5 Congruent Triangles

Chapter Learning Target:
Understand congruent triangles.

Chapter Success Criteria:
- I can classify triangles by sides and angles.
- I can solve problems involving congruent polygons.
- I can explain why triangles are congruent using different theorems.
- I can write a coordinate proof.

Hang Glider *(p. 278)*

Lifeguard Tower *(p. 255)*

Barn *(p. 248)*

Home Decor *(p. 241)*

Painting *(p. 235)*

Maintaining Mathematical Proficiency

Using the Midpoint and Distance Formulas

Example 1 **The endpoints of $\overline{AB}$ are $A(-2, 3)$ and $B(4, 7)$. Find the coordinates of the midpoint M.**

Use the Midpoint Formula.

$$M\left(\frac{-2+4}{2}, \frac{3+7}{2}\right) = M\left(\frac{2}{2}, \frac{10}{2}\right)$$

$$= M(1, 5)$$

▶ The coordinates of the midpoint M are $(1, 5)$.

Example 2 **Find the distance between $C(0, -5)$ and $D(3, 2)$.**

$$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(3 - 0)^2 + [2 - (-5)]^2} \quad \text{Substitute.}$$

$$= \sqrt{3^2 + 7^2} \quad \text{Subtract.}$$

$$= \sqrt{9 + 49} \quad \text{Evaluate powers.}$$

$$= \sqrt{58} \quad \text{Add.}$$

$$\approx 7.6 \quad \text{Use a calculator.}$$

▶ The distance between $C(0, -5)$ and $D(3, 2)$ is about 7.6.

Find the coordinates of the midpoint M of the segment with the given endpoints. Then find the distance between the two points.

1. $P(-4, 1)$ and $Q(0, 7)$

2. $G(3, 6)$ and $H(9, -2)$

3. $U(-1, -2)$ and $V(8, 0)$

Solving Equations with Variables on Both Sides

Example 3 **Solve $2 - 5x = -3x$.**

$$2 - 5x = -3x \quad \text{Write the equation.}$$

$$+5x \quad +5x \quad \text{Add } 5x \text{ to each side.}$$

$$2 = 2x \quad \text{Simplify.}$$

$$\frac{2}{2} = \frac{2x}{2} \quad \text{Divide each side by 2.}$$

$$1 = x \quad \text{Simplify.}$$

▶ The solution is $x = 1$.

Solve the equation.

4. $7x + 12 = 3x$

5. $14 - 6t = t$

6. $5p + 10 = 8p + 1$

7. $w + 13 = 11w - 7$

8. $4x + 1 = 3 - 2x$

9. $z - 2 = 4 + 9z$

10. **ABSTRACT REASONING** Is it possible to find the length of a segment in a coordinate plane without using the Distance Formula? Explain your reasoning.

Mathematical Practices

Mathematically proficient students understand and use given definitions.

Definitions, Postulates, and Theorems

Core Concept

Definitions and Biconditional Statements

A definition is always an "if and only if" statement. Here is an example.

Definition: Two geometric figures are *congruent figures* if and only if there is a rigid motion or a composition of rigid motions that maps one of the figures onto the other.

Because this is a definition, it is a biconditional statement. It implies the following two conditional statements.

1. If two geometric figures are congruent figures, then there is a rigid motion or a composition of rigid motions that maps one of the figures onto the other.
2. If there is a rigid motion or a composition of rigid motions that maps one geometric figure onto another, then the two geometric figures are congruent figures.

Definitions, postulates, and theorems are the building blocks of geometry. In two-column proofs, the statements in the *reason* column are almost always definitions, postulates, or theorems.

EXAMPLE 1 Identifying Definitions, Postulates, and Theorems

Classify each statement as a definition, a postulate, or a theorem.

a. If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.

b. If two coplanar lines have no point of intersection, then the lines are parallel.

c. If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

SOLUTION

a. This is a theorem. It is the Alternate Interior Angles Converse Theorem (Theorem 3.6) studied in Section 3.3.

b. This is the definition of parallel lines.

c. This is a postulate. It is the Parallel Postulate (Postulate 3.1) studied in Section 3.1. In Euclidean geometry, it is assumed, not proved, to be true.

Monitoring Progress

Classify each statement as a definition, a postulate, or a theorem. Explain your reasoning.

1. In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is -1.
2. If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.
3. If two lines intersect to form a right angle, then the lines are perpendicular.
4. Through any two points, there exists exactly one line.

5.1 Angles of Triangles

Essential Question How are the angle measures of a triangle related?

EXPLORATION 1 Writing a Conjecture

Work with a partner.

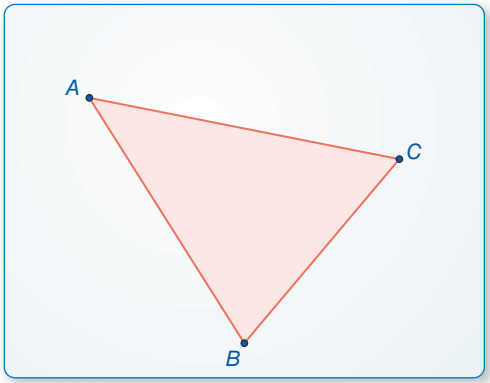
a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

b. Find the measures of the interior angles of the triangle.

c. Find the sum of the interior angle measures.

d. Repeat parts (a)–(c) with several other triangles. Then write a conjecture about the sum of the measures of the interior angles of a triangle.

CONSTRUCTING VIABLE ARGUMENTS
To be proficient in math, you need to reason inductively about data and write conjectures.



Sample

Angles

$m\angle A = 43.67°$

$m\angle B = 81.87°$

$m\angle C = 54.46°$

EXPLORATION 2 Writing a Conjecture

Work with a partner.

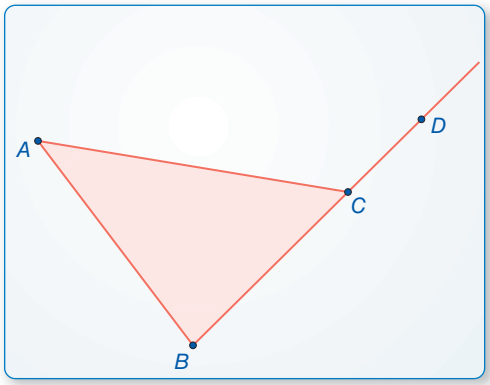
a. Use dynamic geometry software to draw any triangle and label it $\triangle ABC$.

b. Draw an exterior angle at any vertex and find its measure.

c. Find the measures of the two nonadjacent interior angles of the triangle.

d. Find the sum of the measures of the two nonadjacent interior angles. Compare this sum to the measure of the exterior angle.

e. Repeat parts (a)–(d) with several other triangles. Then write a conjecture that compares the measure of an exterior angle with the sum of the measures of the two nonadjacent interior angles.



Sample

Angles

$m\angle A = 43.67°$

$m\angle B = 81.87°$

$m\angle ACD = 125.54°$

Communicate Your Answer

3. How are the angle measures of a triangle related?

4. An exterior angle of a triangle measures 32°. What do you know about the measures of the interior angles? Explain your reasoning.

5.1 Lesson

Core Vocabulary

interior angles, *p. 233*
exterior angles, *p. 233*
corollary to a theorem, *p. 235*

Previous
triangle

What You Will Learn

- Classify triangles by sides and angles.
- Find interior and exterior angle measures of triangles.

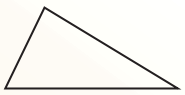
Classifying Triangles by Sides and by Angles

Recall that a *triangle* is a polygon with three sides. You can classify triangles by sides and by angles, as shown below.

Core Concept

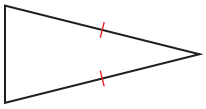
Classifying Triangles by Sides

Scalene Triangle



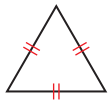
no congruent sides

Isosceles Triangle



at least 2 congruent sides

Equilateral Triangle



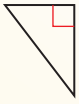
3 congruent sides

Classifying Triangles by Angles

Acute Triangle

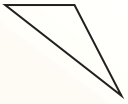
3 acute angles

Right Triangle



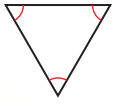
1 right angle

Obtuse Triangle



1 obtuse angle

Equiangular Triangle



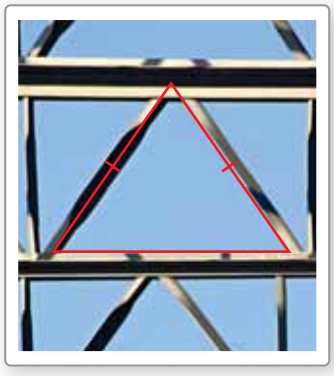
3 congruent angles

READING

Notice that an equilateral triangle is also isosceles. An equiangular triangle is also acute.

EXAMPLE 1 Classifying Triangles by Sides and by Angles

Classify the triangular shape of the support beams in the diagram by its sides and by measuring its angles.



SOLUTION

The triangle has a pair of congruent sides, so it is isosceles. By measuring, the angles are 55°, 55°, and 70°.

- So, it is an acute isosceles triangle.

Monitoring Progress

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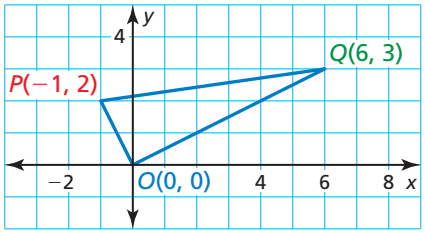
1. Draw an obtuse isosceles triangle and an acute scalene triangle.

EXAMPLE 2 Classifying a Triangle in the Coordinate Plane

Classify $\triangle OPQ$ by its sides. Then determine whether it is a right triangle.



SOLUTION

Step 1 Use the Distance Formula to find the side lengths.

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 0)^2 + (2 - 0)^2} = \sqrt{5} \approx 2.2$$

$$OQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (3 - 0)^2} = \sqrt{45} \approx 6.7$$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{[6 - (-1)]^2 + (3 - 2)^2} = \sqrt{50} \approx 7.1$$

Because no sides are congruent, $\triangle OPQ$ is a scalene triangle.

Step 2 Check for right angles. The slope of $\overline{OP}$ is $\frac{2 - 0}{-1 - 0} = -2$. The slope of $\overline{OQ}$ is $\frac{3 - 0}{6 - 0} = \frac{1}{2}$. The product of the slopes is $-2\left(\frac{1}{2}\right) = -1$. So, $\overline{OP} \perp \overline{OQ}$ and $\angle POQ$ is a right angle.

▶ So, $\triangle OPQ$ is a right scalene triangle.

Monitoring Progress

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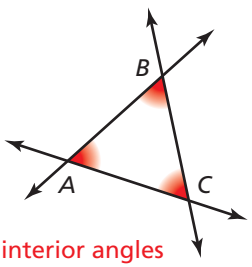
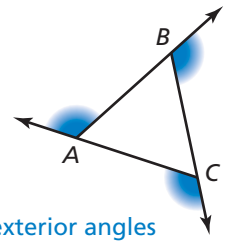
2. $\triangle ABC$ has vertices $A(0, 0)$, $B(3, 3)$, and $C(-3, 3)$. Classify the triangle by its sides. Then determine whether it is a right triangle.

Finding Angle Measures of Triangles

When the sides of a polygon are extended, other angles are formed. The original angles are the **interior angles**. The angles that form linear pairs with the interior angles are the **exterior angles**.



interior angles



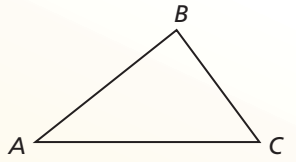
exterior angles

Theorem

Theorem 5.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

Proof p. 234; Ex. 53, p. 238



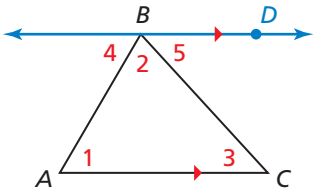
$m\angle A + m\angle B + m\angle C = 180°$

To prove certain theorems, you may need to add a line, a segment, or a ray to a given diagram. An *auxiliary* line is used in the proof of the Triangle Sum Theorem.

PROOF Triangle Sum Theorem

Given $\triangle ABC$

Prove $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$



Plan for Proof

a. Draw an auxiliary line through B that is parallel to $\overline{AC}$.

b. Show that $m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$, $\angle 1 \cong \angle 4$, and $\angle 3 \cong \angle 5$.

c. By substitution, $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$.

Plan in Action

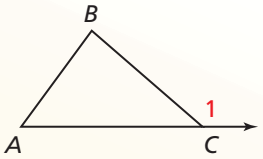
	STATEMENTS	REASONS
a.	**1.** Draw $\overleftrightarrow{BD}$ parallel to $\overline{AC}$.	**1.** Parallel Postulate (Post. 3.1)
b.	**2.** $m\angle 4 + m\angle 2 + m\angle 5 = 180^\circ$	**2.** Angle Addition Postulate (Post. 1.4) and definition of straight angle
	3. $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 5$	**3.** Alternate Interior Angles Theorem (Thm. 3.2)
	4. $m\angle 1 = m\angle 4$, $m\angle 3 = m\angle 5$	**4.** Definition of congruent angles
c.	**5.** $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$	**5.** Substitution Property of Equality

Theorem

Theorem 5.2 Exterior Angle Theorem

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

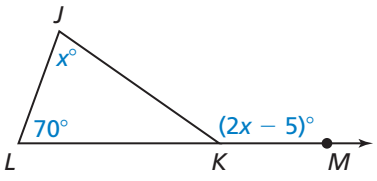
Proof Ex. 42, p. 237



$m\angle 1 = m\angle A + m\angle B$

EXAMPLE 3 Finding an Angle Measure

Find $m\angle JKM$.



SOLUTION

Step 1 Write and solve an equation to find the value of x.

$(2x - 5)^\circ = 70^\circ + x^\circ$ Apply the Exterior Angle Theorem.

$x = 75$ Solve for x.

Step 2 Substitute 75 for x in $2x - 5$ to find $m\angle JKM$.

$2x - 5 = 2 \cdot 75 - 5 = 145$

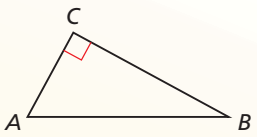
So, the measure of $\angle JKM$ is 145°.

A **corollary to a theorem** is a statement that can be proved easily using the theorem. The corollary below follows from the Triangle Sum Theorem.

Corollary

Corollary 5.1 Corollary to the Triangle Sum Theorem

The acute angles of a right triangle are complementary.

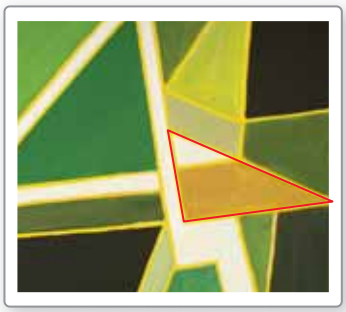


$m\angle A + m\angle B = 90°$

Proof Ex. 41, p. 237

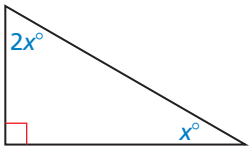
EXAMPLE 4 Modeling with Mathematics

In the painting, the red triangle is a right triangle. The measure of one acute angle in the triangle is twice the measure of the other. Find the measure of each acute angle.



