle B has a side length of 18. What are the possible values for the length of the other side of rectangle B? Select all that apply.

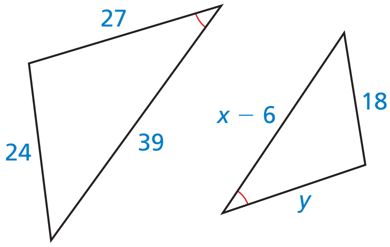
Ⓐ 6 Ⓑ 9 Ⓒ 24 Ⓓ 36

36. DRAWING CONCLUSIONS In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? Explain. If so, find the scale factor of the tennis court to the table.

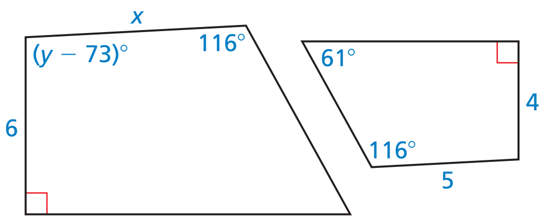


MATHEMATICAL CONNECTIONS In Exercises 37 and 38, the two polygons are similar. Find the values of *x* and *y*.

37.



38.



ATTENDING TO PRECISION In Exercises 39–42, the figures are similar. Find the missing corresponding side length.

39. Figure A has a perimeter of 72 meters and one of the side lengths is 18 meters. Figure B has a perimeter of 120 meters.

40. Figure A has a perimeter of 24 inches. Figure B has a perimeter of 36 inches and one of the side lengths is 12 inches.

41. Figure A has an area of 48 square feet and one of the side lengths is 6 feet. Figure B has an area of 75 square feet.

42. Figure A has an area of 18 square feet. Figure B has an area of 98 square feet and one of the side lengths is 14 feet.

CRITICAL THINKING In Exercises 43–48, tell whether the polygons are *always, sometimes,* or *never* similar.

43. two isosceles triangles

44. two isosceles trapezoids

45. two rhombuses

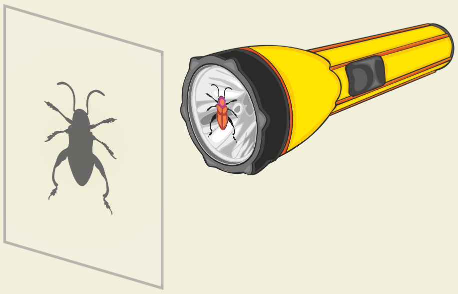
46. two squares

47. two regular polygons

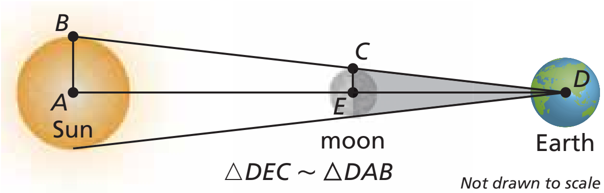
48. a right triangle and an equilateral triangle

49. MAKING AN ARGUMENT Your sister claims that when the side lengths of two rectangles are proportional, the two rectangles must be similar. Is she correct? Explain your reasoning.

50. HOW DO YOU SEE IT? You shine a flashlight directly on an object to project its image onto a parallel screen. Will the object and the image be similar? Explain your reasoning.



51. MODELING WITH MATHEMATICS During a total eclipse of the Sun, the moon is directly in line with the Sun and blocks the Sun's rays. The distance DA between Earth and the Sun is 93,000,000 miles, the distance DE between Earth and the moon is 240,000 miles, and the radius AB of the Sun is 432,500 miles. Use the diagram and the given measurements to estimate the radius EC of the moon.



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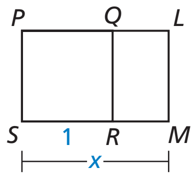
52. PROVING A THEOREM Prove the Perimeters of Similar Polygons Theorem (Theorem 8.1) for similar rectangles. Include a diagram in your proof.

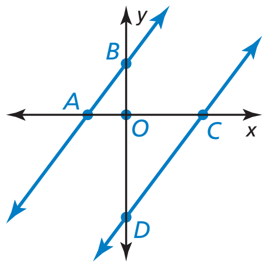
53. PROVING A THEOREM Prove the Areas of Similar Polygons Theorem (Theorem 8.2) for similar rectangles. Include a diagram in your proof.

54. THOUGHT PROVOKING The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. A plane is the surface of the sphere. In spherical geometry, is it possible that two triangles are similar but not congruent? Explain your reasoning.

55. CRITICAL THINKING In the diagram, $PQRS$ is a square, and $PLMS \sim LMRQ$. Find the exact value of x. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.



56. MATHEMATICAL CONNECTIONS The equations of the lines shown are $y = \frac{4}{3}x + 4$ and $y = \frac{4}{3}x - 8$. Show that $\triangle AOB \sim \triangle COD$.

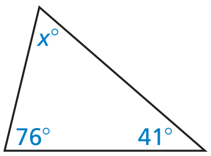


Maintaining Mathematical Proficiency

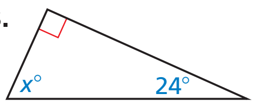
Reviewing what you learned in previous grades and lessons

Find the value of x. *(Section 5.1)*

57.



58.



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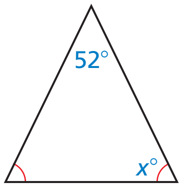
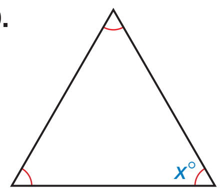


60.



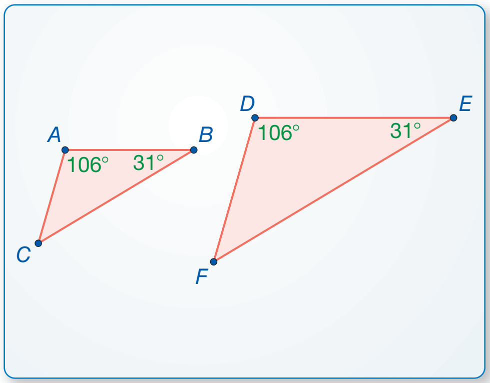
8.2 Proving Triangle Similarity by AA

Essential Question What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?

EXPLORATION 1 Comparing Triangles

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$ and $\triangle DEF$ so that $m\angle A = m\angle D = 106°$, $m\angle B = m\angle E = 31°$, and $\triangle DEF$ is not congruent to $\triangle ABC$.



b. Find the third angle measure and the side lengths of each triangle. Copy the table below and record your results in column 1.

	1.	**2.**	**3.**	**4.**	**5.**	**6.**
$m\angle A$, $m\angle D$	106°	88°	40°			
$m\angle B$, $m\angle E$	31°	42°	65°			
$m\angle C$						
$m\angle F$						
AB						
DE						
BC						
EF						
AC						
DF						

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results in constructing arguments.

c. Are the two triangles similar? Explain.

d. Repeat parts (a)–(c) to complete columns 2 and 3 of the table for the given angle measures.

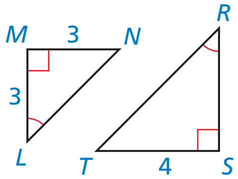
e. Complete each remaining column of the table using your own choice of two pairs of equal corresponding angle measures. Can you construct two triangles in this way that are *not* similar?

f. Make a conjecture about any two triangles with two pairs of congruent corresponding angles.

Communicate Your Answer

2. What can you conclude about two triangles when you know that two pairs of corresponding angles are congruent?

3. Find RS in the figure at the left.



8.2 Lesson

What You Will Learn

- Use the Angle-Angle Similarity Theorem.
- Solve real-life problems.

Core Vocabulary

Previous
similar figures
similarity transformation

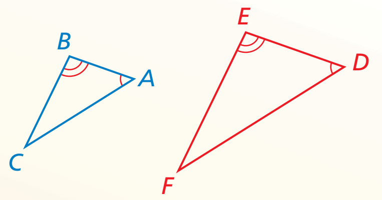
Using the Angle-Angle Similarity Theorem

Theorem

Theorem 8.3 Angle-Angle (AA) Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

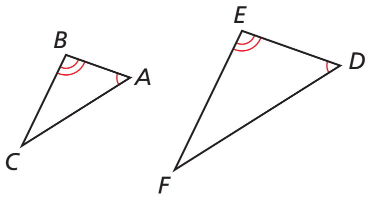


Proof p. 428

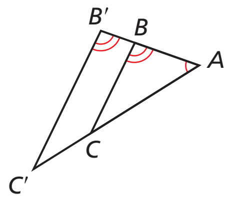
PROOF Angle-Angle (AA) Similarity Theorem

Given $\angle A \cong \angle D$, $\angle B \cong \angle E$

Prove $\triangle ABC \sim \triangle DEF$



Dilate $\triangle ABC$ using a scale factor of $k = \dfrac{DE}{AB}$ and center A. The image of $\triangle ABC$ is $\triangle AB'C'$.



Because a dilation is a similarity transformation, $\triangle ABC \sim \triangle AB'C'$. Because the ratio of corresponding lengths of similar polygons equals the scale factor, $\dfrac{AB'}{AB} = \dfrac{DE}{AB}$. Multiplying each side by AB yields $AB' = DE$. By the definition of congruent segments, $\overline{AB'} \cong \overline{DE}$.

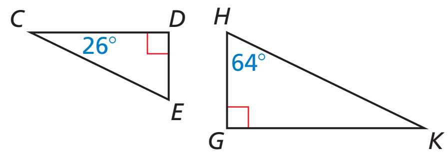
By the Reflexive Property of Congruence (Theorem 2.2), $\angle A \cong \angle A$. Because corresponding angles of similar polygons are congruent, $\angle B' \cong \angle B$. Because $\angle B' \cong \angle B$ and $\angle B \cong \angle E$, $\angle B' \cong \angle E$ by the Transitive Property of Congruence (Theorem 2.2).

Because $\angle A \cong \angle D$, $\angle B' \cong \angle E$, and $\overline{AB'} \cong \overline{DE}$, $\triangle AB'C' \cong \triangle DEF$ by the ASA Congruence Theorem (Theorem 5.10). So, a composition of rigid motions maps $\triangle AB'C'$ to $\triangle DEF$.

Because a dilation followed by a composition of rigid motions maps $\triangle ABC$ to $\triangle DEF$, $\triangle ABC \sim \triangle DEF$.

EXAMPLE 1 Using the AA Similarity Theorem

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.



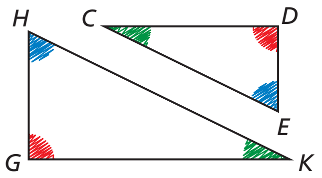
SOLUTION

Because they are both right angles, $\angle D$ and $\angle G$ are congruent.

By the Triangle Sum Theorem (Theorem 5.1), $26° + 90° + m\angle E = 180°$, so $m\angle E = 64°$. So, $\angle E$ and $\angle H$ are congruent.

▶ So, $\triangle CDE \sim \triangle KGH$ by the AA Similarity Theorem.

VISUAL REASONING

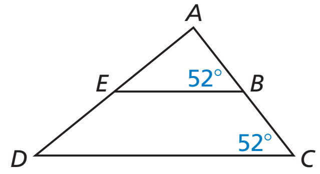


Use colored pencils to show congruent angles. This will help you write similarity statements.

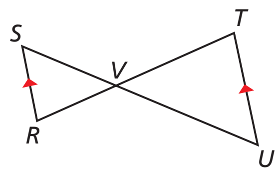
EXAMPLE 2 Using the AA Similarity Theorem

Show that the two triangles are similar.

a. $\triangle ABE \sim \triangle ACD$



b. $\triangle SVR \sim \triangle UVT$

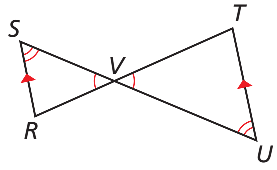


SOLUTION

a. Because $m\angle ABE$ and $m\angle C$ both equal 52°, $\angle ABE \cong \angle C$. By the Reflexive Property of Congruence (Theorem 2.2), $\angle A \cong \angle A$.

▶ So, $\triangle ABE \sim \triangle ACD$ by the AA Similarity Theorem.

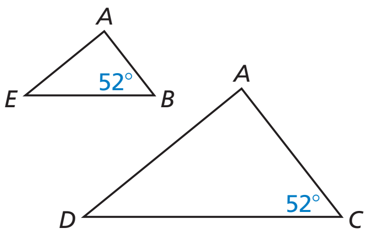
b. You know $\angle SVR \cong \angle UVT$ by the Vertical Angles Congruence Theorem (Theorem 2.6). The diagram shows $\overline{RS} \parallel \overline{UT}$, so $\angle S \cong \angle U$ by the Alternate Interior Angles Theorem (Theorem 3.2).



▶ So, $\triangle SVR \sim \triangle UVT$ by the AA Similarity Theorem.

VISUAL REASONING

You may find it helpful to redraw the triangles separately.



Monitoring Progress

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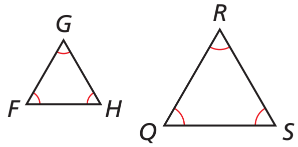
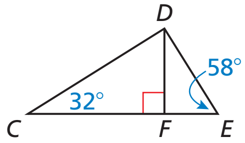
Show that the triangles are similar. Write a similarity statement.