SOLUTION

1. **Understand the Problem** You are given a right triangle and the relationship between the two acute angles in the triangle. You need to find the measure of each acute angle.



2. **Make a Plan** First, sketch a diagram of the situation. You can use the Corollary to the Triangle Sum Theorem and the given relationship between the two acute angles to write and solve an equation to find the measure of each acute angle.
3. **Solve the Problem** Let the measure of the smaller acute angle be $x°$. Then the measure of the larger acute angle is $2x°$. The Corollary to the Triangle Sum Theorem states that the acute angles of a right triangle are complementary.

   Use the corollary to set up and solve an equation.

   $x° + 2x° = 90°$ Corollary to the Triangle Sum Theorem

   $x = 30$ Solve for x.

   ▶ So, the measures of the acute angles are $30°$ and $2(30°) = 60°$.
4. **Look Back** Add the two angles and check that their sum satisfies the Corollary to the Triangle Sum Theorem.

   $30° + 60° = 90°$ ✓

Monitoring Progress

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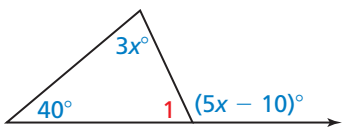
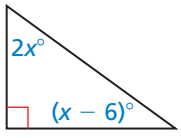
3. Find the measure of $\angle 1$.



4. Find the measure of each acute angle.



5.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

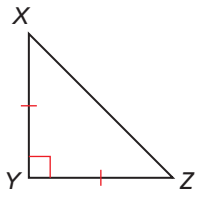
Vocabulary and Core Concept Check

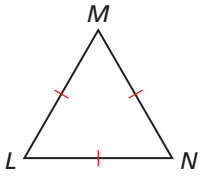
1. **WRITING** Can a right triangle also be obtuse? Explain your reasoning.

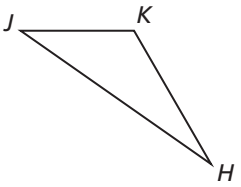
2. **COMPLETE THE SENTENCE** The measure of an exterior angle of a triangle is equal to the sum of the measures of the two _____ interior angles.

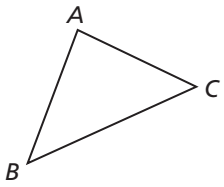
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, classify the triangle by its sides and by measuring its angles. *(See Example 1.)*

3. 

4. 

5. 

6. 

In Exercises 7–10, classify $\triangle ABC$ by its sides. Then determine whether it is a right triangle. *(See Example 2.)*

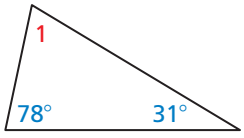
7. $A(2, 3), B(6, 3), C(2, 7)$

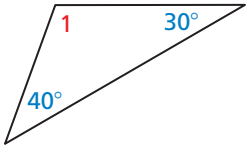
8. $A(3, 3), B(6, 9), C(6, -3)$

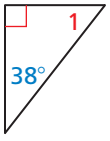
9. $A(1, 9), B(4, 8), C(2, 5)$

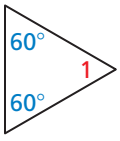
10. $A(-2, 3), B(0, -3), C(3, -2)$

In Exercises 11–14, find $m\angle 1$. Then classify the triangle by its angles.

11. 

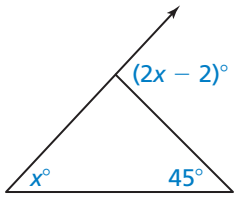
12. 

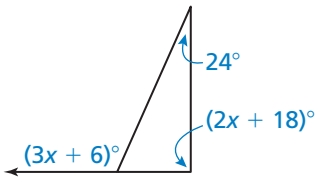
13. 

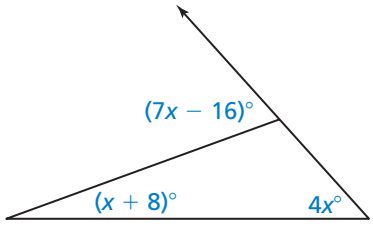
14. 

In Exercises 15–18, find the measure of the exterior angle. *(See Example 3.)*

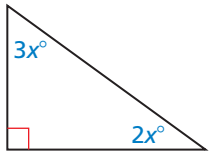
15. 

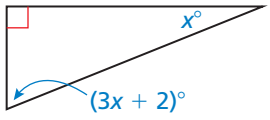
16. 

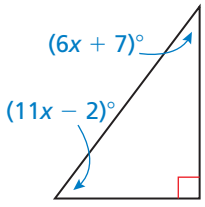
17. 

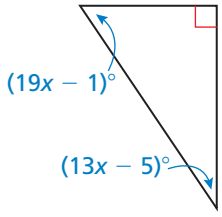
18. 

In Exercises 19–22, find the measure of each acute angle. *(See Example 4.)*

19. 

20. 

21. 

22. 

In Exercises 23–26, find the measure of each acute angle in the right triangle. *(See Example 4.)*

23. The measure of one acute angle is 5 times the measure of the other acute angle.

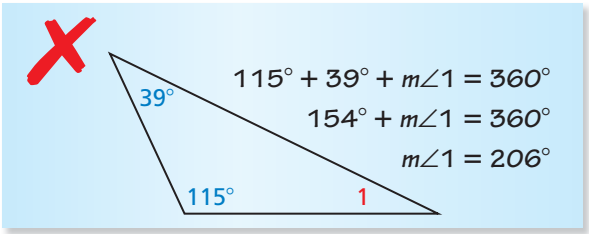
24. The measure of one acute angle is 8 times the measure of the other acute angle.

25. The measure of one acute angle is 3 times the sum of the measure of the other acute angle and 8.

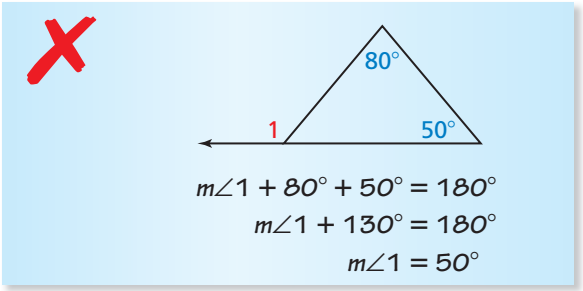
26. The measure of one acute angle is twice the difference of the measure of the other acute angle and 12.

ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in finding $m\angle 1$.**

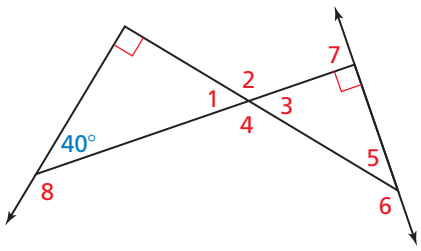
27.



28.



In Exercises 29–36, find the measure of the numbered angle.



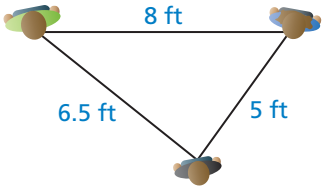
29. $\angle 1$ **30.** $\angle 2$

31. $\angle 3$ **32.** $\angle 4$

33. $\angle 5$ **34.** $\angle 6$

35. $\angle 7$ **36.** $\angle 8$

37. **USING TOOLS** Three people are standing on a stage. The distances between the three people are shown in the diagram. Classify the triangle by its sides and by measuring its angles.



38. **USING STRUCTURE** Which of the following sets of angle measures could form a triangle? Select all that apply.

Ⓐ 100°, 50°, 40° Ⓑ 96°, 74°, 10°

Ⓒ 165°, 113°, 82° Ⓓ 101°, 41°, 38°

Ⓔ 90°, 45°, 45° Ⓕ 84°, 62°, 34°

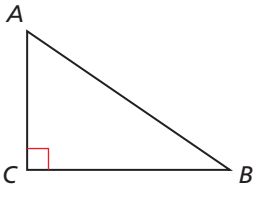
39. **MODELING WITH MATHEMATICS** You are bending a strip of metal into an isosceles triangle for a sculpture. The strip of metal is 20 inches long. The first bend is made 6 inches from one end. Describe two ways you could complete the triangle.

40. **THOUGHT PROVOKING** Find and draw an object (or part of an object) that can be modeled by a triangle and an exterior angle. Describe the relationship between the interior angles of the triangle and the exterior angle in terms of the object.

41. **PROVING A COROLLARY** Prove the Corollary to the Triangle Sum Theorem (Corollary 5.1).

Given $\triangle ABC$ is a right triangle.

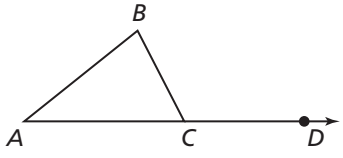
Prove $\angle A$ and $\angle B$ are complementary.



42. **PROVING A THEOREM** Prove the Exterior Angle Theorem (Theorem 5.2).

Given $\triangle ABC$, exterior $\angle BCD$

Prove $m\angle A + m\angle B = m\angle BCD$



43. **CRITICAL THINKING** Is it possible to draw an obtuse isosceles triangle? obtuse equilateral triangle? If so, provide examples. If not, explain why it is not possible.

44. **CRITICAL THINKING** Is it possible to draw a right isosceles triangle? right equilateral triangle? If so, provide an example. If not, explain why it is not possible.

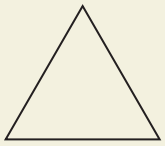
45. **MATHEMATICAL CONNECTIONS** $\triangle ABC$ is isosceles, $AB = x$, and $BC = 2x - 4$.

a. Find two possible values for x when the perimeter of $\triangle ABC$ is 32.

b. How many possible values are there for x when the perimeter of $\triangle ABC$ is 12?

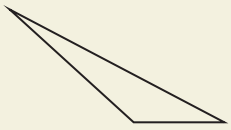
46. **HOW DO YOU SEE IT?** In as many ways as possible, classify each triangle by its appearance.

a.

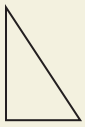


b.

c.



d.



47. **ANALYZING RELATIONSHIPS** Which of the following could represent the measures of an exterior angle and two interior angles of a triangle? Select all that apply.

Ⓐ 100°, 62°, 38° Ⓑ 81°, 57°, 24°

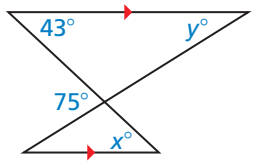
Ⓒ 119°, 68°, 49° Ⓓ 95°, 85°, 28°

Ⓔ 92°, 78°, 68° Ⓕ 149°, 101°, 48°

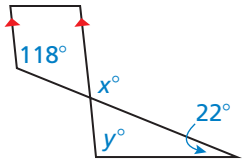
48. **MAKING AN ARGUMENT** Your friend claims the measure of an exterior angle will always be greater than the sum of the nonadjacent interior angle measures. Is your friend correct? Explain your reasoning.

MATHEMATICAL CONNECTIONS **In Exercises 49–52, find the values of x and y.**

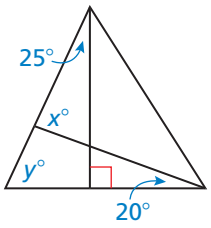
49.



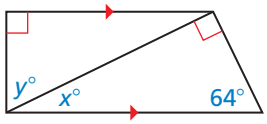
50.



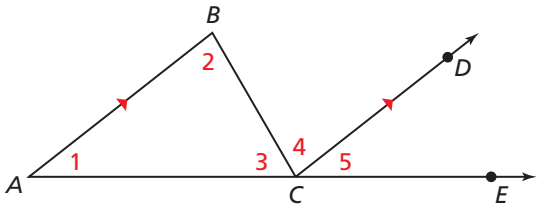
51.



52.



53. **PROVING A THEOREM** Use the diagram to write a proof of the Triangle Sum Theorem (Theorem 5.1). Your proof should be different from the proof of the Triangle Sum Theorem shown in this lesson.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

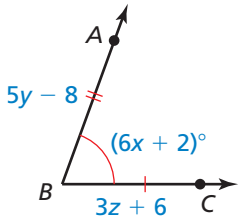
Use the diagram to find the measure of the segment or angle. *(Section 1.2 and Section 1.5)*

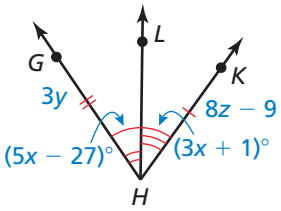
54. $m\angle KHL$

55. $m\angle ABC$

56. GH

57. BC



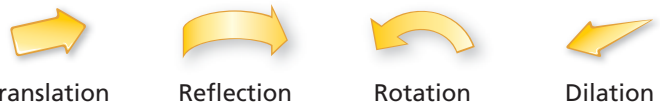


5.2 Congruent Polygons

Essential Question Given two congruent triangles, how can you use rigid motions to map one triangle to the other triangle?

EXPLORATION 1 Describing Rigid Motions

Work with a partner. Of the four transformations you studied in Chapter 4, which are rigid motions? Under a rigid motion, why is the image of a triangle always congruent to the original triangle? Explain your reasoning.



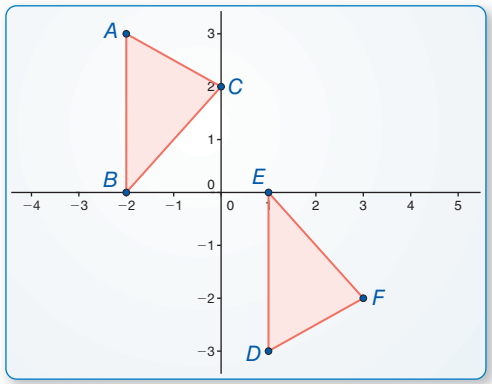
LOOKING FOR STRUCTURE

To be proficient in math, you need to look closely to discern a pattern or structure.

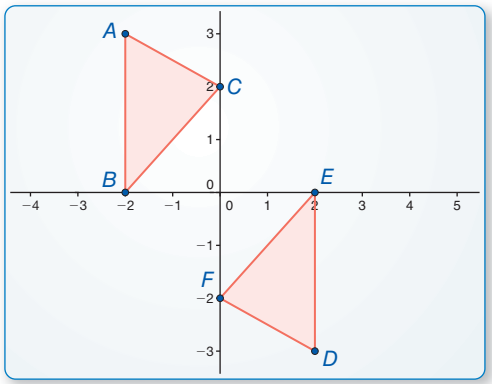
EXPLORATION 2 Finding a Composition of Rigid Motions

Work with a partner. Describe a composition of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Use dynamic geometry software to verify your answer.

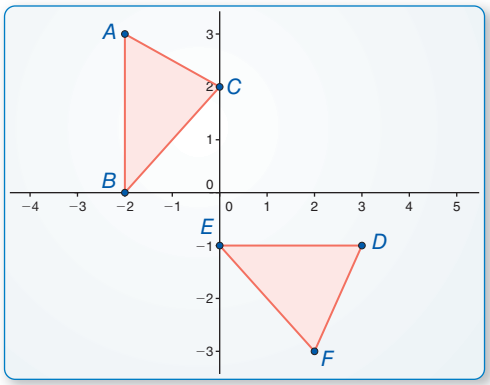
a. $\triangle ABC \cong \triangle DEF$



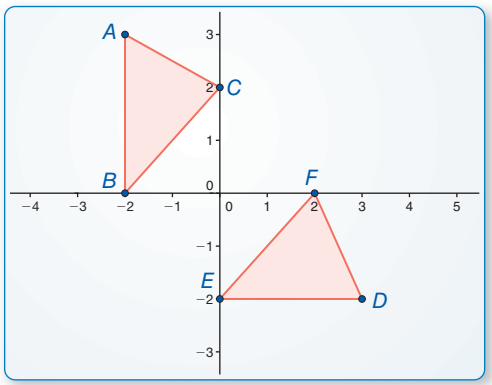
b. $\triangle ABC \cong \triangle DEF$



c. $\triangle ABC \cong \triangle DEF$



d. $\triangle ABC \cong \triangle DEF$



Communicate Your Answer

3. Given two congruent triangles, how can you use rigid motions to map one triangle to the other triangle?

4. The vertices of $\triangle ABC$ are $A(1, 1)$, $B(3, 2)$, and $C(4, 4)$. The vertices of $\triangle DEF$ are $D(2, -1)$, $E(0, 0)$, and $F(-1, 2)$. Describe a composition of rigid motions that maps $\triangle ABC$ to $\triangle DEF$.

5.2 Lesson

Core Vocabulary

corresponding parts, *p. 240*

Previous

congruent figures

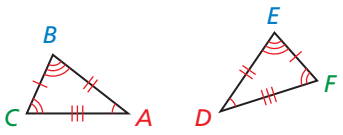
What You Will Learn

- Identify and use corresponding parts.
- Use the Third Angles Theorem.

Identifying and Using Corresponding Parts

Recall that two geometric figures are congruent if and only if a rigid motion or a composition of rigid motions maps one of the figures onto the other. A rigid motion maps each part of a figure to a **corresponding part** of its image. Because rigid motions preserve length and angle measure, corresponding parts of congruent figures are congruent. In congruent polygons, this means that the *corresponding sides* and the *corresponding angles* are congruent.

When $\triangle DEF$ is the image of $\triangle ABC$ after a rigid motion or a composition of rigid motions, you can write congruence statements for the corresponding angles and corresponding sides.



Corresponding angles

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

Corresponding sides

$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$

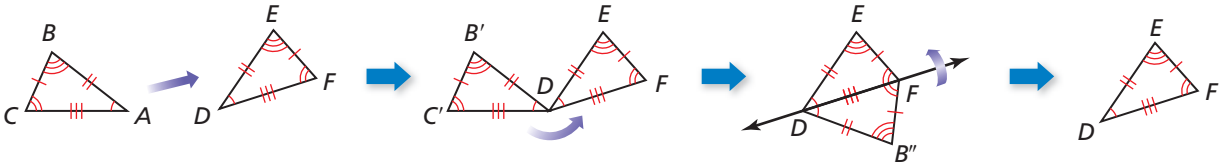
STUDY TIP

Notice that both of the following statements are true.

1. If two triangles are congruent, then all their corresponding parts are congruent.
2. If all the corresponding parts of two triangles are congruent, then the triangles are congruent.

When you write a congruence statement for two polygons, always list the corresponding vertices in the same order. You can write congruence statements in more than one way. Two possible congruence statements for the triangles above are $\triangle ABC \cong \triangle DEF$ or $\triangle BCA \cong \triangle EFD$.

When all the corresponding parts of two triangles are congruent, you can show that the triangles are congruent. Using the triangles above, first translate $\triangle ABC$ so that point A maps to point D. This translation maps $\triangle ABC$ to $\triangle DB'C'$. Next, rotate $\triangle DB'C'$ counterclockwise through $\angle C'DF$ so that the image of $\overrightarrow{DC'}$ coincides with $\overrightarrow{DF}$. Because $\overline{DC'} \cong \overline{DF}$, the rotation maps point C' to point F. So, this rotation maps $\triangle DB'C'$ to $\triangle DB''F$.

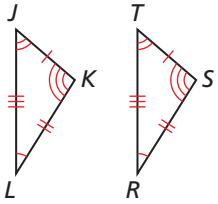


Now, reflect $\triangle DB''F$ in the line through points D and F. This reflection maps the sides and angles of $\triangle DB''F$ to the corresponding sides and corresponding angles of $\triangle DEF$, so $\triangle ABC \cong \triangle DEF$.

So, to show that two triangles are congruent, it is sufficient to show that their corresponding parts are congruent. In general, this is true for all polygons.

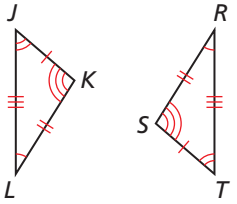
VISUAL REASONING

To help you identify corresponding parts, rotate $\triangle TSR$.



EXAMPLE 1 Identifying Corresponding Parts

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.



SOLUTION

The diagram indicates that $\triangle JKL \cong \triangle TSR$.

Corresponding angles $\angle J \cong \angle T$, $\angle K \cong \angle S$, $\angle L \cong \angle R$

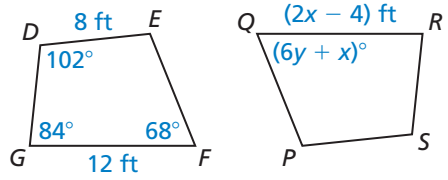
Corresponding sides $\overline{JK} \cong \overline{TS}$, $\overline{KL} \cong \overline{SR}$, $\overline{LJ} \cong \overline{RT}$

EXAMPLE 2 Using Properties of Congruent Figures

In the diagram, $DEFG \cong SPQR$.

a. Find the value of x.

b. Find the value of y.



SOLUTION

a. You know that $\overline{FG} \cong \overline{QR}$.

$FG = QR$

$12 = 2x - 4$

$16 = 2x$

$8 = x$

b. You know that $\angle F \cong \angle Q$.

$m\angle F = m\angle Q$

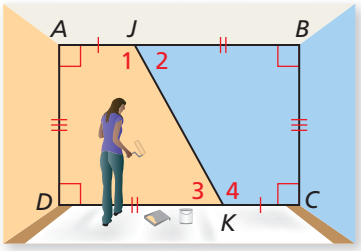
$68° = (6y + x)°$

$68 = 6y + 8$

$10 = y$

EXAMPLE 3 Showing That Figures Are Congruent

You divide the wall into orange and blue sections along $\overline{JK}$. Will the sections of the wall be the same size and shape? Explain.



SOLUTION

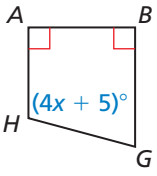
From the diagram, $\angle A \cong \angle C$ and $\angle D \cong \angle B$ because all right angles are congruent. Also, by the Lines Perpendicular to a Transversal Theorem (Thm. 3.12), $\overline{AB} \parallel \overline{DC}$. Then $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$ by the Alternate Interior Angles Theorem (Thm. 3.2). So, all pairs of corresponding angles are congruent. The diagram shows $\overline{AJ} \cong \overline{CK}$, $\overline{KD} \cong \overline{JB}$, and $\overline{DA} \cong \overline{BC}$. By the Reflexive Property of Congruence (Thm. 2.1), $\overline{JK} \cong \overline{KJ}$. So, all pairs of corresponding sides are congruent. Because all corresponding parts are congruent, $AJKD \cong CKJB$.

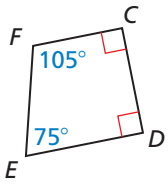
▶ Yes, the two sections will be the same size and shape.

Monitoring Progress

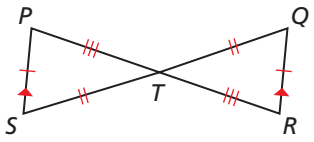
Help in English and Spanish at *BigIdeasMath.com*

In the diagram, $ABGH \cong CDEF$.





1. Identify all pairs of congruent corresponding parts.
2. Find the value of x.
3. In the diagram at the left, show that $\triangle PTS \cong \triangle RTQ$.



Theorem

Theorem 5.3 Properties of Triangle Congruence

Triangle congruence is reflexive, symmetric, and transitive.

Reflexive For any triangle $\triangle ABC$, $\triangle ABC \cong \triangle ABC$.

Symmetric If $\triangle ABC \cong \triangle DEF$, then $\triangle DEF \cong \triangle ABC$.

Transitive If $\triangle ABC \cong \triangle DEF$ and $\triangle DEF \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

Proof *BigIdeasMath.com*

STUDY TIP

The properties of congruence that are true for segments and angles are also true for triangles.

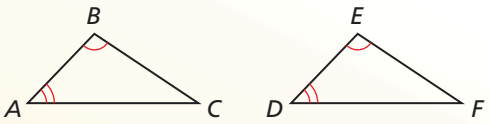
Using the Third Angles Theorem

Theorem

Theorem 5.4 Third Angles Theorem

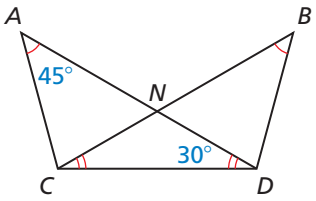
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

Proof Ex. 19, p. 244



If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\angle C \cong \angle F$.

EXAMPLE 4 Using the Third Angles Theorem



Find $m\angle BDC$.

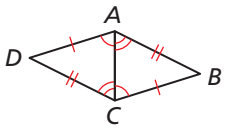
SOLUTION

$\angle A \cong \angle B$ and $\angle ADC \cong \angle BCD$, so by the Third Angles Theorem, $\angle ACD \cong \angle BDC$. By the Triangle Sum Theorem (Theorem 5.1), $m\angle ACD = 180° - 45° - 30° = 105°$.

▶ So, $m\angle BDC = m\angle ACD = 105°$ by the definition of congruent angles.

EXAMPLE 5 Proving That Triangles Are Congruent

Use the information in the figure to prove that $\triangle ACD \cong \triangle CAB$.



SOLUTION

Given $\overline{AD} \cong \overline{CB}, \overline{DC} \cong \overline{BA}, \angle ACD \cong \angle CAB, \angle CAD \cong \angle ACB$

Prove $\triangle ACD \cong \triangle CAB$

Plan for Proof

a. Use the Reflexive Property of Congruence (Thm. 2.1) to show that $\overline{AC} \cong \overline{CA}$.

b. Use the Third Angles Theorem to show that $\angle B \cong \angle D$.

Plan in Action

	STATEMENTS	REASONS
	1. $\overline{AD} \cong \overline{CB}, \overline{DC} \cong \overline{BA}$	**1.** Given
a.	**2.** $\overline{AC} \cong \overline{CA}$	**2.** Reflexive Property of Congruence (Theorem 2.1)
	3. $\angle ACD \cong \angle CAB$, $\angle CAD \cong \angle ACB$	**3.** Given
b.	**4.** $\angle B \cong \angle D$	**4.** Third Angles Theorem
	5. $\triangle ACD \cong \triangle CAB$	**5.** All corresponding parts are congruent.

Monitoring Progress

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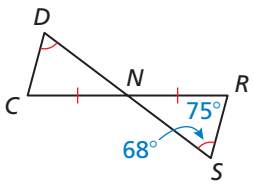


Use the diagram.

4. Find $m\angle DCN$.

5. What additional information is needed to conclude that $\triangle NDC \cong \triangle NSR$?

5.2 Exercises

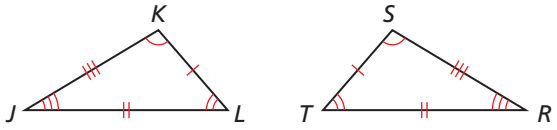
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Based on this lesson, what information do you need to prove that two triangles are congruent? Explain your reasoning.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

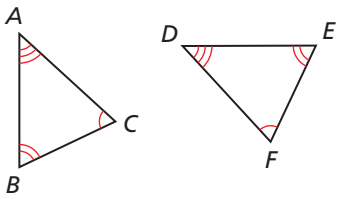
Is $\triangle JKL \cong \triangle RST$?	Is $\triangle KJL \cong \triangle SRT$?
Is $\triangle JLK \cong \triangle STR$?	Is $\triangle LKJ \cong \triangle TSR$?



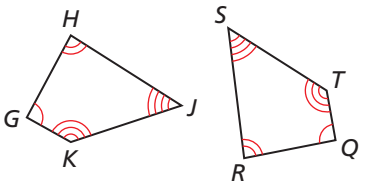
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, identify all pairs of congruent corresponding parts. Then write another congruence statement for the polygons. *(See Example 1.)*

3. $\triangle ABC \cong \triangle DEF$



4. $GHJK \cong QRST$



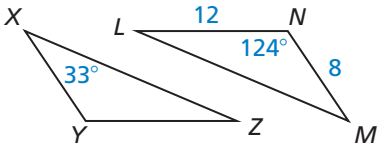
In Exercises 5–8, $\triangle XYZ \cong \triangle MNL$. Copy and complete the statement.

5. $m\angle Y =$ _____

6. $m\angle M =$ _____

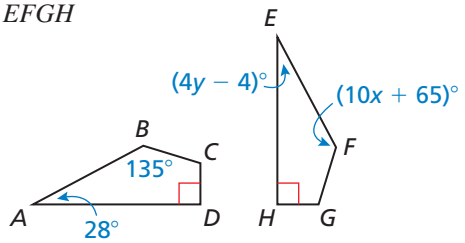
7. $m\angle Z =$ _____

8. $XY =$ _____

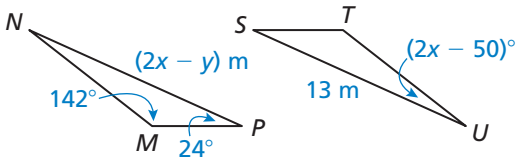


In Exercises 9 and 10, find the values of x and y. *(See Example 2.)*

9. $ABCD \cong EFGH$

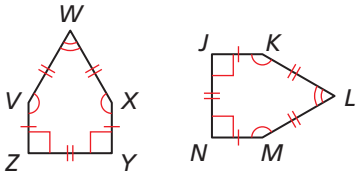


10. $\triangle MNP \cong \triangle TUS$

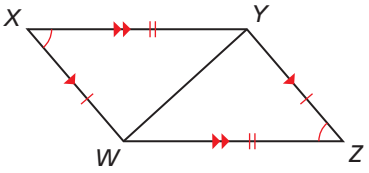


In Exercises 11 and 12, show that the polygons are congruent. Explain your reasoning. *(See Example 3.)*

11.