1. $\triangle FGH$ and $\triangle RQS$



2. $\triangle CDF$ and $\triangle DEF$



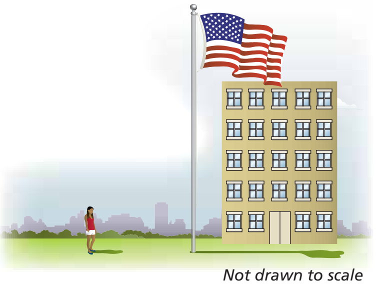
3. **WHAT IF?** Suppose that $\overline{SR} \nparallel \overline{TU}$ in Example 2 part (b). Could the triangles still be similar? Explain.

Solving Real-Life Problems

Previously, you learned a way to use congruent triangles to find measurements indirectly. Another useful way to find measurements indirectly is by using similar triangles.

EXAMPLE 3 **Modeling with Mathematics**

A flagpole casts a shadow that is 50 feet long. At the same time, a woman standing nearby who is 5 feet 4 inches tall casts a shadow that is 40 inches long. How tall is the flagpole to the nearest foot?



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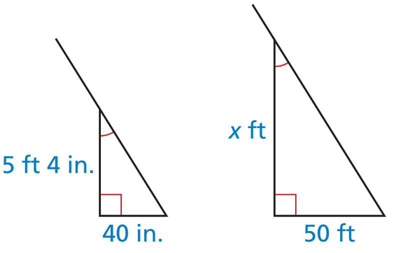
SOLUTION

1. **Understand the Problem** You are given the length of a flagpole's shadow, the height of a woman, and the length of the woman's shadow. You need to find the height of the flagpole.

2. **Make a Plan** Use similar triangles to write a proportion and solve for the height of the flagpole.

3. **Solve the Problem** The flagpole and the woman form sides of two right triangles with the ground. The Sun's rays hit the flagpole and the woman at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Theorem.



You can use a proportion to find the height x. Write 5 feet 4 inches as 64 inches so that you can form two ratios of feet to inches.

$$\frac{x \text{ ft}}{64 \text{ in.}} = \frac{50 \text{ ft}}{40 \text{ in.}}$$ Write proportion of side lengths.

$$40x = 3200$$ Cross Products Property

$$x = 80$$ Solve for x.

▶ The flagpole is 80 feet tall.

4. **Look Back** Attend to precision by checking that your answer has the correct units. The problem asks for the height of the flagpole to the nearest *foot*. Because your answer is 80 feet, the units match.

   Also, check that your answer is reasonable in the context of the problem. A height of 80 feet makes sense for a flagpole. You can estimate that an eight-story building would be about 8(10 feet) = 80 feet, so it is reasonable that a flagpole could be that tall.

Monitoring Progress

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4. **WHAT IF?** A child who is 58 inches tall is standing next to the woman in Example 3. How long is the child's shadow?

5. You are standing outside, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree.

8.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

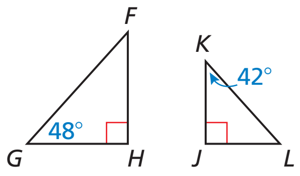
1. **COMPLETE THE SENTENCE** If two angles of one triangle are congruent to two angles of another triangle, then the triangles are ________.

2. **WRITING** Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent? Explain.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning. *(See Example 1.)*

3.



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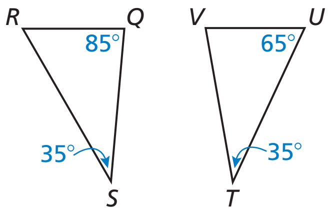

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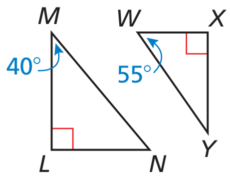

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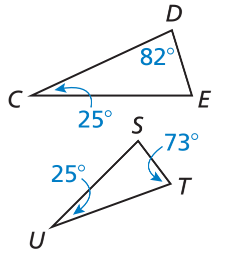
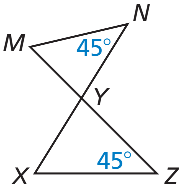


In Exercises 7–10, show that the two triangles are similar. *(See Example 2.)*

7.



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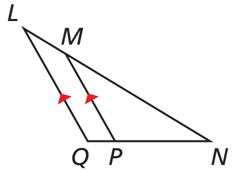

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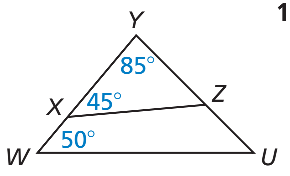

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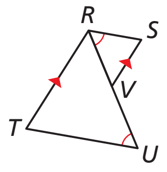
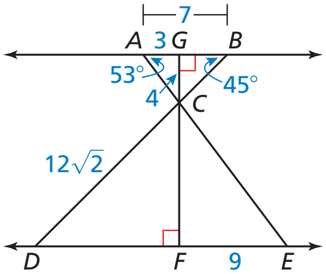


In Exercises 11–18, use the diagram to copy and complete the statement.



11. $\triangle CAG \sim$ ____

12. $\triangle DCF \sim$ ____

13. $\triangle ACB \sim$ ____

14. $m\angle ECF =$ ____

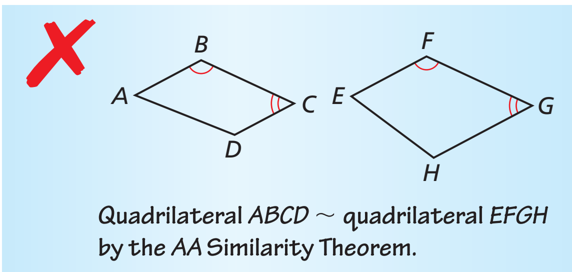
15. $m\angle ECD =$ ____

16. $CF =$ ____

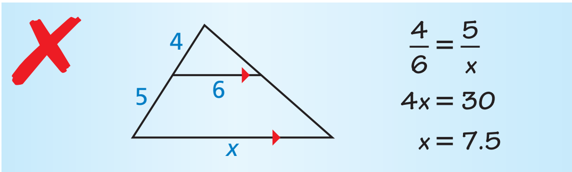
17. $BC =$ ____

18. $DE =$ ____

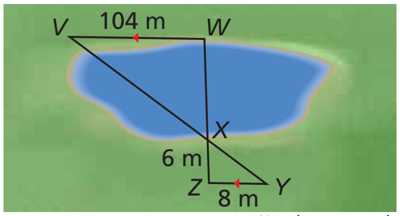
19. **ERROR ANALYSIS** Describe and correct the error in using the AA Similarity Theorem (Theorem 8.3).



20. **ERROR ANALYSIS** Describe and correct the error in finding the value of x.



21. MODELING WITH MATHEMATICS You can measure the width of the lake using a surveying technique, as shown in the diagram. Find the width of the lake, WX. Justify your answer.



Not drawn to scale

22. MAKING AN ARGUMENT You and your cousin are trying to determine the height of a telephone pole. Your cousin tells you to stand in the pole's shadow so that the tip of your shadow coincides with the tip of the pole's shadow. Your cousin claims to be able to use the distance between the tips of the shadows and you, the distance between you and the pole, and your height to estimate the height of the telephone pole. Is this possible? Explain. Include a diagram in your answer.

REASONING In Exercises 23–26, is it possible for $\triangle JKL$ and $\triangle XYZ$ to be similar? Explain your reasoning.

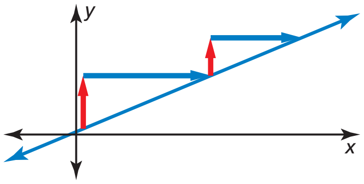
23. $m\angle J = 71°$, $m\angle K = 52°$, $m\angle X = 71°$, and $m\angle Z = 57°$

24. $\triangle JKL$ is a right triangle and $m\angle X + m\angle Y = 150°$.

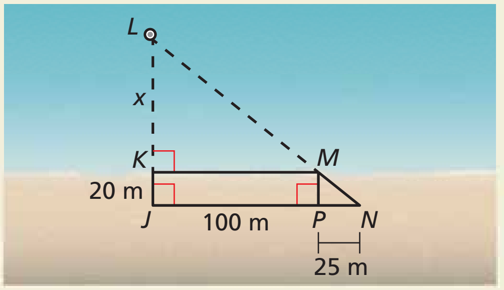
25. $m\angle L = 87°$ and $m\angle Y = 94°$

26. $m\angle J + m\angle K = 85°$ and $m\angle Y + m\angle Z = 80°$

27. MATHEMATICAL CONNECTIONS Explain how you can use similar triangles to show that any two points on a line can be used to find its slope.



28. HOW DO YOU SEE IT? In the diagram, which triangles would you use to find the distance x between the shoreline and the buoy? Explain your reasoning.



29. WRITING Explain why all equilateral triangles are similar.

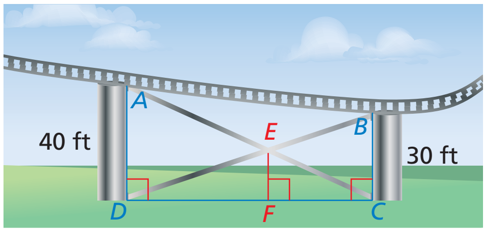
30. THOUGHT PROVOKING Decide whether each is a valid method of showing that two quadrilaterals are similar. Justify your answer.

a. AAA **b.** AAAA

31. PROOF Without using corresponding lengths in similar polygons, prove that the ratio of two corresponding angle bisectors in similar triangles is equal to the scale factor.

32. PROOF Prove that if the lengths of two sides of a triangle are a and b, respectively, then the lengths of the corresponding altitudes to those sides are in the ratio $\frac{b}{a}$.

33. MODELING WITH MATHEMATICS A portion of an amusement park ride is shown. Find EF. Justify your answer.

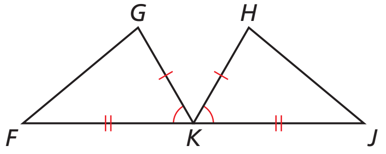


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Determine whether there is enough information to prove that the triangles are congruent. Explain your reasoning. *(Section 5.3, Section 5.5, and Section 5.6)*

34.



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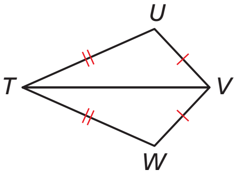

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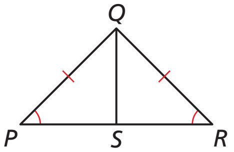


8.1–8.2 What Did You Learn?

Core Concepts

Section 8.1

Corresponding Parts of Similar Polygons, *p. 418*
Corresponding Lengths in Similar Polygons, *p. 419*
Theorem 8.1 Perimeters of Similar Polygons, *p. 420*
Theorem 8.2 Areas of Similar Polygons, *p. 421*

Section 8.2

Theorem 8.3 Angle-Angle (AA) Similarity Theorem, *p. 428*

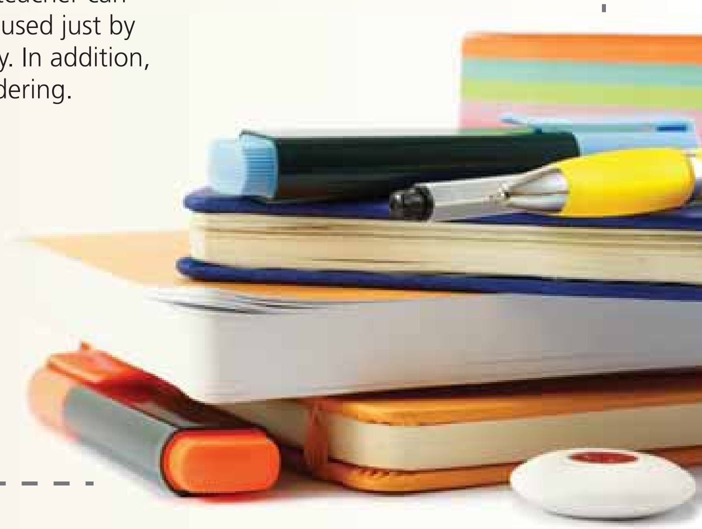
Mathematical Practices

1. In Exercise 35 on page 425, why is there more than one correct answer for the length of the other side?
2. In Exercise 50 on page 426, how could you find the scale factor of the similar figures? Describe any tools that might be helpful.
3. In Exercise 21 on page 432, explain why the surveyor needs V, X, and Y to be collinear and Z, X, and W to be collinear.

Study Skills

Take Control of Your Class Time

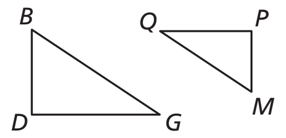
- Sit where you can easily see and hear the teacher, and the teacher can see you. The teacher may be able to tell when you are confused just by the look on your face and may adjust the lesson accordingly. In addition, sitting in this strategic place will keep your mind from wandering.
- Pay attention to what the teacher says about the math, not just what is written on the board. Write problems on the left side of your notes and what the teacher says about the problems on the right side.
- If the teacher is moving through the material too fast, ask a question. Questions help slow the pace for a few minutes and also clarify what is confusing to you.
- Try to memorize new information while learning it. Repeat in your head what you are writing in your notes. That way you are reviewing the information twice.