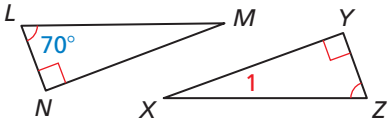


12.

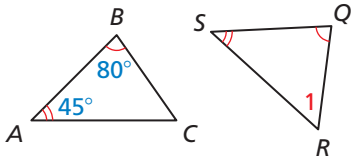


In Exercises 13 and 14, find $m\angle 1$. *(See Example 4.)*

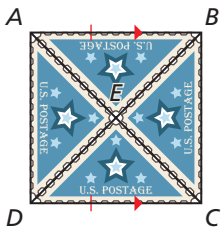
13.



14.



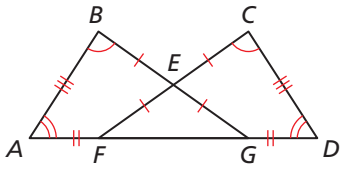
15. PROOF Triangular postage stamps, like the ones shown, are highly valued by stamp collectors. Prove that $\triangle AEB \cong \triangle CED$. *(See Example 5.)*



Given $\overline{AB} \parallel \overline{DC}, \overline{AB} \cong \overline{DC}$, E is the midpoint of $\overline{AC}$ and $\overline{BD}$.

Prove $\triangle AEB \cong \triangle CED$

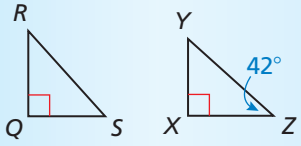
16. PROOF Use the information in the figure to prove that $\triangle ABG \cong \triangle DCF$.



ERROR ANALYSIS **In Exercises 17 and 18, describe and correct the error.**

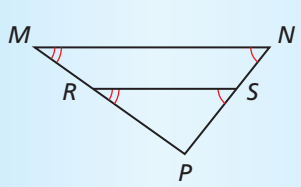
17.

Given $\triangle QRS \cong \triangle XZY$



$\angle S \cong \angle Z$
$m\angle S = m\angle Z$
$m\angle S = 42°$

18.



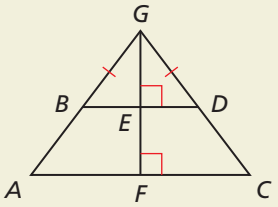
$\triangle MNP \cong \triangle RSP$ because the corresponding angles are congruent.

19. PROVING A THEOREM Prove the Third Angles Theorem (Theorem 5.4) by using the Triangle Sum Theorem (Theorem 5.1).

20. THOUGHT PROVOKING Draw a triangle. Copy the triangle multiple times to create a rug design made of congruent triangles. Which property guarantees that all the triangles are congruent?

21. REASONING $\triangle JKL$ is congruent to $\triangle XYZ$. Identify all pairs of congruent corresponding parts.

22. HOW DO YOU SEE IT? In the diagram, $ABEF \cong CDEF$.



a. Explain how you know that $\overline{BE} \cong \overline{DE}$ and $\angle ABE \cong \angle CDE$.

b. Explain how you know that $\angle GBE \cong \angle GDE$.

c. Explain how you know that $\angle GEB \cong \angle GED$.

d. Do you have enough information to prove that $\triangle BEG \cong \triangle DEG$? Explain.

MATHEMATICAL CONNECTIONS **In Exercises 23 and 24, use the given information to write and solve a system of linear equations to find the values of x and y.**

23. $\triangle LMN \cong \triangle PQR$, $m\angle L = 40°$, $m\angle M = 90°$, $m\angle P = (17x - y)°$, $m\angle R = (2x + 4y)°$

24. $\triangle STU \cong \triangle XYZ$, $m\angle T = 28°$, $m\angle U = (4x + y)°$, $m\angle X = 130°$, $m\angle Y = (8x - 6y)°$

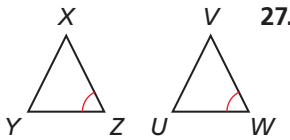
25. PROOF Prove that the criteria for congruent triangles in this lesson is equivalent to the definition of congruence in terms of rigid motions.

Maintaining Mathematical Proficiency

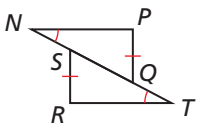
Reviewing what you learned in previous grades and lessons

What can you conclude from the diagram? *(Section 1.6)*

26.



27.



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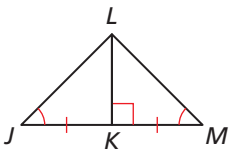

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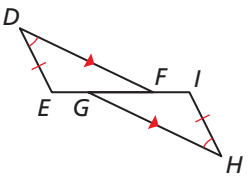


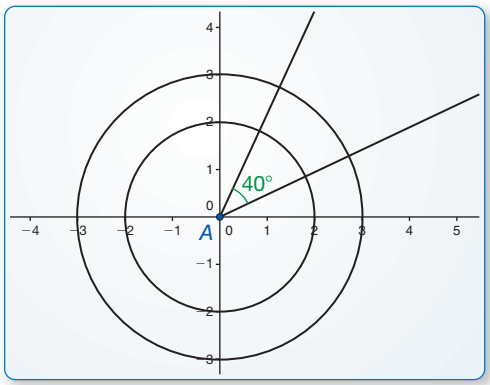
5.3 Proving Triangle Congruence by SAS

Essential Question What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

EXPLORATION 1 Drawing Triangles

Work with a partner. Use dynamic geometry software.

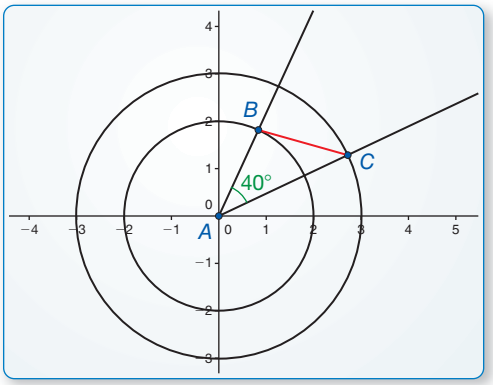
a. Construct circles with radii of 2 units and 3 units centered at the origin. Construct a 40° angle with its vertex at the origin. Label the vertex A.



b. Locate the point where one ray of the angle intersects the smaller circle and label this point B. Locate the point where the other ray of the angle intersects the larger circle and label this point C. Then draw $\triangle ABC$.

c. Find BC, $m\angle B$, and $m\angle C$.

d. Repeat parts (a)–(c) several times, redrawing the angle in different positions. Keep track of your results by copying and completing the table below. What can you conclude?



USING TOOLS STRATEGICALLY

To be proficient in math, you need to use technology to help visualize the results of varying assumptions, explore consequences, and compare predictions with data.

	A	B	C	AB	AC	BC	$m\angle A$	$m\angle B$	$m\angle C$
1.	(0, 0)			2	3		40°		
2.	(0, 0)			2	3		40°		
3.	(0, 0)			2	3		40°		
4.	(0, 0)			2	3		40°		
5.	(0, 0)			2	3		40°		

Communicate Your Answer

2. What can you conclude about two triangles when you know that two pairs of corresponding sides and the corresponding included angles are congruent?

3. How would you prove your conclusion in Exploration 1(d)?

5.3 Lesson

What You Will Learn

- Use the Side-Angle-Side (SAS) Congruence Theorem.
- Solve real-life problems.

Core Vocabulary

Previous
congruent figures
rigid motion

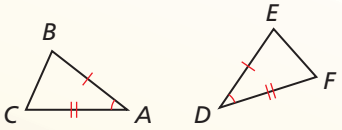
Using the Side-Angle-Side Congruence Theorem

Theorem

Theorem 5.5 Side-Angle-Side (SAS) Congruence Theorem

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.



Proof p. 246

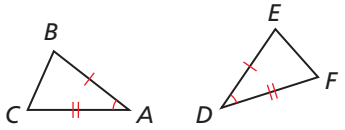
STUDY TIP

The *included angle* of two sides of a triangle is the angle formed by the two sides.

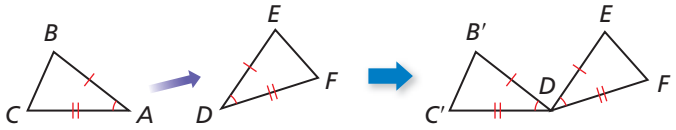
PROOF Side-Angle-Side (SAS) Congruence Theorem

Given $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$

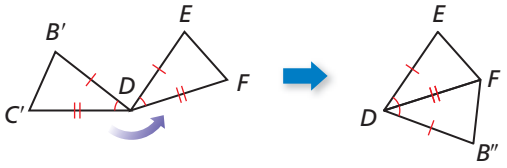
Prove $\triangle ABC \cong \triangle DEF$



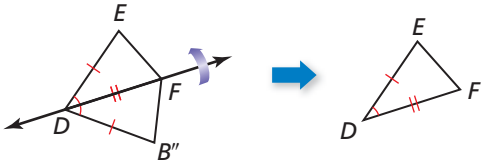
First, translate $\triangle ABC$ so that point A maps to point D, as shown below.



This translation maps $\triangle ABC$ to $\triangle DB'C'$. Next, rotate $\triangle DB'C'$ counterclockwise through $\angle C'DF$ so that the image of $\overrightarrow{DC'}$ coincides with $\overrightarrow{DF}$, as shown below.



Because $\overline{DC'} \cong \overline{DF}$, the rotation maps point C' to point F. So, this rotation maps $\triangle DB'C'$ to $\triangle DB''F$. Now, reflect $\triangle DB''F$ in the line through points D and F, as shown below.



Because points D and F lie on $\overleftrightarrow{DF}$, this reflection maps them onto themselves. Because a reflection preserves angle measure and $\angle B''DF \cong \angle EDF$, the reflection maps $\overrightarrow{DB''}$ to $\overrightarrow{DE}$. Because $\overline{DB''} \cong \overline{DE}$, the reflection maps point B'' to point E. So, this reflection maps $\triangle DB''F$ to $\triangle DEF$.

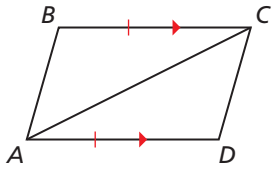
Because you can map $\triangle ABC$ to $\triangle DEF$ using a composition of rigid motions, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 1 Using the SAS Congruence Theorem

Write a proof.

Given $\overline{BC} \cong \overline{DA}, \overline{BC} \parallel \overline{AD}$

Prove $\triangle ABC \cong \triangle CDA$



STUDY TIP

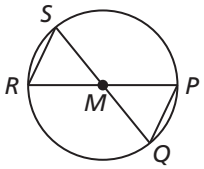
Make your proof easier to read by identifying the steps where you show congruent sides (S) and angles (A).

SOLUTION

	STATEMENTS	REASONS
S	1. $\overline{BC} \cong \overline{DA}$	1. Given
	2. $\overline{BC} \parallel \overline{AD}$	2. Given
A	3. $\angle BCA \cong \angle DAC$	3. Alternate Interior Angles Theorem (Thm. 3.2)
S	4. $\overline{AC} \cong \overline{CA}$	4. Reflexive Property of Congruence (Thm. 2.1)
	5. $\triangle ABC \cong \triangle CDA$	5. SAS Congruence Theorem

EXAMPLE 2 Using SAS and Properties of Shapes

In the diagram, $\overline{QS}$ and $\overline{RP}$ pass through the center M of the circle. What can you conclude about $\triangle MRS$ and $\triangle MPQ$?



SOLUTION

Because they are vertical angles, $\angle PMQ \cong \angle RMS$. All points on a circle are the same distance from the center, so $\overline{MP}, \overline{MQ}, \overline{MR}$, and $\overline{MS}$ are all congruent.

▶ So, $\triangle MRS$ and $\triangle MPQ$ are congruent by the SAS Congruence Theorem.

Monitoring Progress

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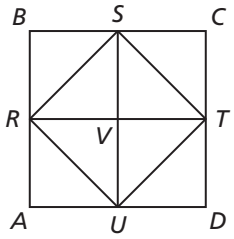
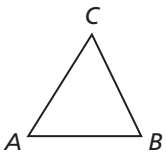
In the diagram, $ABCD$ is a square with four congruent sides and four right angles. R, S, T, and U are the midpoints of the sides of $ABCD$. Also, $\overline{RT} \perp \overline{SU}$ and $\overline{SV} \cong \overline{VU}$.



1. Prove that $\triangle SVR \cong \triangle UVR$.
2. Prove that $\triangle BSR \cong \triangle DUT$.

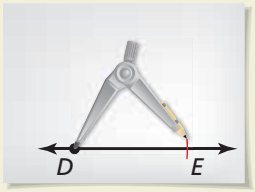
CONSTRUCTION Copying a Triangle Using SAS

Construct a triangle that is congruent to $\triangle ABC$ using the SAS Congruence Theorem. Use a compass and straightedge.



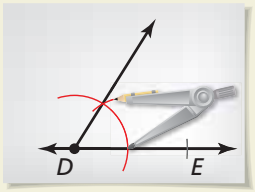
SOLUTION

Step 1



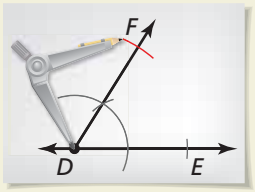
Construct a side
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

Step 2



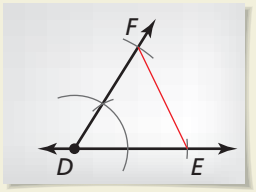
Construct an angle
Construct $\angle D$ with vertex D and side $\overrightarrow{DE}$ so that it is congruent to $\angle A$.

Step 3



Construct a side
Construct $\overline{DF}$ so that it is congruent to $\overline{AC}$.

Step 4

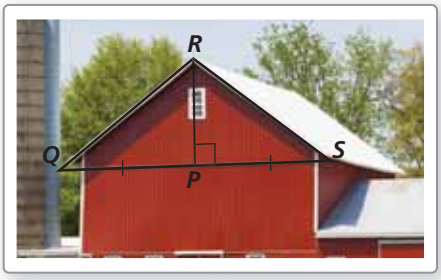


Draw a triangle
Draw $\triangle DEF$. By the SAS Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

Solving Real-Life Problems

EXAMPLE 3 Solving a Real-Life Problem

You are making a canvas sign to hang on the triangular portion of the barn wall shown in the picture. You think you can use two identical triangular sheets of canvas. You know that $\overline{RP} \perp \overline{QS}$ and $\overline{PQ} \cong \overline{PS}$. Use the SAS Congruence Theorem to show that $\triangle PQR \cong \triangle PSR$.



SOLUTION

You are given that $\overline{PQ} \cong \overline{PS}$. By the Reflexive Property of Congruence (Theorem 2.1), $\overline{RP} \cong \overline{RP}$. By the definition of perpendicular lines, both $\angle RPQ$ and $\angle RPS$ are right angles, so they are congruent. So, two pairs of sides and their included angles are congruent.