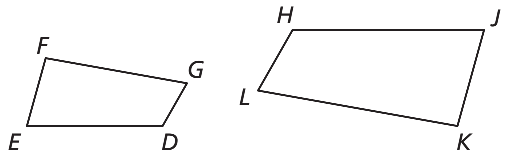
8.1–8.2 Quiz

List all pairs of congruent angles. Then write the ratios of the corresponding side lengths in a statement of proportionality. *(Section 8.1)*

1. $\triangle BDG \sim \triangle MPQ$

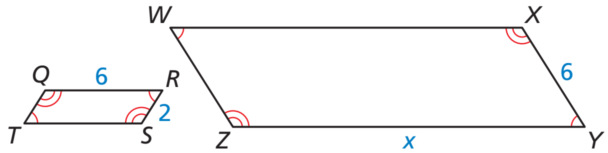


2. $DEFG \sim HJKL$



The polygons are similar. Find the value of *x*. *(Section 8.1)*

3.



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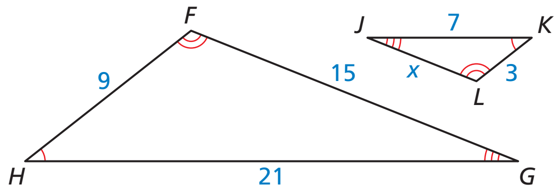
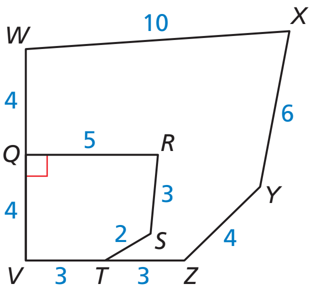


Determine whether the polygons are similar. If they are, write a similarity statement. Explain your reasoning. *(Section 8.1 and Section 8.2)*

5.



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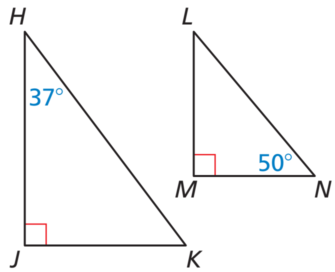

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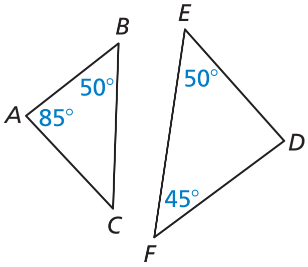
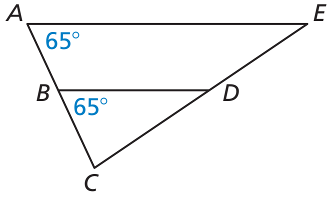


Show that the two triangles are similar. *(Section 8.2)*

8.



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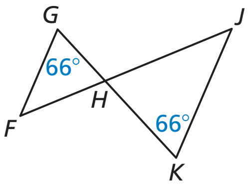
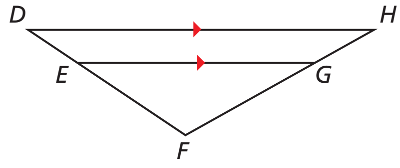


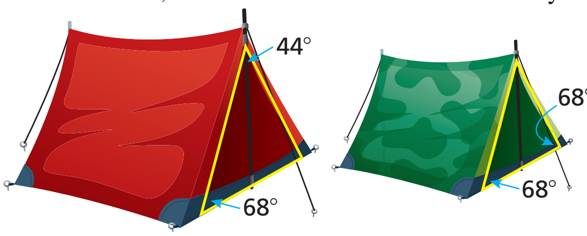
10.



11. The dimensions of an official hockey rink used by the National Hockey League (NHL) are 200 feet by 85 feet. The dimensions of an air hockey table are 96 inches by 40.8 inches. Assume corresponding angles are congruent. *(Section 8.1)*

a. Determine whether the two surfaces are similar.

b. If the surfaces are similar, find the ratio of their perimeters and the ratio of their areas. If not, find the dimensions of an air hockey table that are similar to an NHL hockey rink.



12. You and a friend buy camping tents made by the same company but in different sizes and colors. Use the information given in the diagram to decide whether the triangular faces of the tents are similar. Explain your reasoning. *(Section 8.2)*

8.3 Proving Triangle Similarity by SSS and SAS

Essential Question What are two ways to use corresponding sides of two triangles to determine that the triangles are similar?

EXPLORATION 1 Deciding Whether Triangles Are Similar

Work with a partner. Use dynamic geometry software.

a. Construct $\triangle ABC$ and $\triangle DEF$ with the side lengths given in column 1 of the table below.

	1.	2.	3.	4.	5.	6.	7.
AB	5	5	6	15	9	24	
BC	8	8	8	20	12	18	
AC	10	10	10	10	8	16	
DE	10	15	9	12	12	8	
EF	16	24	12	16	15	6	
DF	20	30	15	8	10	8	
$m\angle A$							
$m\angle B$							
$m\angle C$							
$m\angle D$							
$m\angle E$							
$m\angle F$							

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to analyze situations by breaking them into cases and recognize and use counterexamples.

b. Copy the table and complete column 1.

c. Are the triangles similar? Explain your reasoning.

d. Repeat parts (a)–(c) for columns 2–6 in the table.

e. How are the corresponding side lengths related in each pair of triangles that are similar? Is this true for each pair of triangles that are not similar?

f. Make a conjecture about the similarity of two triangles based on their corresponding side lengths.

g. Use your conjecture to write another set of side lengths of two similar triangles. Use the side lengths to complete column 7 of the table.

EXPLORATION 2 Deciding Whether Triangles Are Similar

Work with a partner. Use dynamic geometry software. Construct any $\triangle ABC$.

a. Find AB, AC, and $m\angle A$. Choose any positive rational number k and construct $\triangle DEF$ so that $DE = k \cdot AB$, $DF = k \cdot AC$, and $m\angle D = m\angle A$.

b. Is $\triangle DEF$ similar to $\triangle ABC$? Explain your reasoning.

c. Repeat parts (a) and (b) several times by changing $\triangle ABC$ and k. Describe your results.

Communicate Your Answer

3. What are two ways to use corresponding sides of two triangles to determine that the triangles are similar?

8.3 Lesson

Core Vocabulary

Previous
similar figures
corresponding parts
slope
parallel lines
perpendicular lines

What You Will Learn

- Use the Side-Side-Side Similarity Theorem.
- Use the Side-Angle-Side Similarity Theorem.
- Prove slope criteria using similar triangles.

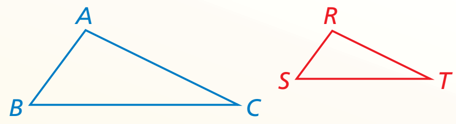
Using the Side-Side-Side Similarity Theorem

In addition to using congruent corresponding angles to show that two triangles are similar, you can use proportional corresponding side lengths.

Theorem

Theorem 8.4 Side-Side-Side (SSS) Similarity Theorem

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

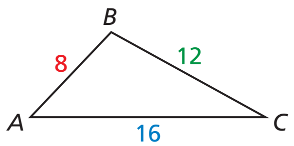


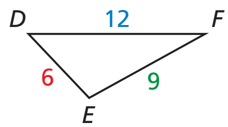
If $\dfrac{AB}{RS} = \dfrac{BC}{ST} = \dfrac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

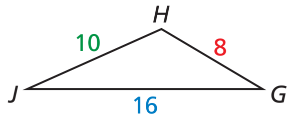
Proof p. 437

EXAMPLE 1 Using the SSS Similarity Theorem

Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?







FINDING AN ENTRY POINT

When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

SOLUTION

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

Shortest sides	Longest sides	Remaining sides
$\dfrac{AB}{DE} = \dfrac{8}{6}$	$\dfrac{CA}{FD} = \dfrac{16}{12}$	$\dfrac{BC}{EF} = \dfrac{12}{9}$
$= \dfrac{4}{3}$	$= \dfrac{4}{3}$	$= \dfrac{4}{3}$

- All the ratios are equal, so $\triangle ABC \sim \triangle DEF$.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

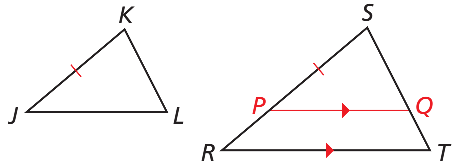
Shortest sides	Longest sides	Remaining sides
$\dfrac{AB}{GH} = \dfrac{8}{8}$	$\dfrac{CA}{JG} = \dfrac{16}{16}$	$\dfrac{BC}{HJ} = \dfrac{12}{10}$
$= 1$	$= 1$	$= \dfrac{6}{5}$

- The ratios are not all equal, so $\triangle ABC$ and $\triangle GHJ$ are not similar.

PROOF SSS Similarity Theorem

Given $\frac{RS}{JK} = \frac{ST}{KL} = \frac{TR}{LJ}$

Prove $\triangle RST \sim \triangle JKL$



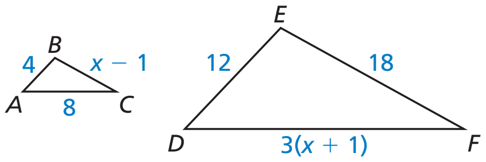
JUSTIFYING STEPS

The Parallel Postulate (Postulate 3.1) allows you to draw an auxiliary line $\overleftrightarrow{PQ}$ in $\triangle RST$. There is only one line through point P parallel to $\overleftrightarrow{RT}$, so you are able to draw it.

Locate P on $\overline{RS}$ so that $PS = JK$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$ by the AA Similarity Theorem (Theorem 8.3), and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$. You can use the given proportion and the fact that $PS = JK$ to deduce that $SQ = KL$ and $QP = LJ$. By the SSS Congruence Theorem (Theorem 5.8), it follows that $\triangle PSQ \cong \triangle JKL$. Finally, use the definition of congruent triangles and the AA Similarity Theorem (Theorem 8.3) to conclude that $\triangle RST \sim \triangle JKL$.

EXAMPLE 2 Using the SSS Similarity Theorem

Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



FINDING AN ENTRY POINT

You can use either $\frac{AB}{DE} = \frac{BC}{EF}$ or $\frac{AB}{DE} = \frac{AC}{DF}$ in Step 1.

SOLUTION

Step 1 Find the value of x that makes corresponding side lengths proportional.

$$\frac{AB}{DE} = \frac{BC}{EF}$$ Write proportion.

$$\frac{4}{12} = \frac{x-1}{18}$$ Substitute.

$$4 \cdot 18 = 12(x-1)$$ Cross Products Property

$$72 = 12x - 12$$ Simplify.

$$7 = x$$ Solve for x.

Step 2 Check that the side lengths are proportional when $x = 7$.

$BC = x - 1 = 6$

$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{12} = \frac{6}{18}$ ✓

$DF = 3(x + 1) = 24$

$\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF} \rightarrow \frac{4}{12} = \frac{8}{24}$ ✓

▶ When $x = 7$, the triangles are similar by the SSS Similarity Theorem.

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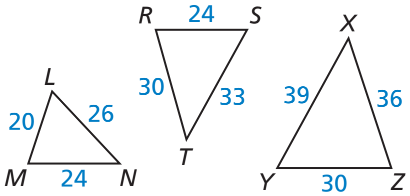
Use the diagram.



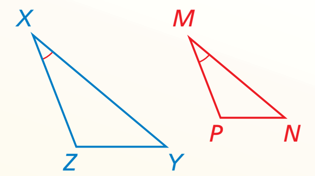
1. Which of the three triangles are similar? Write a similarity statement.
2. The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle.

Using the Side-Angle-Side Similarity Theorem

Theorem

Theorem 8.5 Side-Angle-Side (SAS) Similarity Theorem

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

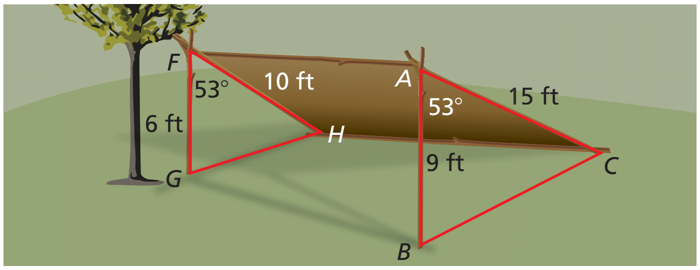


If $\angle X \cong \angle M$ and $\dfrac{ZX}{PM} = \dfrac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

Proof Ex. 33, p. 443

EXAMPLE 3 Using the SAS Similarity Theorem

You are building a lean-to shelter starting from a tree branch, as shown. Can you construct the right end so it is similar to the left end using the angle measure and lengths shown?



SOLUTION

Both $m\angle A$ and $m\angle F$ equal 53°, so $\angle A \cong \angle F$. Next, compare the ratios of the lengths of the sides that include $\angle A$ and $\angle F$.

Shorter sides

$$\frac{AB}{FG} = \frac{9}{6} = \frac{3}{2}$$

Longer sides

$$\frac{AC}{FH} = \frac{15}{10} = \frac{3}{2}$$

The lengths of the sides that include $\angle A$ and $\angle F$ are proportional. So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle FGH$.

▶ Yes, you can make the right end similar to the left end of the shelter.

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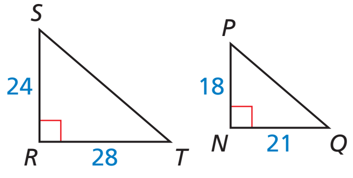
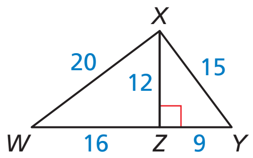
Explain how to show that the indicated triangles are similar.