▶ $\triangle PQR$ and $\triangle PSR$ are congruent by the SAS Congruence Theorem.

Monitoring Progress

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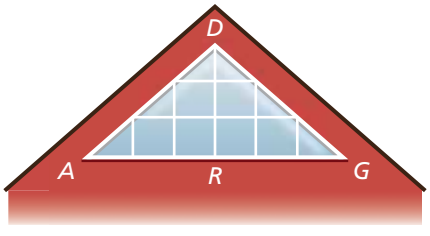
3. You are designing the window shown in the photo. You want to make $\triangle DRA$ congruent to $\triangle DRG$. You design the window so that $\overline{DA} \cong \overline{DG}$ and $\angle ADR \cong \angle GDR$. Use the SAS Congruence Theorem to prove $\triangle DRA \cong \triangle DRG$.



5.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

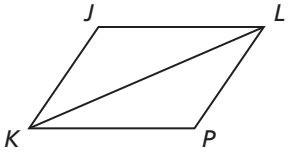
Vocabulary and Core Concept Check

1. **WRITING** What is an included angle?

2. **COMPLETE THE SENTENCE** If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then _____.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, name the included angle between the pair of sides given.



3. $\overline{JK}$ and $\overline{KL}$

4. $\overline{PK}$ and $\overline{LK}$

5. $\overline{LP}$ and $\overline{LK}$

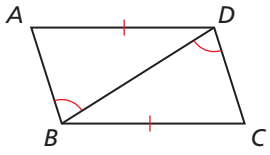
6. $\overline{JL}$ and $\overline{JK}$

7. $\overline{KL}$ and $\overline{JL}$

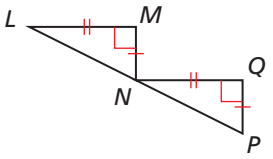
8. $\overline{KP}$ and $\overline{PL}$

In Exercises 9–14, decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Theorem (Theorem 5.5). Explain.

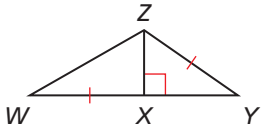
9. $\triangle ABD$, $\triangle CDB$



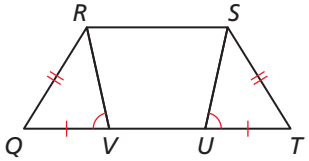
10. $\triangle LMN$, $\triangle NQP$



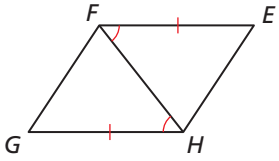
11. $\triangle YXZ$, $\triangle WXZ$



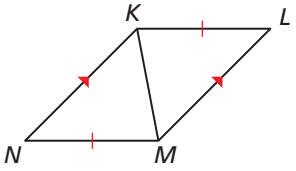
12. $\triangle QRV$, $\triangle TSU$



13. $\triangle EFH$, $\triangle GHF$

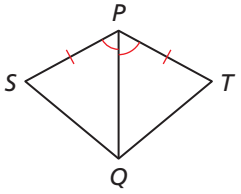


14. $\triangle KLM$, $\triangle MNK$

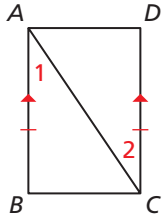


In Exercises 15–18, write a proof. *(See Example 1.)*

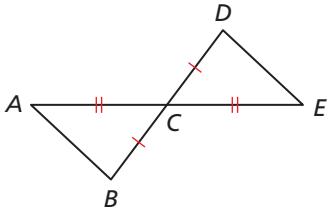
15. **Given** $\overline{PQ}$ bisects $\angle SPT$, $\overline{SP} \cong \overline{TP}$
Prove $\triangle SPQ \cong \triangle TPQ$



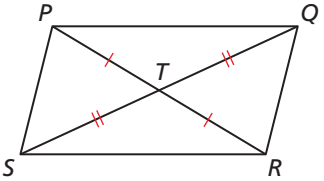
16. **Given** $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$
Prove $\triangle ABC \cong \triangle CDA$



17. **Given** C is the midpoint of $\overline{AE}$ and $\overline{BD}$.
Prove $\triangle ABC \cong \triangle EDC$

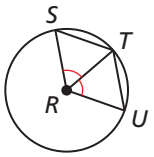


18. **Given** $\overline{PT} \cong \overline{RT}$, $\overline{QT} \cong \overline{ST}$
Prove $\triangle PQT \cong \triangle RST$

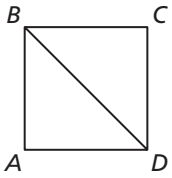


In Exercises 19–22, use the given information to name two triangles that are congruent. Explain your reasoning. *(See Example 2.)*

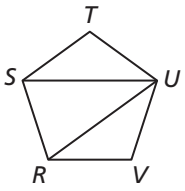
19. $\angle SRT \cong \angle URT$, and R is the center of the circle.



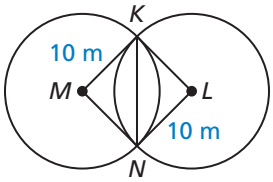
20. $ABCD$ is a square with four congruent sides and four congruent angles.



21. $RSTUV$ is a regular pentagon.

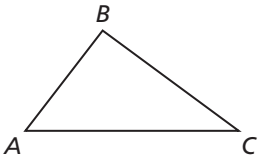


22. $\overline{MK} \perp \overline{MN}$, $\overline{KL} \perp \overline{NL}$, and M and L are centers of circles.

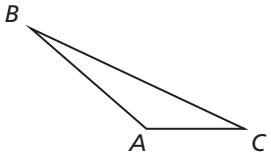


CONSTRUCTION **In Exercises 23 and 24, construct a triangle that is congruent to $\triangle ABC$ using the SAS Congruence Theorem (Theorem 5.5).**

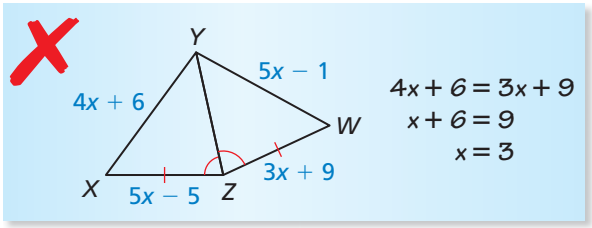
23.



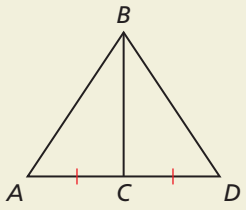
24.



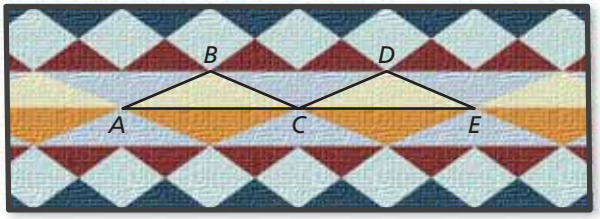
25. ERROR ANALYSIS Describe and correct the error in finding the value of x.



26. HOW DO YOU SEE IT? What additional information do you need to prove that $\triangle ABC \cong \triangle DBC$?

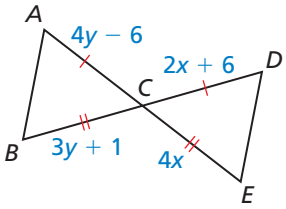


27. PROOF The Navajo rug is made of isosceles triangles. You know $\angle B \cong \angle D$. Use the SAS Congruence Theorem (Theorem 5.5) to show that $\triangle ABC \cong \triangle CDE$. *(See Example 3.)*

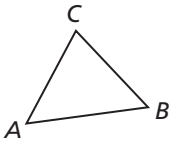


28. THOUGHT PROVOKING There are six possible subsets of three sides or angles of a triangle: SSS, SAS, SSA, AAA, ASA, and AAS. Which of these correspond to congruence theorems? For those that do not, give a counterexample.

29. MATHEMATICAL CONNECTIONS Prove that $\triangle ABC \cong \triangle DEC$. Then find the values of x and y.



30. MAKING AN ARGUMENT Your friend claims it is possible to construct a triangle congruent to $\triangle ABC$ by first constructing $\overline{AB}$ and $\overline{AC}$, and then copying $\angle C$. Is your friend correct? Explain your reasoning.



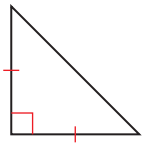
31. PROVING A THEOREM Prove the Reflections in Intersecting Lines Theorem (Theorem 4.3).

Maintaining Mathematical Proficiency

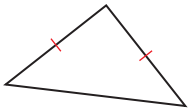
Reviewing what you learned in previous grades and lessons

Classify the triangle by its sides and by measuring its angles. *(Section 5.1)*

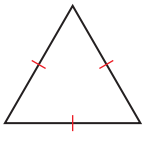
32.



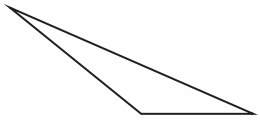
33.



34.



35.



5.4 Equilateral and Isosceles Triangles

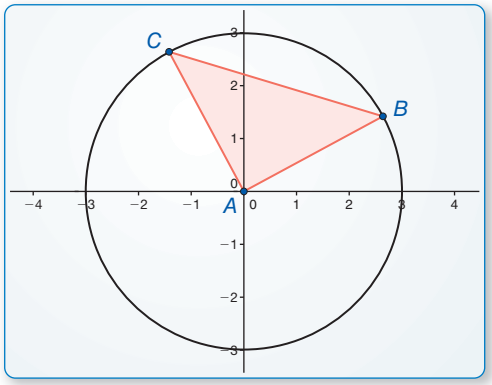
Essential Question What conjectures can you make about the side lengths and angle measures of an isosceles triangle?

EXPLORATION 1 Writing a Conjecture about Isosceles Triangles

Work with a partner. Use dynamic geometry software.

a. Construct a circle with a radius of 3 units centered at the origin.

b. Construct $\triangle ABC$ so that B and C are on the circle and A is at the origin.



Sample

Points
$A(0, 0)$
$B(2.64, 1.42)$
$C(-1.42, 2.64)$
Segments
$AB = 3$
$AC = 3$
$BC = 4.24$
Angles
$m\angle A = 90°$
$m\angle B = 45°$
$m\angle C = 45°$

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to make conjectures and build a logical progression of statements to explore the truth of your conjectures.

c. Recall that a triangle is *isosceles* if it has at least two congruent sides. Explain why $\triangle ABC$ is an isosceles triangle.

d. What do you observe about the angles of $\triangle ABC$?

e. Repeat parts (a)–(d) with several other isosceles triangles using circles of different radii. Keep track of your observations by copying and completing the table below. Then write a conjecture about the angle measures of an isosceles triangle.

Sample

	A	B	C	AB	AC	BC	$m\angle A$	$m\angle B$	$m\angle C$
1.	(0, 0)	(2.64, 1.42)	(−1.42, 2.64)	3	3	4.24	90°	45°	45°
2.	(0, 0)								
3.	(0, 0)								
4.	(0, 0)								
5.	(0, 0)								

f. Write the converse of the conjecture you wrote in part (e). Is the converse true?

Communicate Your Answer

2. What conjectures can you make about the side lengths and angle measures of an isosceles triangle?

3. How would you prove your conclusion in Exploration 1(e)? in Exploration 1(f)?

5.4 Lesson

Core Vocabulary

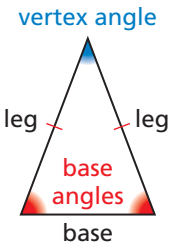
legs, *p. 252*
vertex angle, *p. 252*
base, *p. 252*
base angles, *p. 252*

What You Will Learn

- Use the Base Angles Theorem.
- Use isosceles and equilateral triangles.

Using the Base Angles Theorem

A triangle is isosceles when it has at least two congruent sides. When an isosceles triangle has exactly two congruent sides, these two sides are the **legs**. The angle formed by the legs is the **vertex angle**. The third side is the **base** of the isosceles triangle. The two angles adjacent to the base are called **base angles**.



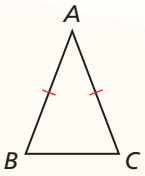
Theorems

Theorem 5.6 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

Proof p. 252

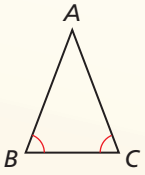


Theorem 5.7 Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

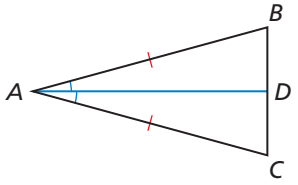
Proof Ex. 27, p. 275



PROOF Base Angles Theorem

Given $\overline{AB} \cong \overline{AC}$

Prove $\angle B \cong \angle C$



Plan for Proof

a. Draw $\overline{AD}$ so that it bisects $\angle CAB$.

b. Use the SAS Congruence Theorem to show that $\triangle ADB \cong \triangle ADC$.

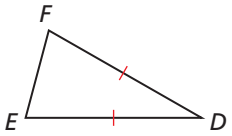
c. Use properties of congruent triangles to show that $\angle B \cong \angle C$.

Plan in Action

	STATEMENTS	REASONS
a.	**1.** Draw $\overline{AD}$, the angle bisector of $\angle CAB$.	**1.** Construction of angle bisector
	2. $\angle CAD \cong \angle BAD$	**2.** Definition of angle bisector
	3. $\overline{AB} \cong \overline{AC}$	**3.** Given
	4. $\overline{DA} \cong \overline{DA}$	**4.** Reflexive Property of Congruence (Thm. 2.1)
b.	**5.** $\triangle ADB \cong \triangle ADC$	**5.** SAS Congruence Theorem (Thm. 5.5)
c.	**6.** $\angle B \cong \angle C$	**6.** Corresponding parts of congruent triangles are congruent.

EXAMPLE 1 Using the Base Angles Theorem

In $\triangle DEF$, $\overline{DE} \cong \overline{DF}$. Name two congruent angles.



SOLUTION

▶ $\overline{DE} \cong \overline{DF}$, so by the Base Angles Theorem, $\angle E \cong \angle F$.

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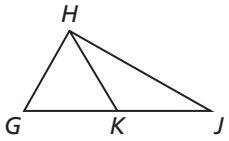
Copy and complete the statement.

1. If $\overline{HG} \cong \overline{HK}$, then $\angle$____ $\cong \angle$____.
2. If $\angle KHJ \cong \angle KJH$, then ____ $\cong$ ____.



Recall that an equilateral triangle has three congruent sides.

Corollaries

READING

The corollaries state that a triangle is *equilateral* if and only if it is *equiangular*.

Corollary 5.2 Corollary to the Base Angles Theorem

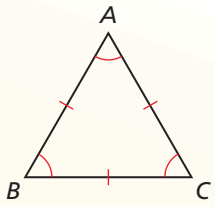
If a triangle is equilateral, then it is equiangular.

Proof Ex. 37, p. 258; Ex. 10, p. 353

Corollary 5.3 Corollary to the Converse of the Base Angles Theorem

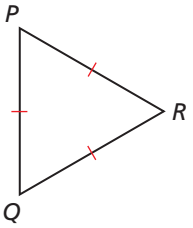
If a triangle is equiangular, then it is equilateral.

Proof Ex. 39, p. 258



EXAMPLE 2 Finding Measures in a Triangle

Find the measures of $\angle P$, $\angle Q$, and $\angle R$.



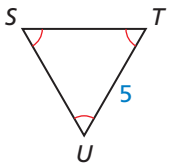
SOLUTION

The diagram shows that $\triangle PQR$ is equilateral. So, by the Corollary to the Base Angles Theorem, $\triangle PQR$ is equiangular. So, $m\angle P = m\angle Q = m\angle R$.

$3(m\angle P) = 180^\circ$ Triangle Sum Theorem (Theorem 5.1)

$m\angle P = 60^\circ$ Divide each side by 3.

▶ The measures of $\angle P$, $\angle Q$, and $\angle R$ are all 60°.



Monitoring Progress

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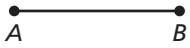
3. Find the length of $\overline{ST}$ for the triangle at the left.

Using Isosceles and Equilateral Triangles

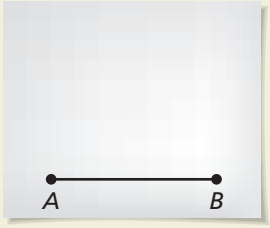
CONSTRUCTION Constructing an Equilateral Triangle

Construct an equilateral triangle that has side lengths congruent to $\overline{AB}$. Use a compass and straightedge.



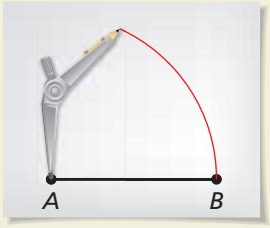
SOLUTION

Step 1



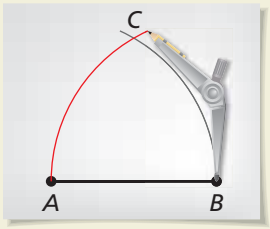
Copy a segment Copy $\overline{AB}$.

Step 2



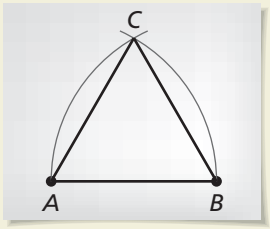
Draw an arc Draw an arc with center A and radius AB.

Step 3



Draw an arc Draw an arc with center B and radius AB. Label the intersection of the arcs from Steps 2 and 3 as C.

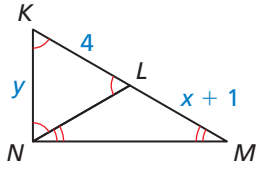
Step 4



Draw a triangle Draw $\triangle ABC$. Because $\overline{AB}$ and $\overline{AC}$ are radii of the same circle, $\overline{AB} \cong \overline{AC}$. Because $\overline{AB}$ and $\overline{BC}$ are radii of the same circle, $\overline{AB} \cong \overline{BC}$. By the Transitive Property of Congruence (Theorem 2.1), $\overline{AC} \cong \overline{BC}$. So, $\triangle ABC$ is equilateral.

EXAMPLE 3 Using Isosceles and Equilateral Triangles

Find the values of x and y in the diagram.



COMMON ERROR

You cannot use N to refer to $\angle LNM$ because three angles have N as their vertex.

SOLUTION

Step 1 Find the value of y. Because $\triangle KLN$ is equiangular, it is also equilateral and $\overline{KN} \cong \overline{KL}$. So, $y = 4$.

Step 2 Find the value of x. Because $\angle LNM \cong \angle LMN$, $\overline{LN} \cong \overline{LM}$, and $\triangle LMN$ is isosceles. You also know that $LN = 4$ because $\triangle KLN$ is equilateral.

$LN = LM$	Definition of congruent segments
$4 = x + 1$	Substitute 4 for LN and $x + 1$ for LM.
$3 = x$	Subtract 1 from each side.

EXAMPLE 4 Solving a Multi-Step Problem

In the lifeguard tower, $\overline{PS} \cong \overline{QR}$ and $\angle QPS \cong \angle PQR$.

a. Explain how to prove that $\triangle QPS \cong \triangle PQR$.

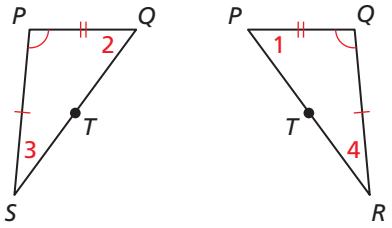
b. Explain why $\triangle PQT$ is isosceles.

COMMON ERROR

When you redraw the triangles so that they do not overlap, be careful to copy all given information and labels correctly.

SOLUTION

a. Draw and label $\triangle QPS$ and $\triangle PQR$ so that they do not overlap. You can see that $\overline{PQ} \cong \overline{QP}$, $\overline{PS} \cong \overline{QR}$, and $\angle QPS \cong \angle PQR$. So, by the SAS Congruence Theorem (Theorem 5.5), $\triangle QPS \cong \triangle PQR$.

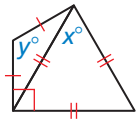


b. From part (a), you know that $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent. By the Converse of the Base Angles Theorem, $\overline{PT} \cong \overline{QT}$, and $\triangle PQT$ is isosceles.

Monitoring Progress

Help in English and Spanish at *BigIdeasMath.com*

4. Find the values of x and y in the diagram.



5. In Example 4, show that $\triangle PTS \cong \triangle QTR$.

5.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

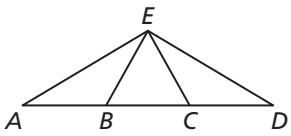
Vocabulary and Core Concept Check

1. **VOCABULARY** Describe how to identify the *vertex angle* of an isosceles triangle.

2. **WRITING** What is the relationship between the base angles of an isosceles triangle? Explain.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, copy and complete the statement. State which theorem you used. *(See Example 1.)*



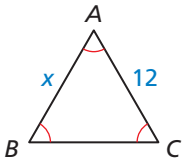
3. If $\overline{AE} \cong \overline{DE}$, then $\angle$___ $\cong \angle$___.

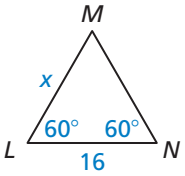
4. If $\overline{AB} \cong \overline{EB}$, then $\angle$___ $\cong \angle$___.

5. If $\angle D \cong \angle CED$, then ___ $\cong$ ___.

6. If $\angle EBC \cong \angle ECB$, then ___ $\cong$ ___.

In Exercises 7–10, find the value of x. *(See Example 2.)*

7.


8.


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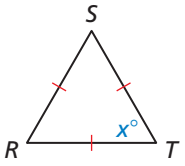
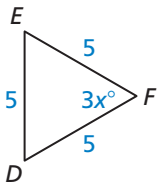
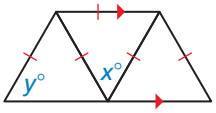


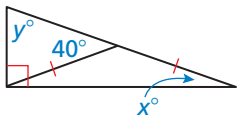
10.


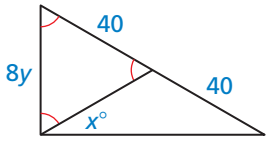
11. **MODELING WITH MATHEMATICS** The dimensions of a sports pennant are given in the diagram. Find the values of x and y.

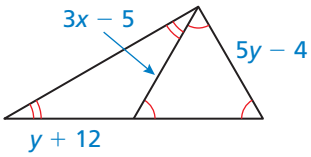
12. **MODELING WITH MATHEMATICS** A logo in an advertisement is an equilateral triangle with a side length of 7 centimeters. Sketch the logo and give the measure of each side.

In Exercises 13–16, find the values of x and y. *(See Example 3.)*

13.


14.


15.


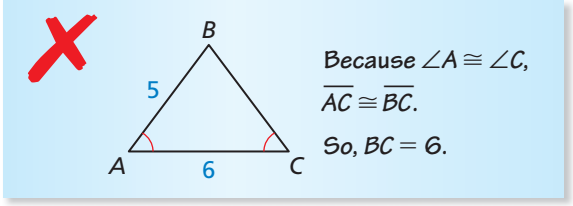
16.


CONSTRUCTION **In Exercises 17 and 18, construct an equilateral triangle whose sides are the given length.**

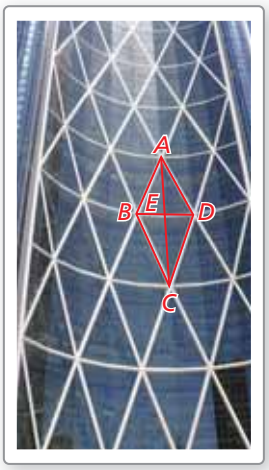
17. 3 inches

18. 1.25 inches

19. **ERROR ANALYSIS** Describe and correct the error in finding the length of $\overline{BC}$.



20. PROBLEM SOLVING The diagram represents part of the exterior of the Bow Tower in Calgary, Alberta, Canada. In the diagram, $\triangle ABD$ and $\triangle CBD$ are congruent equilateral triangles. *(See Example 4.)*



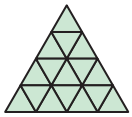
a. Explain why $\triangle ABC$ is isosceles.

b. Explain why $\angle BAE \cong \angle BCE$.

c. Show that $\triangle ABE$ and $\triangle CBE$ are congruent.

d. Find the measure of $\angle BAE$.

21. FINDING A PATTERN In the pattern shown, each small triangle is an equilateral triangle with an area of 1 square unit.

Triangle	Area
	1 square unit
	
	
	

a. Explain how you know that any triangle made out of equilateral triangles is equilateral.

b. Find the areas of the first four triangles in the pattern.

c. Describe any patterns in the areas. Predict the area of the seventh triangle in the pattern. Explain your reasoning.

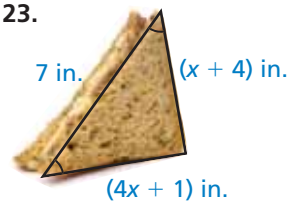
22. REASONING The base of isosceles $\triangle XYZ$ is $\overline{YZ}$. What can you prove? Select all that apply.

Ⓐ $\overline{XY} \cong \overline{XZ}$ Ⓑ $\angle X \cong \angle Y$

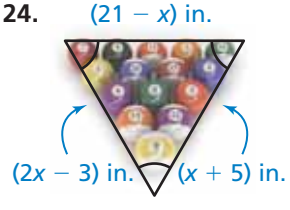
Ⓒ $\angle Y \cong \angle Z$ Ⓓ $\overline{YZ} \cong \overline{ZX}$

In Exercises 23 and 24, find the perimeter of the triangle.

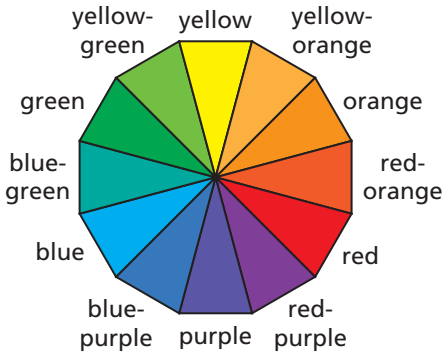
23.



24.



MODELING WITH MATHEMATICS In Exercises 25–28, use the diagram based on the color wheel. The 12 triangles in the diagram are isosceles triangles with congruent vertex angles.



25. Complementary colors lie directly opposite each other on the color wheel. Explain how you know that the yellow triangle is congruent to the purple triangle.

26. The measure of the vertex angle of the yellow triangle is 30°. Find the measures of the base angles.

27. Trace the color wheel. Then form a triangle whose vertices are the midpoints of the bases of the red, yellow, and blue triangles. (These colors are the *primary colors*.) What type of triangle is this?

28. Other triangles can be formed on the color wheel that are congruent to the triangle in Exercise 27. The colors on the vertices of these triangles are called *triads*. What are the possible triads?

29. CRITICAL THINKING Are isosceles triangles always acute triangles? Explain your reasoning.

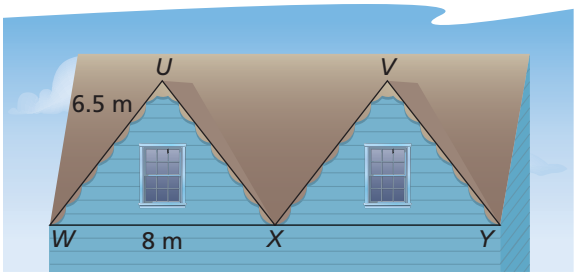
30. CRITICAL THINKING Is it possible for an equilateral triangle to have an angle measure other than 60°? Explain your reasoning.

31. MATHEMATICAL CONNECTIONS The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$. Find the values of t that make the triangle isosceles. Explain your reasoning.

32. MATHEMATICAL CONNECTIONS The measure of an exterior angle of an isosceles triangle is $x°$. Write expressions representing the possible angle measures of the triangle in terms of x.

33. WRITING Explain why the measure of the vertex angle of an isosceles triangle must be an even number of degrees when the measures of all the angles of the triangle are whole numbers.

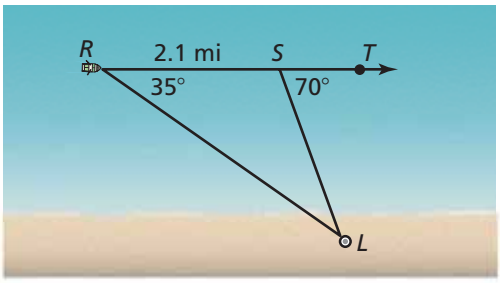
34. PROBLEM SOLVING The triangular faces of the peaks on a roof are congruent isosceles triangles with vertex angles U and V.



a. Name two angles congruent to $\angle WUX$. Explain your reasoning.

b. Find the distance between points U and V.

35. PROBLEM SOLVING A boat is traveling parallel to the shore along $\overrightarrow{RT}$. When the boat is at point R, the captain measures the angle to the lighthouse as 35°. After the boat has traveled 2.1 miles, the captain measures the angle to the lighthouse to be 70°.



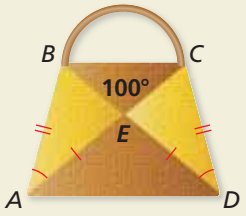
a. Find SL. Explain your reasoning.

b. Explain how to find the distance between the boat and the shoreline.

36. THOUGHT PROVOKING The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, do all equiangular triangles have the same angle measures? Justify your answer.

37. PROVING A COROLLARY Prove that the Corollary to the Base Angles Theorem (Corollary 5.2) follows from the Base Angles Theorem (Theorem 5.6).

38. HOW DO YOU SEE IT? You are designing fabric purses to sell at the school fair.



a. Explain why $\triangle ABE \cong \triangle DCE$.

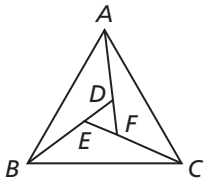
b. Name the isosceles triangles in the purse.

c. Name three angles that are congruent to $\angle EAD$.