3. $\triangle SRT \sim \triangle PNQ$



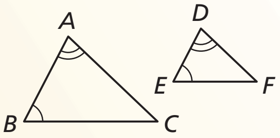
4. $\triangle XZW \sim \triangle YZX$



Concept Summary

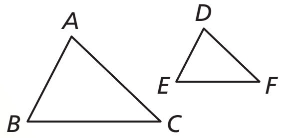
Triangle Similarity Theorems

AA Similarity Theorem



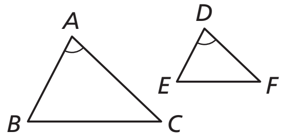
If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

SSS Similarity Theorem



If $\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

SAS Similarity Theorem



If $\angle A \cong \angle D$ and $\dfrac{AB}{DE} = \dfrac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

Proving Slope Criteria Using Similar Triangles

You can use similar triangles to prove the Slopes of Parallel Lines Theorem (Theorem 3.13). Because the theorem is biconditional, you must prove both parts.

1. If two nonvertical lines are parallel, then they have the same slope.
2. If two nonvertical lines have the same slope, then they are parallel.

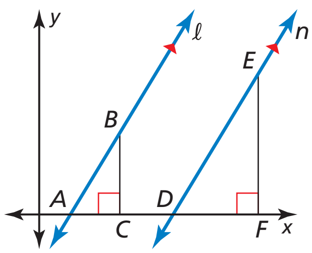
The first part is proved below. The second part is proved in the exercises.

PROOF Part of Slopes of Parallel Lines Theorem (Theorem 3.13)

Given $\ell \parallel n$, ℓ and n are nonvertical.

Prove $m_\ell = m_n$

First, consider the case where ℓ and n are horizontal. Because all horizontal lines are parallel and have a slope of 0, the statement is true for horizontal lines.



For the case of nonhorizontal, nonvertical lines, draw two such parallel lines, ℓ and n, and label their x-intercepts A and D, respectively. Draw a vertical segment $\overline{BC}$ parallel to the y-axis from point B on line ℓ to point C on the x-axis. Draw a vertical segment $\overline{EF}$ parallel to the y-axis from point E on line n to point F on the x-axis. Because vertical and horizontal lines are perpendicular, $\angle BCA$ and $\angle EFD$ are right angles.

STATEMENTS	REASONS
1. $\ell \parallel n$	**1.** Given
2. $\angle BAC \cong \angle EDF$	**2.** Corresponding Angles Theorem (Thm. 3.1)
3. $\angle BCA \cong \angle EFD$	**3.** Right Angles Congruence Theorem (Thm. 2.3)
4. $\triangle ABC \sim \triangle DEF$	**4.** AA Similarity Theorem (Thm. 8.3)
5. $\dfrac{BC}{EF} = \dfrac{AC}{DF}$	**5.** Corresponding sides of similar figures are proportional.
6. $\dfrac{BC}{AC} = \dfrac{EF}{DF}$	**6.** Rewrite proportion.
7. $m_\ell = \dfrac{BC}{AC}$, $m_n = \dfrac{EF}{DF}$	**7.** Definition of slope
8. $m_n = \dfrac{BC}{AC}$	**8.** Substitution Property of Equality
9. $m_\ell = m_n$	**9.** Transitive Property of Equality

To prove the Slopes of Perpendicular Lines Theorem (Theorem 3.14), you must prove both parts.

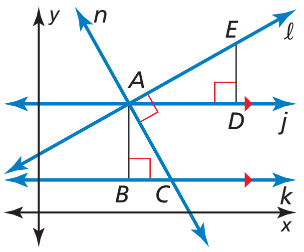
1. If two nonvertical lines are perpendicular, then the product of their slopes is -1.
2. If the product of the slopes of two nonvertical lines is -1, then the lines are perpendicular.

The first part is proved below. The second part is proved in the exercises.

PROOF **Part of Slopes of Perpendicular Lines Theorem (Theorem 3.14)**

Given $\ell \perp n$, ℓ and n are nonvertical.

Prove $m_\ell m_n = -1$



Draw two nonvertical, perpendicular lines, ℓ and n, that intersect at point A. Draw a horizontal line j parallel to the x-axis through point A. Draw a horizontal line k parallel to the x-axis through point C on line n. Because horizontal lines are parallel, $j \parallel k$. Draw a vertical segment $\overline{AB}$ parallel to the y-axis from point A to point B on line k. Draw a vertical segment $\overline{ED}$ parallel to the y-axis from point E on line ℓ to point D on line j. Because horizontal and vertical lines are perpendicular, $\angle ABC$ and $\angle ADE$ are right angles.

STATEMENTS	REASONS
1. $\ell \perp n$	1. Given
2. $m\angle CAE = 90°$	2. $\ell \perp n$
3. $m\angle CAE = m\angle DAE + m\angle CAD$	3. Angle Addition Postulate (Post. 1.4)
4. $m\angle DAE + m\angle CAD = 90°$	4. Transitive Property of Equality
5. $\angle BCA \cong \angle CAD$	5. Alternate Interior Angles Theorem (Thm. 3.2)
6. $m\angle BCA = m\angle CAD$	6. Definition of congruent angles
7. $m\angle DAE + m\angle BCA = 90°$	7. Substitution Property of Equality
8. $m\angle DAE = 90° - m\angle BCA$	8. Solve statement 7 for $m\angle DAE$.
9. $m\angle BCA + m\angle BAC + 90° = 180°$	9. Triangle Sum Theorem (Thm. 5.1)
10. $m\angle BAC = 90° - m\angle BCA$	10. Solve statement 9 for $m\angle BAC$.
11. $m\angle DAE = m\angle BAC$	11. Transitive Property of Equality
12. $\angle DAE \cong \angle BAC$	12. Definition of congruent angles
13. $\angle ABC \cong \angle ADE$	13. Right Angles Congruence Theorem (Thm. 2.3)
14. $\triangle ABC \sim \triangle ADE$	14. AA Similarity Theorem (Thm. 8.3)
15. $\frac{AD}{AB} = \frac{DE}{BC}$	15. Corresponding sides of similar figures are proportional.
16. $\frac{AD}{DE} = \frac{AB}{BC}$	16. Rewrite proportion.
17. $m_\ell = \frac{DE}{AD}$, $m_n = -\frac{AB}{BC}$	17. Definition of slope
18. $m_\ell m_n = \frac{DE}{AD} \cdot \left(-\frac{AB}{BC}\right)$	18. Substitution Property of Equality
19. $m_\ell m_n = \frac{DE}{AD} \cdot \left(-\frac{AD}{DE}\right)$	19. Substitution Property of Equality
20. $m_\ell m_n = -1$	20. Simplify.

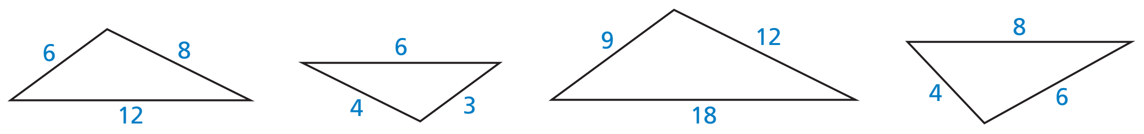
8.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** You plan to show that $\triangle QRS$ is similar to $\triangle XYZ$ by the SSS Similarity Theorem (Theorem 8.4). Copy and complete the proportion that you will use: $\frac{QR}{\square} = \frac{\square}{YZ} = \frac{QS}{\square}$.

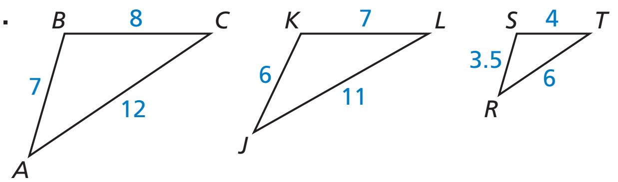
2. **WHICH ONE DOESN'T BELONG?** Which triangle does *not* belong with the other three? Explain your reasoning.



Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, determine whether $\triangle JKL$ or $\triangle RST$ is similar to $\triangle ABC$. *(See Example 1.)*

3.



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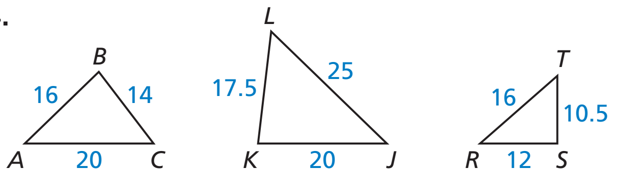
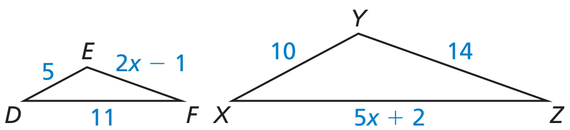


In Exercises 5 and 6, find the value of x that makes $\triangle DEF \sim \triangle XYZ$. *(See Example 2.)*

5.



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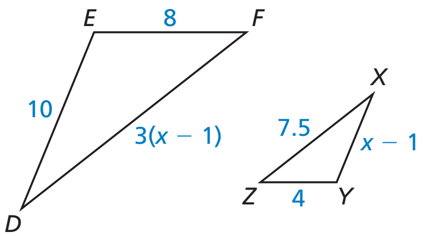


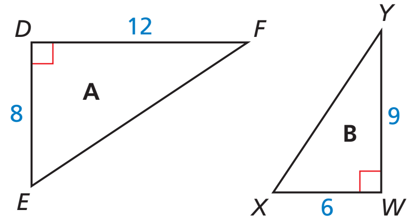
In Exercises 7 and 8, verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.

7. $\triangle ABC$: $BC = 18$, $AB = 15$, $AC = 12$
$\triangle DEF$: $EF = 12$, $DE = 10$, $DF = 8$

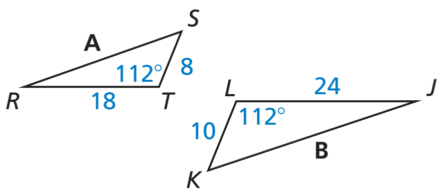
8. $\triangle ABC$: $AB = 10$, $BC = 16$, $CA = 20$
$\triangle DEF$: $DE = 25$, $EF = 40$, $FD = 50$

In Exercises 9 and 10, determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of triangle B to triangle A. *(See Example 3.)*

9.



10.



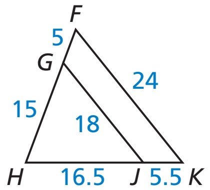
In Exercises 11 and 12, sketch the triangles using the given description. Then determine whether the two triangles can be similar.

11. In $\triangle RST$, $RS = 20$, $ST = 32$, and $m\angle S = 16°$. In $\triangle FGH$, $GH = 30$, $HF = 48$, and $m\angle H = 24°$.

12. The side lengths of $\triangle ABC$ are 24, $8x$, and 48, and the side lengths of $\triangle DEF$ are 15, 25, and $6x$.

In Exercises 13–16, show that the triangles are similar and write a similarity statement. Explain your reasoning.

13.



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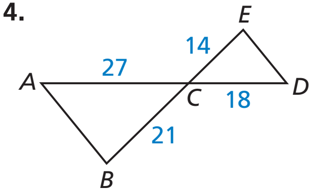

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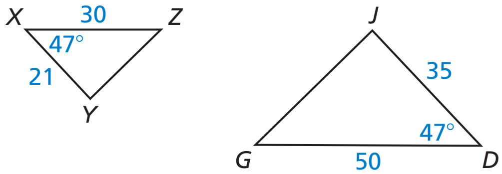

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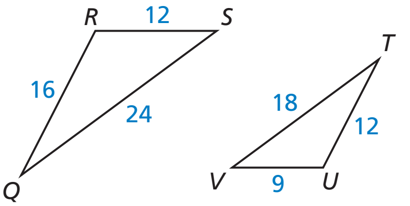
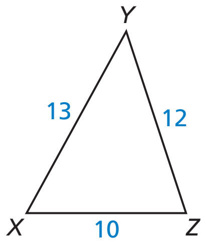


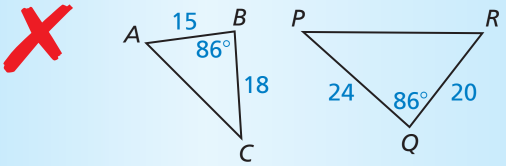
In Exercises 17 and 18, use $\triangle XYZ$.



17. The shortest side of a triangle similar to $\triangle XYZ$ is 20 units long. Find the other side lengths of the triangle.

18. The longest side of a triangle similar to $\triangle XYZ$ is 39 units long. Find the other side lengths of the triangle.

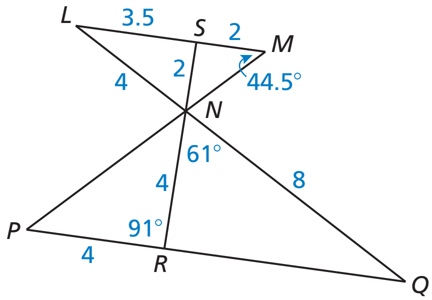
19. ERROR ANALYSIS Describe and correct the error in writing a similarity statement.



$\triangle ABC \sim \triangle PQR$ by the SAS Similarity Theorem (Theorem 8.5).

20. MATHEMATICAL CONNECTIONS Find the value of n that makes $\triangle DEF \sim \triangle XYZ$ when $DE = 4$, $EF = 5$, $XY = 4(n + 1)$, $YZ = 7n - 1$, and $\angle E \cong \angle Y$. Include a sketch.

ATTENDING TO PRECISION **In Exercises 21–26, use the diagram to copy and complete the statement.**



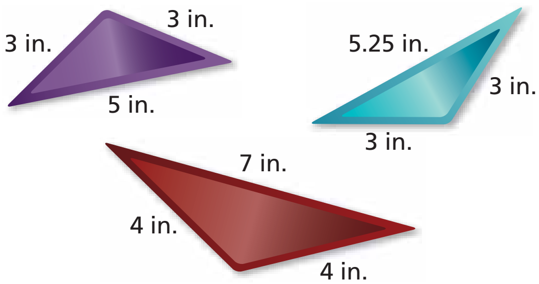
21. $m\angle LNS =$ ☐ **22.** $m\angle NRQ =$ ☐

23. $m\angle NQR =$ ☐ **24.** $RQ =$ ☐

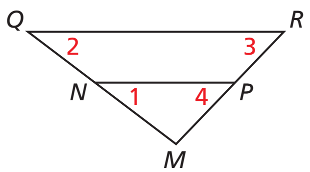
25. $m\angle NSM =$ ☐ **26.** $m\angle NPR =$ ☐

27. MAKING AN ARGUMENT Your friend claims that $\triangle JKL \sim \triangle MNO$ by the SAS Similarity Theorem (Theorem 8.5) when $JK = 18$, $m\angle K = 130°$, $KL = 16$, $MN = 9$, $m\angle N = 65°$, and $NO = 8$. Do you support your friend's claim? Explain your reasoning.

28. ANALYZING RELATIONSHIPS Certain sections of stained glass are sold in triangular, beveled pieces. Which of the three beveled pieces, if any, are similar?



29. ATTENDING TO PRECISION In the diagram, $\dfrac{MN}{MR} = \dfrac{MP}{MQ}$. Which of the statements must be true? Select all that apply. Explain your reasoning.



Ⓐ $\angle 1 \cong \angle 2$ Ⓑ $\overline{QR} \parallel \overline{NP}$

Ⓒ $\angle 1 \cong \angle 4$ Ⓓ $\triangle MNP \sim \triangle MRQ$

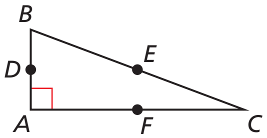
30. WRITING Are any two right triangles similar? Explain.

31. MODELING WITH MATHEMATICS In the portion of the shuffleboard court shown, $\frac{BC}{AC} = \frac{BD}{AE}$.

a. What additional information do you need to show that $\triangle BCD \sim \triangle ACE$ using the SSS Similarity Theorem (Theorem 8.4)?

b. What additional information do you need to show that $\triangle BCD \sim \triangle ACE$ using the SAS Similarity Theorem (Theorem 8.5)?

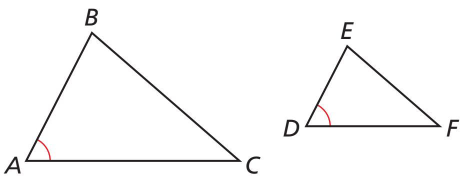
32. PROOF Given that $\triangle BAC$ is a right triangle and D, E, and F are midpoints, prove that $m\angle DEF = 90°$.



33. PROVING A THEOREM Write a two-column proof of the SAS Similarity Theorem (Theorem 8.5).

Given $\angle A \cong \angle D$, $\frac{AB}{DE} = \frac{AC}{DF}$

Prove $\triangle ABC \sim \triangle DEF$



34. CRITICAL THINKING You are given two right triangles with one pair of corresponding legs and the pair of hypotenuses having the same length ratios.

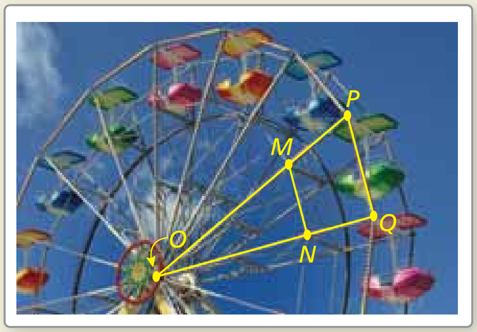
a. The lengths of the given pair of corresponding legs are 6 and 18, and the lengths of the hypotenuses are 10 and 30. Use the Pythagorean Theorem to find the lengths of the other pair of corresponding legs. Draw a diagram.

b. Write the ratio of the lengths of the second pair of corresponding legs.

c. Are these triangles similar? Does this suggest a Hypotenuse-Leg Similarity Theorem for right triangles? Explain.

35. WRITING Can two triangles have all three ratios of corresponding angle measures equal to a value greater than 1? less than 1? Explain.

36. HOW DO YOU SEE IT? Which theorem could you use to show that $\triangle OPQ \sim \triangle OMN$ in the portion of the Ferris wheel shown when $PM = QN = 5$ feet and $MO = NO = 10$ feet?



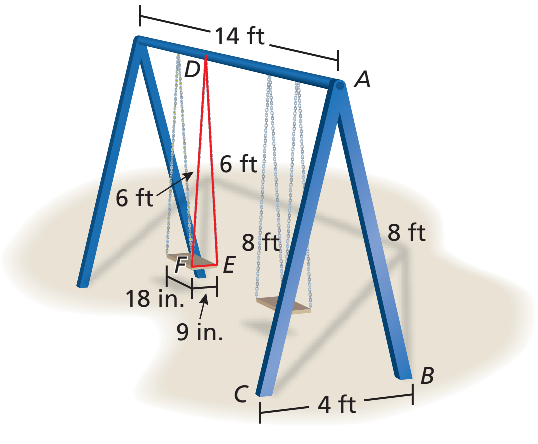
37. DRAWING CONCLUSIONS Explain why it is not necessary to have an Angle-Side-Angle Similarity Theorem.

38. THOUGHT PROVOKING Decide whether each is a valid method of showing that two quadrilaterals are similar. Justify your answer.

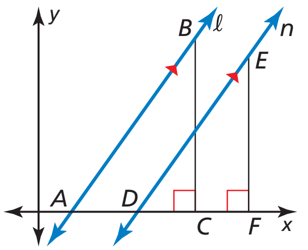
a. SASA **b.** SASAS **c.** SSSS **d.** SASSS

39. MULTIPLE REPRESENTATIONS Use a diagram to show why there is no Side-Side-Angle Similarity Theorem.

40. MODELING WITH MATHEMATICS The dimensions of an actual swing set are shown. You want to create a scale model of the swing set for a dollhouse using similar triangles. Sketch a drawing of your swing set and label each side length. Write a similarity statement for each pair of similar triangles. State the scale factor you used to create the scale model.



41. PROVING A THEOREM Copy and complete the paragraph proof of the second part of the Slopes of Parallel Lines Theorem (Theorem 3.13) from page 439.



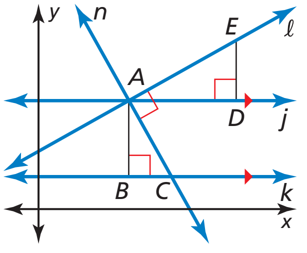
Given $m_\ell = m_n$, ℓ and n are nonvertical.

Prove $\ell \parallel n$

You are given that $m_\ell = m_n$. By the definition of slope, $m_\ell = \frac{BC}{AC}$ and $m_n = \frac{EF}{DF}$. By ____________________, $\frac{BC}{AC} = \frac{EF}{DF}$. Rewriting this proportion yields __________.

By the Right Angles Congruence Theorem (Thm. 2.3), _______________. So, $\triangle ABC \sim \triangle DEF$ by ____________________. Because corresponding angles of similar triangles are congruent, $\angle BAC \cong \angle EDF$. By __________________, $\ell \parallel n$.

42. PROVING A THEOREM Copy and complete the two-column proof of the second part of the Slopes of Perpendicular Lines Theorem (Theorem 3.14) from page 440.



Given $m_\ell m_n = -1$, ℓ and n are nonvertical.

Prove $\ell \perp n$

STATEMENTS	REASONS
1. $m_\ell m_n = -1$	**1.** Given
2. $m_\ell = \frac{DE}{AD}$, $m_n = -\frac{AB}{BC}$	**2.** Definition of slope
3. $\frac{DE}{AD} \cdot -\frac{AB}{BC} = -1$	**3.** ____________________
4. $\frac{DE}{AD} = \frac{BC}{AB}$	**4.** Multiply each side of statement 3 by $-\frac{BC}{AB}$.
5. $\frac{DE}{BC} = \underline{\quad}$	**5.** Rewrite proportion.
6. ____________________	**6.** Right Angles Congruence Theorem (Thm. 2.3)
7. $\triangle ABC \sim \triangle ADE$	**7.** ____________________
8. $\angle BAC \cong \angle DAE$	**8.** Corresponding angles of similar figures are congruent.
9. $\angle BCA \cong \angle CAD$	**9.** Alternate Interior Angles Theorem (Thm. 3.2)
10. $m\angle BAC = m\angle DAE$, $m\angle BCA = m\angle CAD$	**10.** ____________________
11. $m\angle BAC + m\angle BCA + 90^\circ = 180^\circ$	**11.** ____________________
12. ____________________	**12.** Subtraction Property of Equality
13. $m\angle CAD + m\angle DAE = 90^\circ$	**13.** Substitution Property of Equality
14. $m\angle CAE = m\angle DAE + m\angle CAD$	**14.** Angle Addition Postulate (Post. 1.4)
15. $m\angle CAE = 90^\circ$	**15.** ____________________
16. ____________________	**16.** Definition of perpendicular lines

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the coordinates of point P along the directed line segment AB so that AP to PB is the given ratio. *(Section 3.5)*

43. $A(-3, 6)$, $B(2, 1)$; 3 to 2 **44.** $A(-3, -5)$, $B(9, -1)$; 1 to 3 **45.** $A(1, -2)$, $B(8, 12)$; 4 to 3

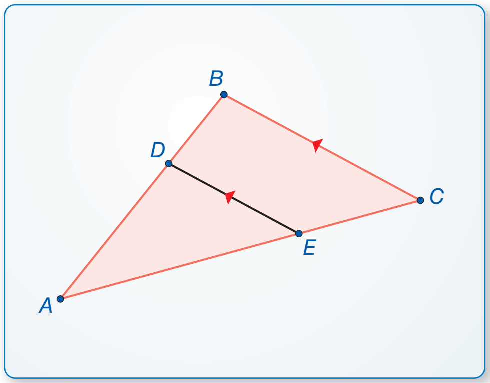
8.4 Proportionality Theorems

Essential Question What proportionality relationships exist in a triangle intersected by an angle bisector or by a line parallel to one of the sides?

EXPLORATION 1 Discovering a Proportionality Relationship

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$.

a. Construct $\overline{DE}$ parallel to $\overline{BC}$ with endpoints on $\overline{AB}$ and $\overline{AC}$, respectively.



LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

b. Compare the ratios of AD to BD and AE to CE.

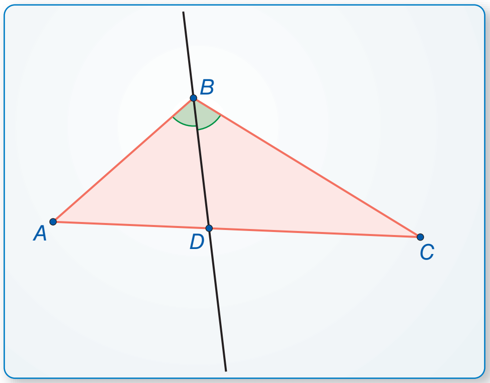
c. Move $\overline{DE}$ to other locations parallel to $\overline{BC}$ with endpoints on $\overline{AB}$ and $\overline{AC}$, and repeat part (b).

d. Change $\triangle ABC$ and repeat parts (a)–(c) several times. Write a conjecture that summarizes your results.

EXPLORATION 2 Discovering a Proportionality Relationship

Work with a partner. Use dynamic geometry software to draw any $\triangle ABC$.

a. Bisect $\angle B$ and plot point D at the intersection of the angle bisector and $\overline{AC}$.



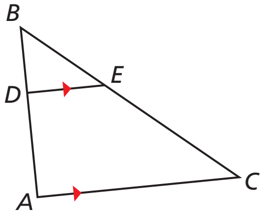
b. Compare the ratios of AD to DC and BA to BC.

c. Change $\triangle ABC$ and repeat parts (a) and (b) several times. Write a conjecture that summarizes your results.

Communicate Your Answer

3. What proportionality relationships exist in a triangle intersected by an angle bisector or by a line parallel to one of the sides?

4. Use the figure at the right to write a proportion.



8.4 Lesson

Core Vocabulary

Previous
corresponding angles
ratio
proportion

What You Will Learn

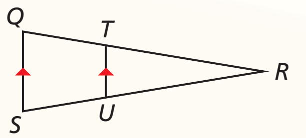
- Use the Triangle Proportionality Theorem and its converse.
- Use other proportionality theorems.

Using the Triangle Proportionality Theorem

Theorems

Theorem 8.6 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

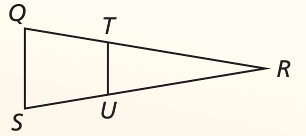


If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

Proof Ex. 27, p. 451

Theorem 8.7 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

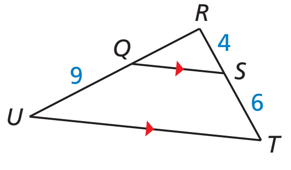


If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

Proof Ex. 28, p. 451

EXAMPLE 1 Finding the Length of a Segment

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of $\overline{RQ}$?