39. PROVING A COROLLARY Prove that the Corollary to the Converse of the Base Angles Theorem (Corollary 5.3) follows from the Converse of the Base Angles Theorem (Theorem 5.7).

40. MAKING AN ARGUMENT The coordinates of two points are $T(0, 6)$ and $U(6, 0)$. Your friend claims that points T, U, and V will always be the vertices of an isosceles triangle when V is any point on the line $y = x$. Is your friend correct? Explain your reasoning.

41. PROOF Use the diagram to prove that $\triangle DEF$ is equilateral.



Given $\triangle ABC$ is equilateral.
$\angle CAD \cong \angle ABE \cong \angle BCF$

Prove $\triangle DEF$ is equilateral.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the given property to complete the statement. *(Section 2.5)*

42. Reflexive Property of Congruence (Theorem 2.1): ____ $\cong \overline{SE}$

43. Symmetric Property of Congruence (Theorem 2.1): If ____ $\cong$ ____, then $\overline{RS} \cong \overline{JK}$.

44. Transitive Property of Congruence (Theorem 2.1): If $\overline{EF} \cong \overline{PQ}$, and $\overline{PQ} \cong \overline{UV}$, then ____ $\cong$ ____.

5.1–5.4 What Did You Learn?

Core Vocabulary

interior angles, *p. 233*
exterior angles, *p. 233*
corollary to a theorem, *p. 235*
corresponding parts, *p. 240*
legs (of an isosceles triangle), *p. 252*
vertex angle (of an isosceles triangle), *p. 252*
base (of an isosceles triangle), *p. 252*
base angles (of an isosceles triangle), *p. 252*

Core Concepts

Classifying Triangles by Sides, *p. 232*
Classifying Triangles by Angles, *p. 232*
Theorem 5.1 Triangle Sum Theorem, *p. 233*
Theorem 5.2 Exterior Angle Theorem, *p. 234*
Corollary 5.1 Corollary to the Triangle Sum Theorem, *p. 235*
Identifying and Using Corresponding Parts, *p. 240*
Theorem 5.3 Properties of Triangle Congruence, *p. 241*
Theorem 5.4 Third Angles Theorem, *p. 242*
Theorem 5.5 Side-Angle-Side (SAS) Congruence Theorem, *p. 246*
Theorem 5.6 Base Angles Theorem, *p. 252*
Theorem 5.7 Converse of the Base Angles Theorem, *p. 252*
Corollary 5.2 Corollary to the Base Angles Theorem, *p. 253*
Corollary 5.3 Corollary to the Converse of the Base Angles Theorem, *p. 253*

Mathematical Practices

1. In Exercise 37 on page 237, what are you given? What relationships are present? What is your goal?
2. Explain the relationships present in Exercise 23 on page 244.
3. Describe at least three different patterns created using triangles for the picture in Exercise 20 on page 257.

Study Skills

Visual Learners

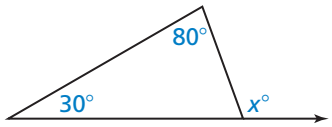
Draw a picture of a word problem.

- Draw a picture of a word problem before starting to solve the problem. You do not have to be an artist.
- When making a review card for a word problem, include a picture. This will help you recall the information while taking a test.
- Make sure your notes are visually neat for easy recall.

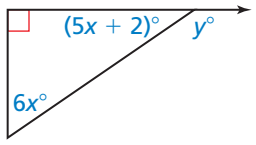
5.1–5.4 Quiz

Find the measure of the exterior angle. *(Section 5.1)*

1.



2.



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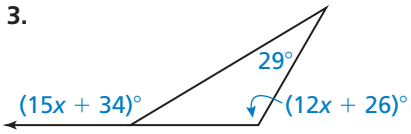
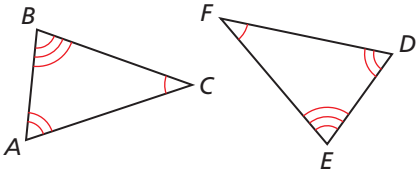
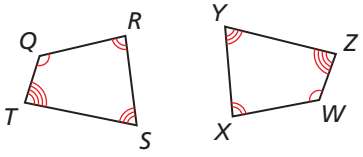


Identify all pairs of congruent corresponding parts. Then write another congruence statement for the polygons. *(Section 5.2)*

4. $\triangle ABC \cong \triangle DEF$

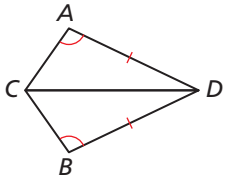


5. $QRST \cong WXYZ$

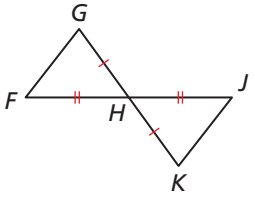


Decide whether enough information is given to prove that the triangles are congruent using the SAS Congruence Theorem (Thm. 5.5). If so, write a proof. If not, explain why. *(Section 5.3)*

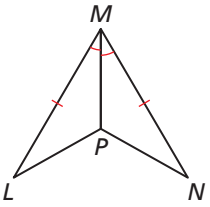
6. $\triangle CAD, \triangle CBD$



7. $\triangle GHF, \triangle KHJ$



8. $\triangle LMP, \triangle NMP$



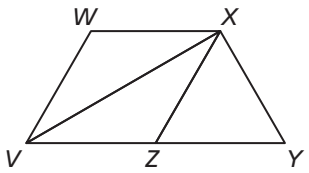
Copy and complete the statement. State which theorem you used. *(Section 5.4)*

9. If $VW \cong WX$, then $\angle$___ $\cong \angle$___.

10. If $XZ \cong XY$, then $\angle$___ $\cong \angle$___.

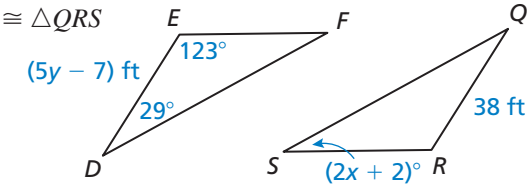
11. If $\angle ZVX \cong \angle ZXV$, then ___ $\cong$ ___.

12. If $\angle XYZ \cong \angle ZXY$, then ___ $\cong$ ___.

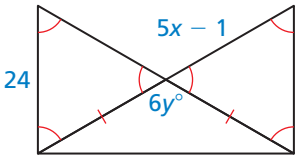


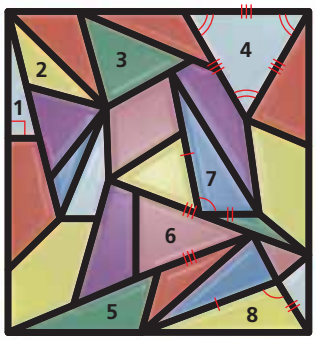
Find the values of x and y. *(Section 5.2 and Section 5.4)*

13. $\triangle DEF \cong \triangle QRS$



14.





15. In a right triangle, the measure of one acute angle is 4 times the difference of the measure of the other acute angle and 5. Find the measure of each acute angle in the triangle. *(Section 5.1)*

16. The figure shows a stained glass window. *(Section 5.1 and Section 5.3)*

a. Classify triangles 1–4 by their angles.

b. Classify triangles 4–6 by their sides.

c. Is there enough information given to prove that $\triangle 7 \cong \triangle 8$? If so, label the vertices and write a proof. If not, determine what additional information is needed.

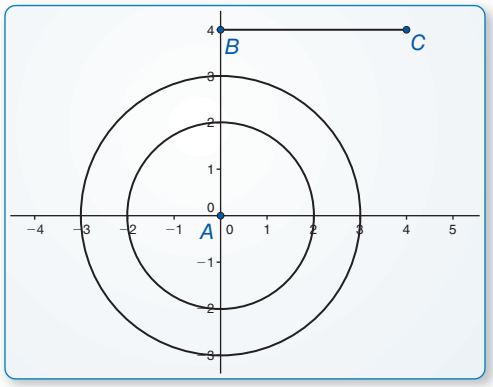
5.5 Proving Triangle Congruence by SSS

Essential Question What can you conclude about two triangles when you know the corresponding sides are congruent?

EXPLORATION 1 Drawing Triangles

Work with a partner. Use dynamic geometry software.

a. Construct circles with radii of 2 units and 3 units centered at the origin. Label the origin A. Then draw $\overline{BC}$ of length 4 units.

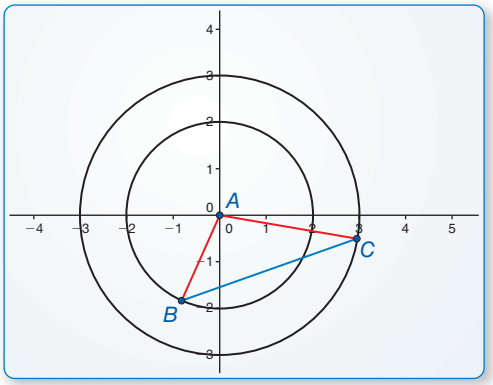


b. Move $\overline{BC}$ so that B is on the smaller circle and C is on the larger circle. Then draw $\triangle ABC$.

c. Explain why the side lengths of $\triangle ABC$ are 2, 3, and 4 units.

d. Find $m\angle A$, $m\angle B$, and $m\angle C$.

e. Repeat parts (b) and (d) several times, moving $\overline{BC}$ to different locations. Keep track of your results by copying and completing the table below. What can you conclude?



USING TOOLS STRATEGICALLY

To be proficient in math, you need to use technology to help visualize the results of varying assumptions, explore consequences, and compare predictions with data.

	A	B	C	AB	AC	BC	$m\angle A$	$m\angle B$	$m\angle C$
1.	(0, 0)			2	3	4			
2.	(0, 0)			2	3	4			
3.	(0, 0)			2	3	4			
4.	(0, 0)			2	3	4			
5.	(0, 0)			2	3	4			

Communicate Your Answer

2. What can you conclude about two triangles when you know the corresponding sides are congruent?

3. How would you prove your conclusion in Exploration 1(e)?

5.5 Lesson

Core Vocabulary

legs, *p. 264*
hypotenuse, *p. 264*

Previous
congruent figures
rigid motion

What You Will Learn

- Use the Side-Side-Side (SSS) Congruence Theorem.
- Use the Hypotenuse-Leg (HL) Congruence Theorem.

Using the Side-Side-Side Congruence Theorem

Theorem

Theorem 5.8 Side-Side-Side (SSS) Congruence Theorem

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

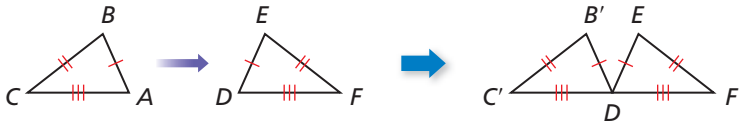
If $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.

PROOF **Side-Side-Side (SSS) Congruence Theorem**

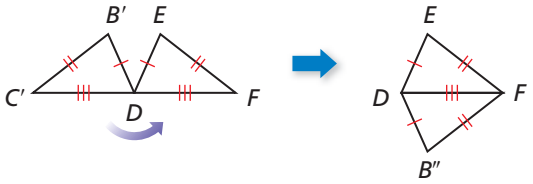
Given $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $\overline{AC} \cong \overline{DF}$

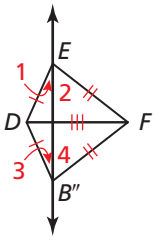
Prove $\triangle ABC \cong \triangle DEF$

First, translate $\triangle ABC$ so that point A maps to point D, as shown below.



This translation maps $\triangle ABC$ to $\triangle DB'C'$. Next, rotate $\triangle DB'C'$ counterclockwise through $\angle C'DF$ so that the image of $\overrightarrow{DC'}$ coincides with $\overrightarrow{DF}$, as shown below.





Because $\overline{DC'} \cong \overline{DF}$, the rotation maps point C' to point F. So, this rotation maps $\triangle DB'C'$ to $\triangle DB''F$. Draw an auxiliary line through points E and B''. This line creates $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$, as shown at the left.

Because $\overline{DE} \cong \overline{DB''}$, $\triangle DEB''$ is an isosceles triangle. Because $\overline{FE} \cong \overline{FB''}$, $\triangle FEB''$ is an isosceles triangle. By the Base Angles Theorem (Thm. 5.6), $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. By the definition of congruence, $m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 4$. By construction, $m\angle DEF = m\angle 1 + m\angle 2$ and $m\angle DB''F = m\angle 3 + m\angle 4$. You can now use the Substitution Property of Equality to show $m\angle DEF = m\angle DB''F$.

$m\angle DEF = m\angle 1 + m\angle 2$ — Angle Addition Postulate (Postulate 1.4)

$= m\angle 3 + m\angle 4$ — Substitute $m\angle 3$ for $m\angle 1$ and $m\angle 4$ for $m\angle 2$.

$= m\angle DB''F$ — Angle Addition Postulate (Postulate 1.4)

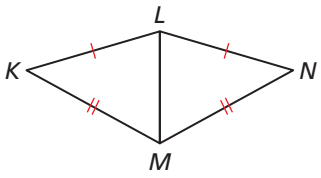
By the definition of congruence, $\angle DEF \cong \angle DB''F$. So, two pairs of sides and their included angles are congruent. By the SAS Congruence Theorem (Thm. 5.5), $\triangle DB''F \cong \triangle DEF$. So, a composition of rigid motions maps $\triangle DB''F$ to $\triangle DEF$. Because a composition of rigid motions maps $\triangle ABC$ to $\triangle DB''F$ and a composition of rigid motions maps $\triangle DB''F$ to $\triangle DEF$, a composition of rigid motions maps $\triangle ABC$ to $\triangle DEF$. So, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 1 Using the SSS Congruence Theorem

Write a proof.

Given $\overline{KL} \cong \overline{NL}$, $\overline{KM} \cong \overline{NM}$

Prove $\triangle KLM \cong \triangle NLM$



SOLUTION

STATEMENTS	REASONS
S 1. $\overline{KL} \cong \overline{NL}$	1. Given
S 2. $\overline{KM} \cong \overline{NM}$	2. Given
S 3. $\overline{LM} \cong \overline{LM}$	3. Reflexive Property of Congruence (Thm. 2.1)
4. $\triangle KLM \cong \triangle NLM$	4. SSS Congruence Theorem

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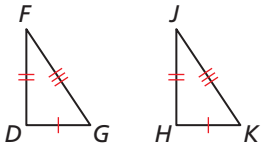
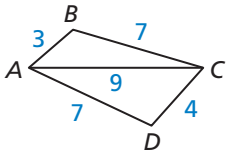
Decide whether the congruence statement is true. Explain your reasoning.