SOLUTION

$\frac{RQ}{QU} = \frac{RS}{ST}$ Triangle Proportionality Theorem

$\frac{RQ}{9} = \frac{4}{6}$ Substitute.

$RQ = 6$ Multiply each side by 9 and simplify.

- The length of $\overline{RQ}$ is 6 units.

Monitoring Progress

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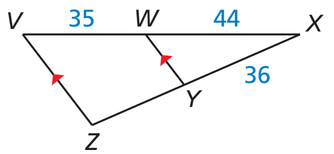
1. Find the length of $\overline{YZ}$.



The theorems on the previous page also imply the following:

Contrapositive of the Triangle Proportionality Theorem

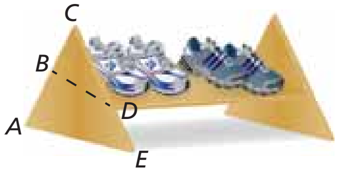
If $\frac{RT}{TQ} \neq \frac{RU}{US}$, then $\overline{TU} \nparallel \overline{QS}$.

Inverse of the Triangle Proportionality Theorem

If $\overline{TU} \nparallel \overline{QS}$, then $\frac{RT}{TQ} \neq \frac{RU}{US}$.

EXAMPLE 2 Solving a Real-Life Problem

On the shoe rack shown, $BA = 33$ centimeters, $CB = 27$ centimeters, $CD = 44$ centimeters, and $DE = 25$ centimeters. Explain why the shelf is not parallel to the floor.



SOLUTION

Find and simplify the ratios of the lengths.

$$\frac{CD}{DE} = \frac{44}{25} \qquad \frac{CB}{BA} = \frac{27}{33} = \frac{9}{11}$$

▶ Because $\frac{44}{25} \neq \frac{9}{11}$, $\overline{BD}$ is not parallel to $\overline{AE}$. So, the shelf is not parallel to the floor.

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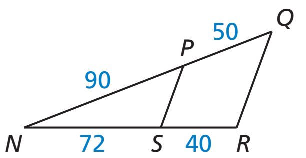


2. Determine whether $\overline{PS} \parallel \overline{QR}$.

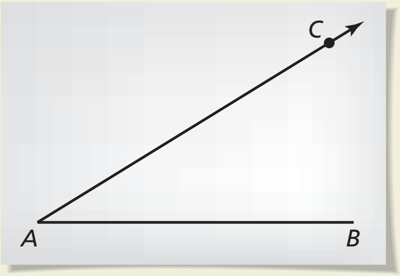
Recall that you partitioned a directed line segment in the coordinate plane in Section 3.5. You can apply the Triangle Proportionality Theorem to construct a point along a directed line segment that partitions the segment in a given ratio.

CONSTRUCTION Constructing a Point along a Directed Line Segment

Construct the point L on $\overline{AB}$ so that the ratio of AL to LB is 3 to 1.

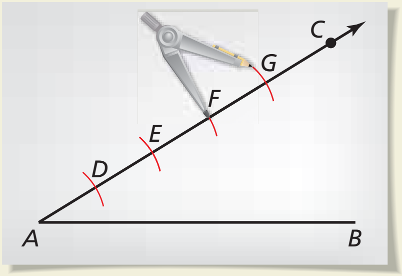
SOLUTION

Step 1



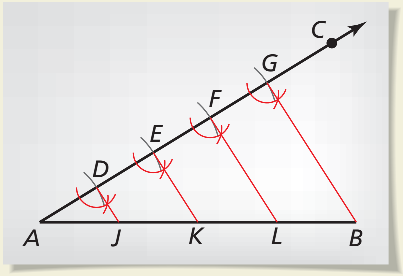
Draw a segment and a ray Draw $\overline{AB}$ of any length. Choose any point C not on $\overleftrightarrow{AB}$. Draw $\overrightarrow{AC}$.

Step 2



Draw arcs Place the point of a compass at A and make an arc of any radius intersecting $\overrightarrow{AC}$. Label the point of intersection D. Using the same compass setting, make three more arcs on $\overrightarrow{AC}$, as shown. Label the points of intersection E, F, and G and note that $AD = DE = EF = FG$.

Step 3



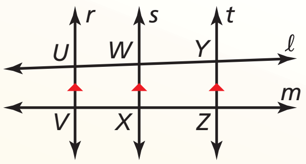
Draw a segment Draw $\overline{GB}$. Copy $\angle AGB$ and construct congruent angles at D, E, and F with sides that intersect $\overline{AB}$ at J, K, and L. Sides $\overline{DJ}$, $\overline{EK}$, and $\overline{FL}$ are all parallel, and they divide $\overline{AB}$ equally. So, $AJ = JK = KL = LB$. Point L divides directed line segment AB in the ratio 3 to 1.

Using Other Proportionality Theorems

Theorem

Theorem 8.8 Three Parallel Lines Theorem

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

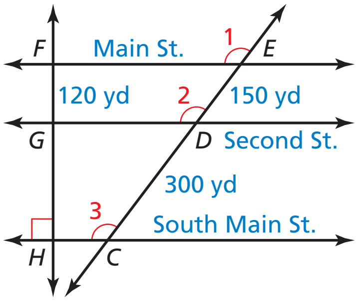


$$\frac{UW}{WY} = \frac{VX}{XZ}$$

Proof Ex. 32, p. 451

EXAMPLE 3 Using the Three Parallel Lines Theorem

In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent, $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.



SOLUTION

Corresponding angles are congruent, so $\overleftrightarrow{FE}$, $\overleftrightarrow{GD}$, and $\overleftrightarrow{HC}$ are parallel. There are different ways you can write a proportion to find HG.

Method 1 Use the Three Parallel Lines Theorem to set up a proportion.

$$\frac{HG}{GF} = \frac{CD}{DE}$$ Three Parallel Lines Theorem

$$\frac{HG}{120} = \frac{300}{150}$$ Substitute.

$$HG = 240$$ Multiply each side by 120 and simplify.

By the Segment Addition Postulate (Postulate 1.2), $HF = HG + GF = 240 + 120 = 360$.

▶ The distance between Main Street and South Main Street is 360 yards.

Method 2 Set up a proportion involving total and partial distances.

Step 1 Make a table to compare the distances.

	$\overleftrightarrow{CE}$	$\overleftrightarrow{HF}$
Total distance	$CE = 300 + 150 = 450$	HF
Partial distance	$DE = 150$	$GF = 120$

Step 2 Write and solve a proportion.

$$\frac{450}{150} = \frac{HF}{120}$$ Write proportion.

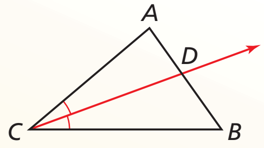
$$360 = HF$$ Multiply each side by 120 and simplify.

▶ The distance between Main Street and South Main Street is 360 yards.

Theorem

Theorem 8.9 Triangle Angle Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

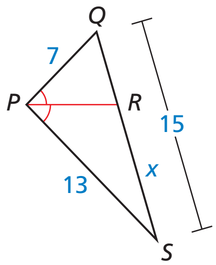


$$\frac{AD}{DB} = \frac{CA}{CB}$$

Proof Ex. 35, p. 452

EXAMPLE 4 Using the Triangle Angle Bisector Theorem

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.



SOLUTION

Because $\overrightarrow{PR}$ is an angle bisector of $\angle QPS$, you can apply the Triangle Angle Bisector Theorem. Let $RS = x$. Then $RQ = 15 - x$.

$\frac{RQ}{RS} = \frac{PQ}{PS}$	Triangle Angle Bisector Theorem
$\frac{15 - x}{x} = \frac{7}{13}$	Substitute.
$195 - 13x = 7x$	Cross Products Property
$9.75 = x$	Solve for x.

▶ The length of $\overline{RS}$ is 9.75 units.

Monitoring Progress

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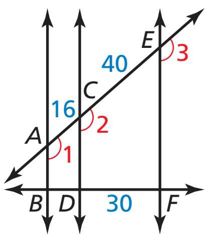
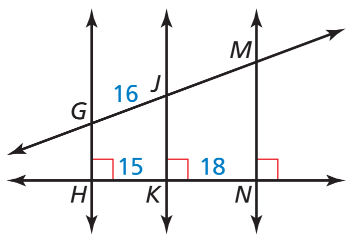
Find the length of the given line segment.

3. $\overline{BD}$

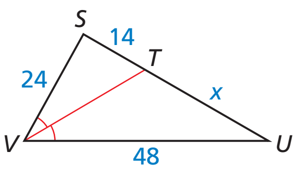


4. $\overline{JM}$

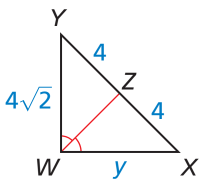


Find the value of the variable.

5.



6.



8.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

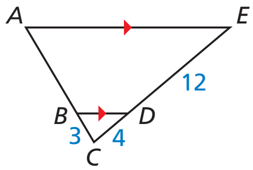
Vocabulary and Core Concept Check

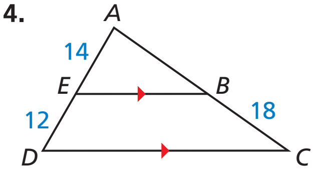
1. **COMPLETE THE STATEMENT** If a line divides two sides of a triangle proportionally, then it is __________ to the third side. This theorem is known as the ____________.

2. **VOCABULARY** In $\triangle ABC$, point R lies on $\overline{BC}$ and $\overrightarrow{AR}$ bisects $\angle CAB$. Write the proportionality statement for the triangle that is based on the Triangle Angle Bisector Theorem (Theorem 8.9).

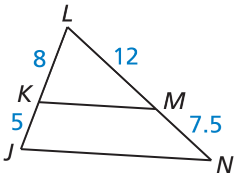
Monitoring Progress and Modeling with Mathematics

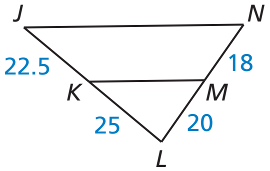
In Exercises 3 and 4, find the length of $\overline{AB}$. *(See Example 1.)*

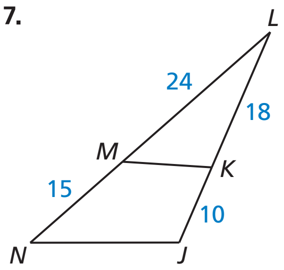
3. 

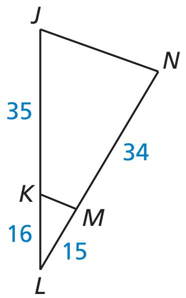
4. 

In Exercises 5–8, determine whether $\overline{KM} \parallel \overline{JN}$. *(See Example 2.)*

5. 

6. 

7. 

8. 

CONSTRUCTION **In Exercises 9–12, draw a segment with the given length. Construct the point that divides the segment in the given ratio.**

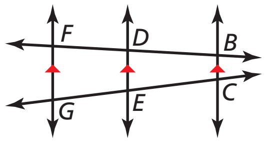
9. 3 in.; 1 to 4

10. 2 in.; 2 to 3

11. 12 cm; 1 to 3

12. 9 cm; 2 to 5

In Exercises 13–16, use the diagram to complete the proportion.



13. $\dfrac{BD}{BF} = \dfrac{\square}{CG}$

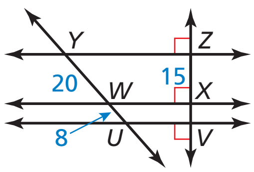
14. $\dfrac{CG}{\square} = \dfrac{BF}{DF}$

15. $\dfrac{EG}{CE} = \dfrac{DF}{\square}$

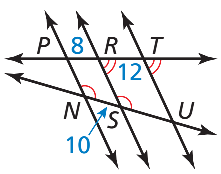
16. $\dfrac{\square}{BD} = \dfrac{CG}{CE}$

In Exercises 17 and 18, find the length of the indicated line segment. *(See Example 3.)*

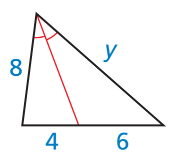
17. $\overline{VX}$

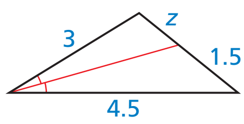


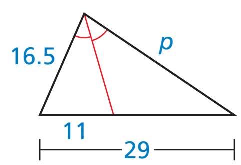
18. $\overline{SU}$

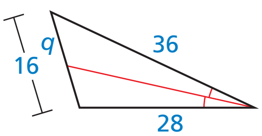


In Exercises 19–22, find the value of the variable. *(See Example 4.)*

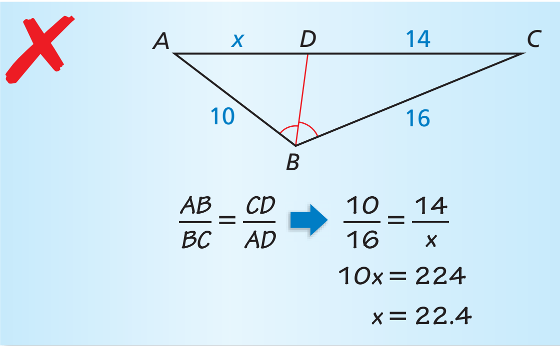
19. 

20. 

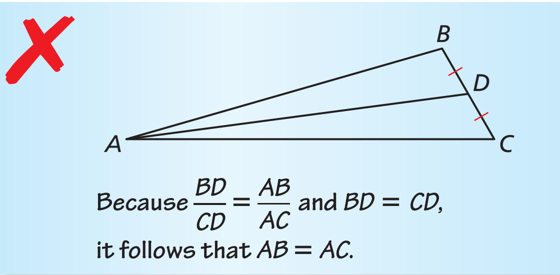
21. 

22. 

23. ERROR ANALYSIS Describe and correct the error in solving for x.

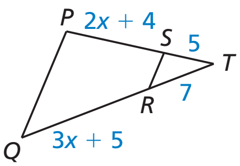


24. ERROR ANALYSIS Describe and correct the error in the student's reasoning.

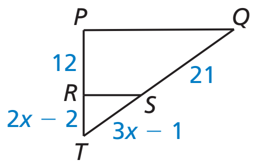


MATHEMATICAL CONNECTIONS In Exercises 25 and 26, find the value of x for which $\overline{PQ} \parallel \overline{RS}$.

25.



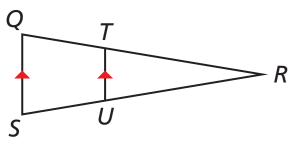
26.



27. PROVING A THEOREM Prove the Triangle Proportionality Theorem (Theorem 8.6).

Given $\overline{QS} \parallel \overline{TU}$

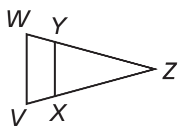
Prove $\dfrac{QT}{TR} = \dfrac{SU}{UR}$



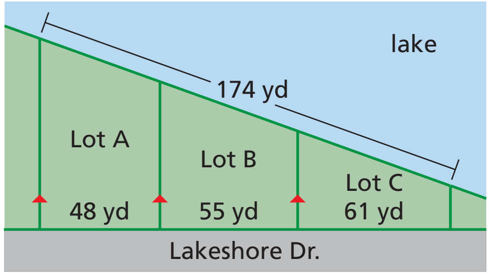
28. PROVING A THEOREM Prove the Converse of the Triangle Proportionality Theorem (Theorem 8.7).

Given $\dfrac{ZY}{YW} = \dfrac{ZX}{XV}$

Prove $\overline{YX} \parallel \overline{WV}$



29. MODELING WITH MATHEMATICS The real estate term *lake frontage* refers to the distance along the edge of a piece of property that touches a lake.

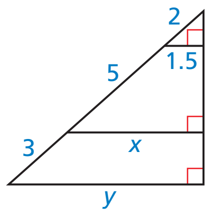


a. Find the lake frontage (to the nearest tenth) of each lot shown.

b. In general, the more lake frontage a lot has, the higher its selling price. Which lot(s) should be listed for the highest price?

c. Suppose that lot prices are in the same ratio as lake frontages. If the least expensive lot is \$250,000, what are the prices of the other lots? Explain your reasoning.

30. USING STRUCTURE Use the diagram to find the values of x and y.

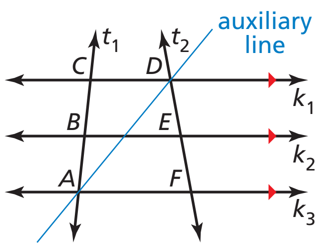


31. REASONING In the construction on page 447, explain why you can apply the Triangle Proportionality Theorem (Theorem 8.6) in Step 3.

32. PROVING A THEOREM Use the diagram with the auxiliary line drawn to write a paragraph proof of the Three Parallel Lines Theorem (Theorem 8.8).

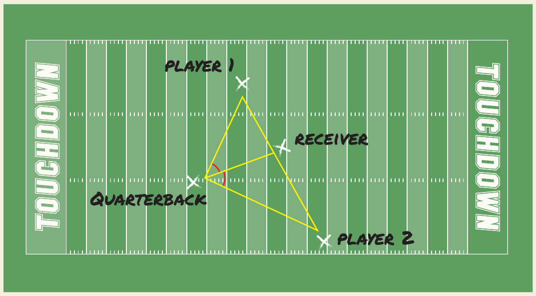
Given $k_1 \parallel k_2 \parallel k_3$

Prove $\dfrac{CB}{BA} = \dfrac{DE}{EF}$



33. **CRITICAL THINKING** In $\triangle LMN$, the angle bisector of $\angle M$ also bisects $\overline{LN}$. Classify $\triangle LMN$ as specifically as possible. Justify your answer.

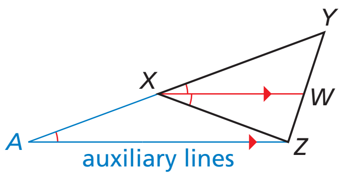
34. **HOW DO YOU SEE IT?** During a football game, the quarterback throws the ball to the receiver. The receiver is between two defensive players, as shown. If Player 1 is closer to the quarterback when the ball is thrown and both defensive players move at the same speed, which player will reach the receiver first? Explain your reasoning.



35. **PROVING A THEOREM** Use the diagram with the auxiliary lines drawn to write a paragraph proof of the Triangle Angle Bisector Theorem (Theorem 8.9).

Given $\angle YXW \cong \angle WXZ$

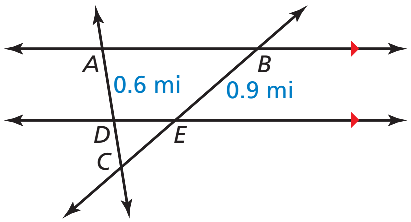
Prove $\dfrac{YW}{WZ} = \dfrac{XY}{XZ}$



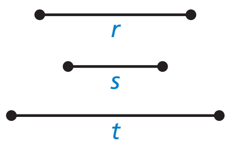
36. **THOUGHT PROVOKING** Write the converse of the Triangle Angle Bisector Theorem (Theorem 8.9). Is the converse true? Justify your answer.

37. **REASONING** How is the Triangle Midsegment Theorem (Theorem 6.8) related to the Triangle Proportionality Theorem (Theorem 8.6)? Explain your reasoning.

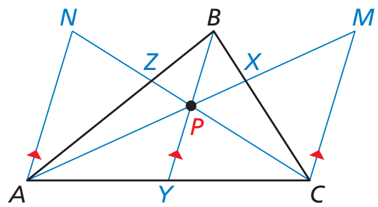
38. **MAKING AN ARGUMENT** Two people leave points A and B at the same time. They intend to meet at point C at the same time. The person who leaves point A walks at a speed of 3 miles per hour. You and a friend are trying to determine how fast the person who leaves point B must walk. Your friend claims you need to know the length of $\overline{AC}$. Is your friend correct? Explain your reasoning.



39. **CONSTRUCTION** Given segments with lengths r, s, and t, construct a segment of length x, such that $\frac{r}{s} = \frac{t}{x}$.



40. **PROOF** Prove *Ceva's Theorem*: If P is any point inside $\triangle ABC$, then $\dfrac{AY}{YC} \cdot \dfrac{CX}{XB} \cdot \dfrac{BZ}{ZA} = 1$.



(*Hint*: Draw segments parallel to $\overline{BY}$ through A and C, as shown. Apply the Triangle Proportionality Theorem (Theorem 8.6) to $\triangle ACM$. Show that $\triangle APN \sim \triangle MPC$, $\triangle CXM \sim \triangle BXP$, and $\triangle BZP \sim \triangle AZN$.)

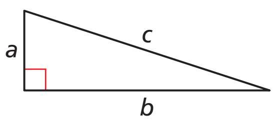
Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the triangle. *(Section 5.5)*

41. Which sides are the legs?

42. Which side is the hypotenuse?



Solve the equation. *(Skills Review Handbook)*

43. $x^2 = 121$

44. $x^2 + 16 = 25$

45. $36 + x^2 = 85$

8.3–8.4 What Did You Learn?

Core Concepts

Section 8.3

Theorem 8.4 Side-Side-Side (SSS) Similarity Theorem, *p. 436*
Theorem 8.5 Side-Angle-Side (SAS) Similarity Theorem, *p. 438*
Proving Slope Criteria Using Similar Triangles, *p. 439*

Section 8.4

Theorem 8.6 Triangle Proportionality Theorem, *p. 446*
Theorem 8.7 Converse of the Triangle Proportionality Theorem, *p. 446*
Theorem 8.8 Three Parallel Lines Theorem, *p. 448*
Theorem 8.9 Triangle Angle Bisector Theorem, *p. 449*

Mathematical Practices

1. In Exercise 17 on page 442, why must you be told which side is 20 units long?

2. In Exercise 42 on page 444, analyze the given statement. Describe the relationship between the slopes of the lines.

3. In Exercise 4 on page 450, is it better to use $\frac{7}{6}$ or 1.17 as your ratio of the lengths when finding the length of $\overline{AB}$? Explain your reasoning.

Performance Task

Judging the Math Fair

You have been selected to be one of the judges for the Middle School Math Fair. In one competition, seventh-grade students were asked to create scale drawings or scale models of real-life objects. As a judge, you need to verify that the objects are scaled correctly in at least two different ways. How will you verify that the entries are scaled correctly?

To explore the answers to this question and more, go to ***BigIdeasMath.com***.

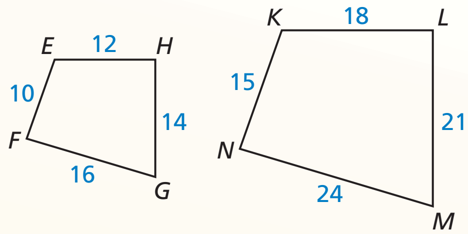
8 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

8.1 Similar Polygons *(pp. 417–426)*

In the diagram, *EHGF* ~ *KLMN*. Find the scale factor from *EHGF* to *KLMN*. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.

From the diagram, you can see that $\overline{EH}$ and $\overline{KL}$ are corresponding sides. So, the scale factor of *EHGF* to *KLMN* is $\frac{KL}{EH} = \frac{18}{12} = \frac{3}{2}$.

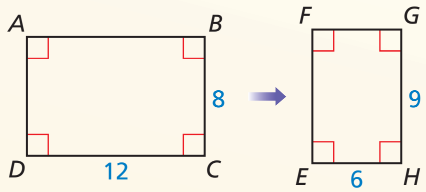


$\angle E \cong \angle K$, $\angle H \cong \angle L$, $\angle G \cong \angle M$, and $\angle F \cong \angle N$.

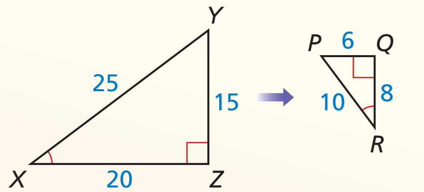
$$\frac{KL}{EH} = \frac{LM}{HG} = \frac{MN}{GF} = \frac{NK}{FE}$$

Find the scale factor. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality.

1. *ABCD* ~ *EFGH*



2. $\triangle XYZ \sim \triangle RPQ$



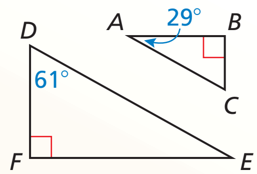
3. Two similar triangles have a scale factor of 3 : 5. The altitude of the larger triangle is 24 inches. What is the altitude of the smaller triangle?

4. Two similar triangles have a pair of corresponding sides of length 12 meters and 8 meters. The larger triangle has a perimeter of 48 meters and an area of 180 square meters. Find the perimeter and area of the smaller triangle.

8.2 Proving Triangle Similarity by AA *(pp. 427–432)*

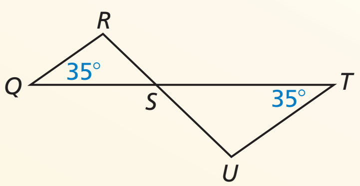
Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

Because they are both right angles, $\angle F$ and $\angle B$ are congruent. By the Triangle Sum Theorem (Theorem 5.1), $61° + 90° + m\angle E = 180°$, so $m\angle E = 29°$. So, $\angle E$ and $\angle A$ are congruent. So, $\triangle DFE \sim \triangle CBA$ by the AA Similarity Theorem (Theorem 8.3).

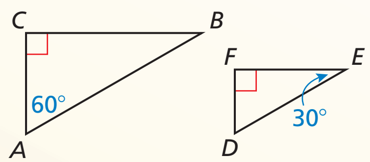


Show that the triangles are similar. Write a similarity statement.

5.



6.

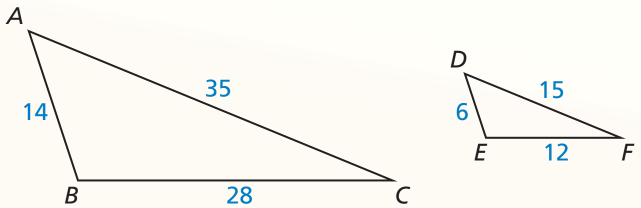


7. A cellular telephone tower casts a shadow that is 72 feet long, while a nearby tree that is 27 feet tall casts a shadow that is 6 feet long. How tall is the tower?

8.3 Proving Triangle Similarity by SSS and SAS (pp. 435–444)

Show that the triangles are similar.

a.



Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

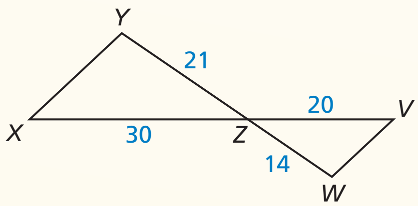
Shortest sides $\frac{AB}{DE} = \frac{14}{6} = \frac{7}{3}$

Longest sides $\frac{AC}{DF} = \frac{35}{15} = \frac{7}{3}$

Remaining sides $\frac{BC}{EF} = \frac{28}{12} = \frac{7}{3}$

All the ratios are equal, so $\triangle ABC \sim \triangle DEF$ by the SSS Similarity Theorem (Theorem 8.4).

b.



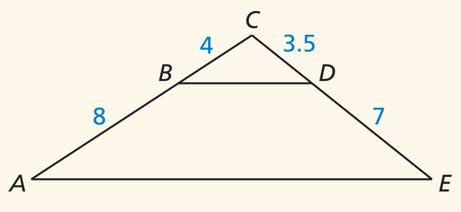
$\angle YZX \cong \angle WZV$ by the Vertical Angles Congruence Theorem (Theorem 2.6). Next, compare the ratios of the corresponding side lengths of $\triangle YZX$ and $\triangle WZV$.

$\frac{WZ}{YZ} = \frac{14}{21} = \frac{2}{3}$ $\qquad$ $\frac{VZ}{XZ} = \frac{20}{30} = \frac{2}{3}$

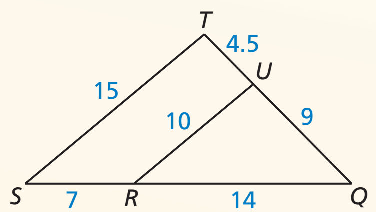
► So, by the SAS Similarity Theorem (Theorem 8.5), $\triangle YZX \sim \triangle WZV$.

Use the SSS Similarity Theorem (Theorem 8.4) or the SAS Similarity Theorem (Theorem 8.5) to show that the triangles are similar.

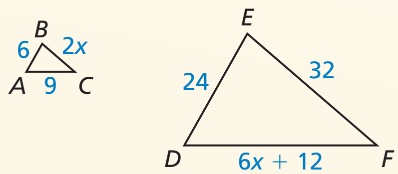
8.



9.



10. Find the value of x that makes $\triangle ABC \sim \triangle DEF$.



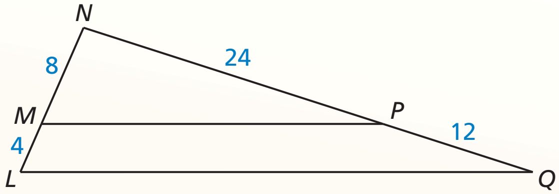
8.4 Proportionality Theorems (pp. 445–452)

a. Determine whether $\overline{MP} \parallel \overline{LQ}$.

Begin by finding and simplifying ratios of lengths determined by $\overline{MP}$.

$$\frac{NM}{ML} = \frac{8}{4} = \frac{2}{1} = 2$$

$$\frac{NP}{PQ} = \frac{24}{12} = \frac{2}{1} = 2$$



Because $\frac{NM}{ML} = \frac{NP}{PQ}$, $\overline{MP}$ is parallel to $\overline{LQ}$ by the Converse of the Triangle Proportionality Theorem (Theorem 8.7).

b. In the diagram, $\overline{AD}$ bisects $\angle CAB$. Find the length of $\overline{DB}$.

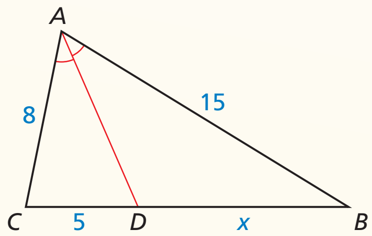
Because $\overline{AD}$ is an angle bisector of $\angle CAB$, you can apply the Triangle Angle Bisector Theorem (Theorem 8.9).

$$\frac{DB}{DC} = \frac{AB}{AC}$$ Triangle Angle Bisector Theorem

$$\frac{x}{5} = \frac{15}{8}$$ Substitute.

$$8x = 75$$ Cross Products Property

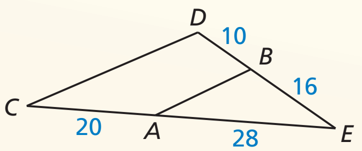
$$9.375 = x$$ Solve for x.



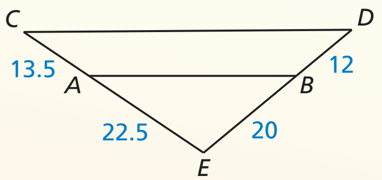
▶ The length of $\overline{DB}$ is 9.375 units.

Determine whether $\overline{AB} \parallel \overline{CD}$.

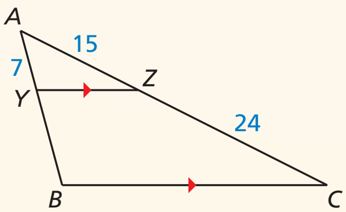
11.



12.

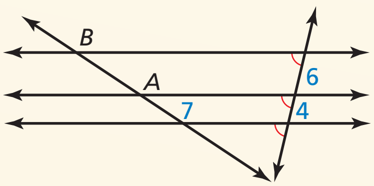


13. Find the length of $\overline{YB}$.

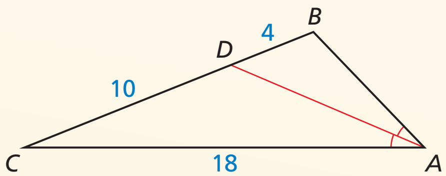


Find the length of $\overline{AB}$.

14.



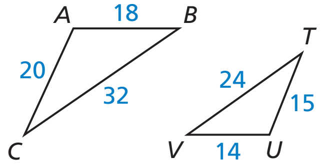
15.



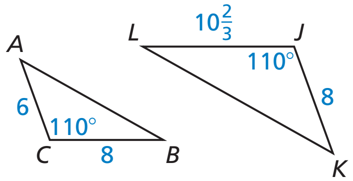
8 Chapter Test

Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.

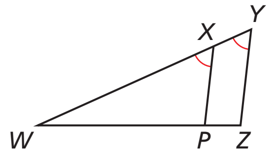
1.



2.

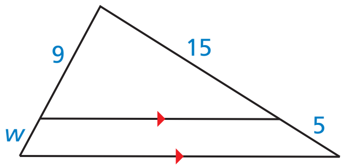


3.

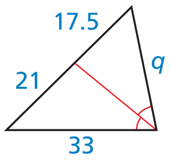


Find the value of the variable.

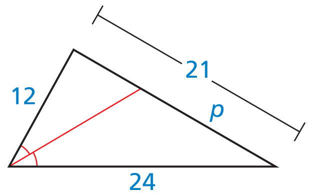
4.



5.

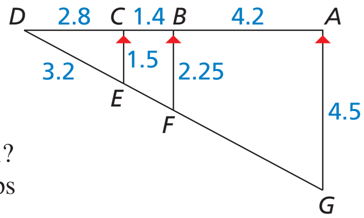


6.



7. Given $\triangle QRS \sim \triangle MNP$, list all pairs of congruent angles. Then write the ratios of the corresponding side lengths in a statement of proportionality.

Use the diagram.

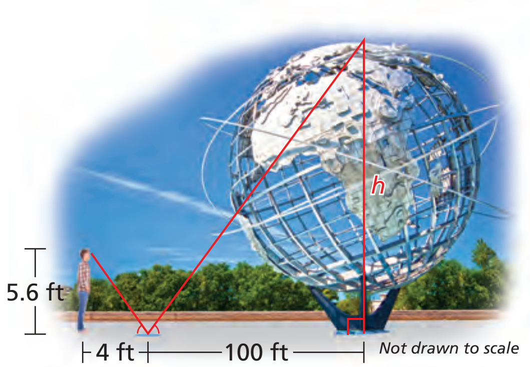


8. Find the length of $\overline{EF}$.

9. Find the length of $\overline{FG}$.

10. Is quadrilateral *FECB* similar to quadrilateral *GFBA*? If so, what is the scale factor of the dilation that maps quadrilateral *FECB* to quadrilateral *GFBA*?

11. You are visiting the Unisphere at Flushing Meadows Corona Park in New York. To estimate the height of the stainless steel model of Earth, you place a mirror on the ground and stand where you can see the top of the model in the mirror. Use the diagram to estimate the height of the model. Explain why this method works.

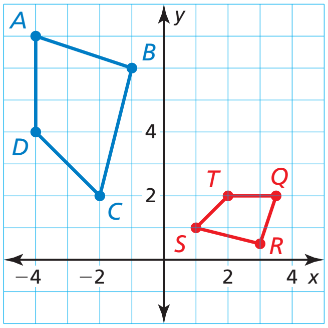


12. You are making a scale model of a rectangular park for a school project. Your model has a length of 2 feet and a width of 1.4 feet. The actual park is 800 yards long. What are the perimeter and area of the actual park?

13. In a *perspective drawing*, lines that are parallel in real life must meet at a vanishing point on the horizon. To make the train cars in the drawing appear equal in length, they are drawn so that the lines connecting the opposite corners of each car are parallel. Find the length of the bottom edge of the drawing of Car 2.

8 Cumulative Assessment

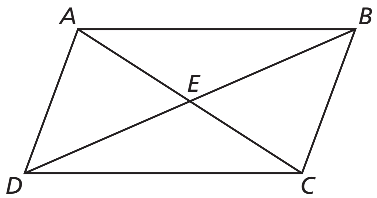
1. Use the graph of quadrilaterals *ABCD* and *QRST*.



a. Write a composition of transformations that maps quadrilateral *ABCD* to quadrilateral *QRST*.

b. Are the quadrilaterals similar? Explain your reasoning.

2. In the diagram, *ABCD* is a parallelogram. Which congruence theorem(s) could you use to show that $\triangle AED \cong \triangle CEB$? Select all that apply.



SAS Congruence Theorem (Theorem 5.5)

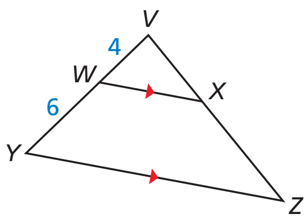
SSS Congruence Theorem (Theorem 5.8)

HL Congruence Theorem (Theorem 5.9)

ASA Congruence Theorem (Theorem 5.10)

AAS Congruence Theorem (Theorem 5.11)

3. By the Triangle Proportionality Theorem (Theorem 8.6), $\frac{VW}{WY} = \frac{VX}{XZ}$. In the diagram, $VX > VW$ and $XZ > WY$. List three possible values for *VX* and *XZ*.



4. The slope of line ℓ is $-\frac{3}{4}$. The slope of line *n* is $\frac{4}{3}$. What must be true about lines ℓ and *n*?

Ⓐ Lines ℓ and *n* are parallel.

Ⓑ Lines ℓ and *n* are perpendicular.

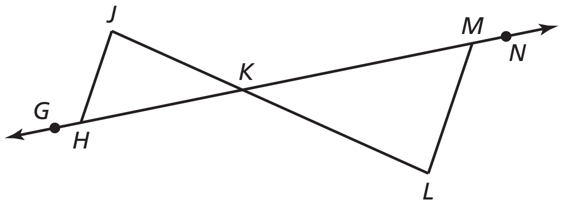
Ⓒ Lines ℓ and *n* are skew.

Ⓓ Lines ℓ and *n* are the same line.

5. Enter a statement or reason in each blank to complete the two-column proof.

Given $\frac{KJ}{KL} = \frac{KH}{KM}$

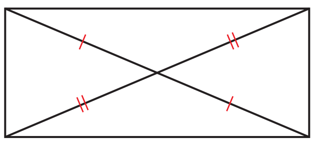
Prove $\angle LMN \cong \angle JHG$



STATEMENTS	REASONS
1. $\frac{KJ}{KL} = \frac{KH}{KM}$	1. Given
2. $\angle JKH \cong \angle LKM$	2. ________
3. $\triangle JKH \sim \triangle LKM$	3. ________
4. $\angle KHJ \cong \angle KML$	4. ________
5. ________	5. Definition of congruent angles
6. $m\angle KHJ + m\angle JHG = 180°$	6. Linear Pair Postulate (Post. 2.8)
7. $m\angle JHG = 180° - m\angle KHJ$	7. ________
8. $m\angle KML + m\angle LMN = 180°$	8. ________
9. ________	9. Subtraction Property of Equality
10. $m\angle LMN = 180° - m\angle KHJ$	10. ________
11. ________	11. Transitive Property of Equality
12. $\angle LMN \cong \angle JHG$	12. ________

6. The coordinates of the vertices of $\triangle DEF$ are $D(-8, 5)$, $E(-5, 8)$, and $F(-1, 4)$. The coordinates of the vertices of $\triangle JKL$ are $J(16, -10)$, $K(10, -16)$, and $L(2, -8)$. $\angle D \cong \angle J$. Can you show that $\triangle DEF \sim \triangle JKL$ by using the AA Similarity Theorem (Theorem 8.3)? If so, do so by listing the congruent corresponding angles and writing a similarity transformation that maps $\triangle DEF$ to $\triangle JKL$. If not, explain why not.

7. Classify the quadrilateral using the most specific name.



rectangle | square | parallelogram | rhombus

8. Your friend makes the statement "Quadrilateral *PQRS* is similar to quadrilateral *WXYZ*." Describe the relationships between corresponding angles and between corresponding sides that make this statement true.