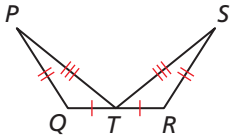
1. $\triangle DFG \cong \triangle HJK$



2. $\triangle ACB \cong \triangle CAD$

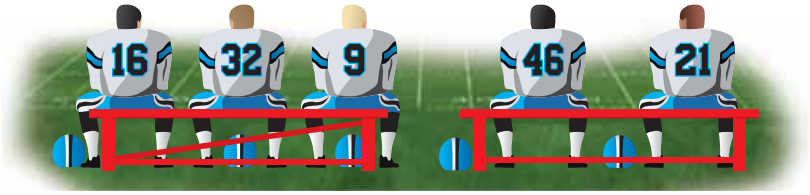


3. $\triangle QPT \cong \triangle RST$



EXAMPLE 2 Solving a Real-Life Problem

Explain why the bench with the diagonal support is stable, while the one without the support can collapse.



SOLUTION

The bench with the diagonal support forms triangles with fixed side lengths. By the SSS Congruence Theorem, these triangles cannot change shape, so the bench is stable. The bench without the diagonal support is not stable because there are many possible quadrilaterals with the given side lengths.

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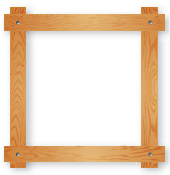
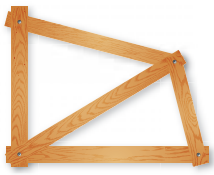
Determine whether the figure is stable. Explain your reasoning.

4.



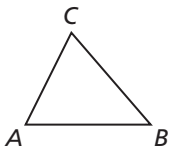
5.



6.

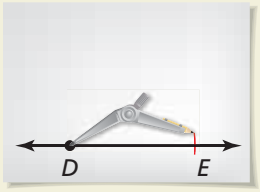
CONSTRUCTION **Copying a Triangle Using SSS**

Construct a triangle that is congruent to $\triangle ABC$ using the SSS Congruence Theorem. Use a compass and straightedge.



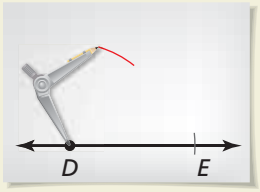
SOLUTION

Step 1



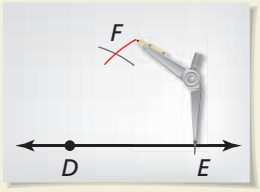
Construct a side
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

Step 2



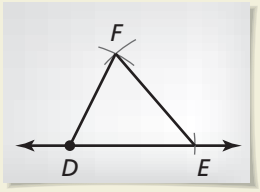
Draw an arc
Open your compass to the length AC. Use this length to draw an arc with center D.

Step 3



Draw an arc
Draw an arc with radius BC and center E that intersects the arc from Step 2. Label the intersection point F.

Step 4

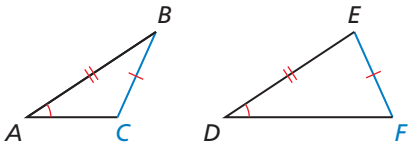


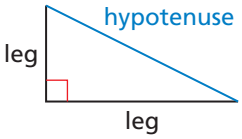
Draw a triangle
Draw $\triangle DEF$. By the SSS Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

Using the Hypotenuse-Leg Congruence Theorem

You know that SAS and SSS are valid methods for proving that triangles are congruent. What about SSA?

In general, SSA is *not* a valid method for proving that triangles are congruent. In the triangles below, two pairs of sides and a pair of angles not included between them are congruent, but the triangles are not congruent.





While SSA is not valid in general, there is a special case for right triangles.

In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.

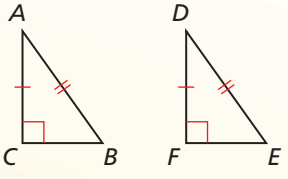
Theorem

Theorem 5.9 Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $m\angle C = m\angle F = 90°$, then $\triangle ABC \cong \triangle DEF$.

Proof Ex. 38, p. 470; *BigIdeasMath.com*

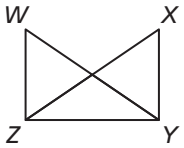


EXAMPLE 3 Using the Hypotenuse-Leg Congruence Theorem

Write a proof.

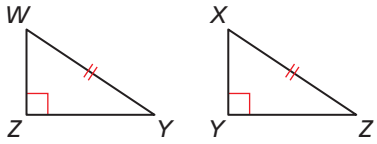
Given $\overline{WY} \cong \overline{XZ}$, $\overline{WZ} \perp \overline{ZY}$, $\overline{XY} \perp \overline{ZY}$

Prove $\triangle WYZ \cong \triangle XZY$



SOLUTION

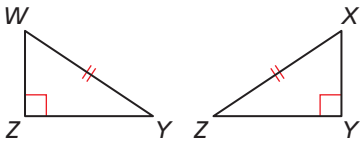
Redraw the triangles so they are side by side with corresponding parts in the same position. Mark the given information in the diagram.



STATEMENTS	REASONS
H 1. $\overline{WY} \cong \overline{XZ}$	1. Given
2. $\overline{WZ} \perp \overline{ZY}, \overline{XY} \perp \overline{ZY}$	2. Given
3. $\angle Z$ and $\angle Y$ are right angles.	3. Definition of $\perp$ lines
4. $\triangle WYZ$ and $\triangle XZY$ are right triangles.	4. Definition of a right triangle
L 5. $\overline{ZY} \cong \overline{YZ}$	5. Reflexive Property of Congruence (Thm. 2.1)
6. $\triangle WYZ \cong \triangle XZY$	6. HL Congruence Theorem

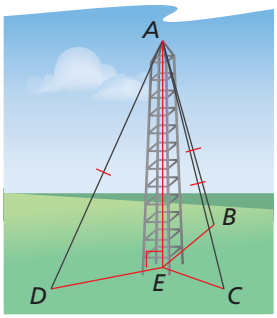
STUDY TIP

If you have trouble matching vertices to letters when you separate the overlapping triangles, leave the triangles in their original orientations.



EXAMPLE 4 Using the Hypotenuse-Leg Congruence Theorem

The television antenna is perpendicular to the plane containing points B, C, D, and E. Each of the cables running from the top of the antenna to B, C, and D has the same length. Prove that $\triangle AEB$, $\triangle AEC$, and $\triangle AED$ are congruent.



Given $\overline{AE} \perp \overline{EB}$, $\overline{AE} \perp \overline{EC}$, $\overline{AE} \perp \overline{ED}$, $\overline{AB} \cong \overline{AC} \cong \overline{AD}$

Prove $\triangle AEB \cong \triangle AEC \cong \triangle AED$

SOLUTION

You are given that $\overline{AE} \perp \overline{EB}$ and $\overline{AE} \perp \overline{EC}$. So, $\angle AEB$ and $\angle AEC$ are right angles by the definition of perpendicular lines. By definition, $\triangle AEB$ and $\triangle AEC$ are right triangles. You are given that the hypotenuses of these two triangles, $\overline{AB}$ and $\overline{AC}$, are congruent. Also, $\overline{AE}$ is a leg for both triangles, and $\overline{AE} \cong \overline{AE}$ by the Reflexive Property of Congruence (Thm. 2.1). So, by the Hypotenuse-Leg Congruence Theorem, $\triangle AEB \cong \triangle AEC$. You can use similar reasoning to prove that $\triangle AEC \cong \triangle AED$.

So, by the Transitive Property of Triangle Congruence (Thm. 5.3), $\triangle AEB \cong \triangle AEC \cong \triangle AED$.

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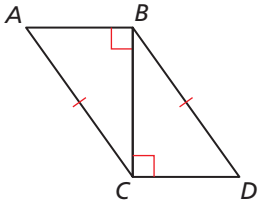
Use the diagram.



7. Redraw $\triangle ABC$ and $\triangle DCB$ side by side with corresponding parts in the same position.
8. Use the information in the diagram to prove that $\triangle ABC \cong \triangle DCB$.

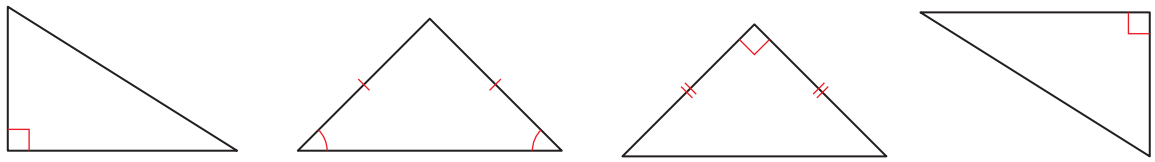
5.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. COMPLETE THE SENTENCE The side opposite the right angle is called the _____ of the right triangle.

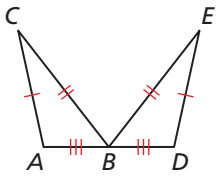
2. WHICH ONE DOESN'T BELONG? Which triangle's legs do *not* belong with the other three? Explain your reasoning.



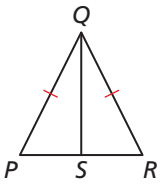
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, decide whether enough information is given to prove that the triangles are congruent using the SSS Congruence Theorem (Theorem 5.8). Explain.

3. $\triangle ABC, \triangle DBE$

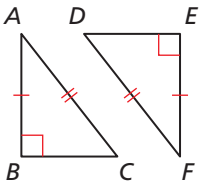


4. $\triangle PQS, \triangle RQS$

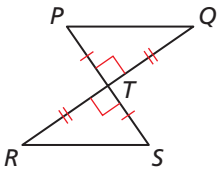


In Exercises 5 and 6, decide whether enough information is given to prove that the triangles are congruent using the HL Congruence Theorem (Theorem 5.9). Explain.

5. $\triangle ABC, \triangle FED$

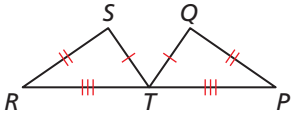


6. $\triangle PQT, \triangle SRT$

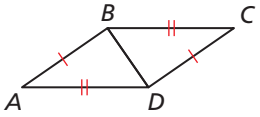


In Exercises 7–10, decide whether the congruence statement is true. Explain your reasoning. *(See Example 1.)*

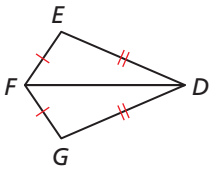
7. $\triangle RST \cong \triangle TQP$



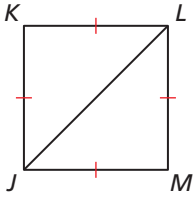
8. $\triangle ABD \cong \triangle CDB$



9. $\triangle DEF \cong \triangle DGF$

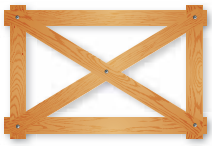


10. $\triangle JKL \cong \triangle LJM$

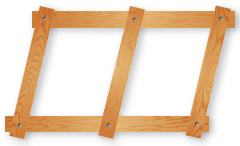


In Exercises 11 and 12, determine whether the figure is stable. Explain your reasoning. *(See Example 2.)*

11.



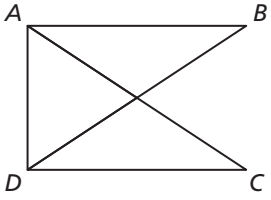
12.



In Exercises 13 and 14, redraw the triangles so they are side by side with corresponding parts in the same position. Then write a proof. *(See Example 3.)*

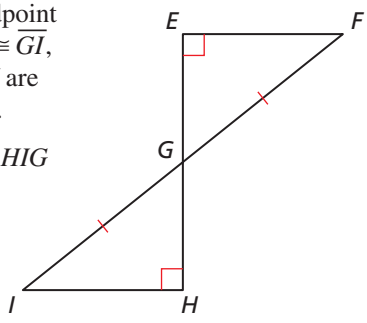
13. Given $\overline{AC} \cong \overline{BD}$, $\overline{AB} \perp \overline{AD}$, $\overline{CD} \perp \overline{AD}$

Prove $\triangle BAD \cong \triangle CDA$



14. Given G is the midpoint of $\overline{EH}$, $\overline{FG} \cong \overline{GI}$, $\angle E$ and $\angle H$ are right angles.

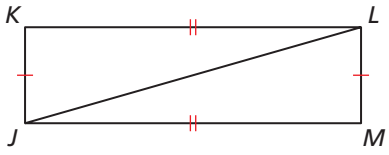
Prove $\triangle EFG \cong \triangle HIG$



In Exercises 15 and 16, write a proof.

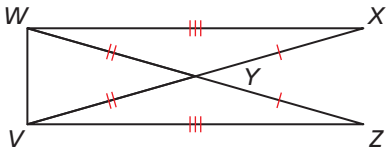
15. **Given** $\overline{LM} \cong \overline{JK}, \overline{MJ} \cong \overline{KL}$

Prove $\triangle LMJ \cong \triangle JKL$



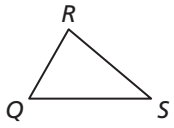
16. **Given** $\overline{WX} \cong \overline{VZ}, \overline{WY} \cong \overline{VY}, \overline{YZ} \cong \overline{YX}$

Prove $\triangle VWX \cong \triangle WVZ$

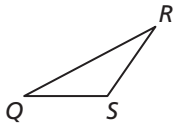


CONSTRUCTION **In Exercises 17 and 18, construct a triangle that is congruent to $\triangle QRS$ using the SSS Congruence Theorem (Theorem 5.8).**

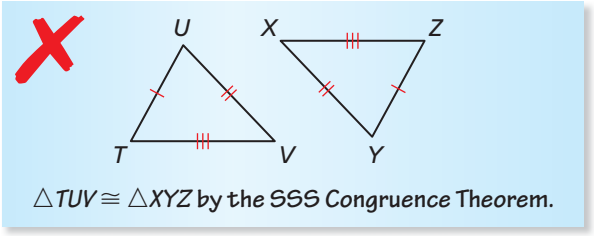
17.



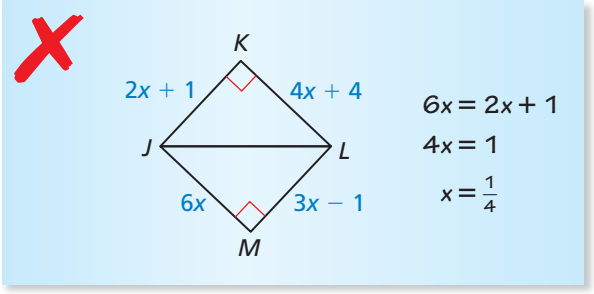
18.



19. **ERROR ANALYSIS** Describe and correct the error in identifying congruent triangles.

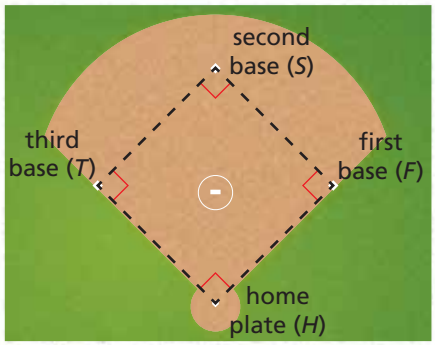


20. **ERROR ANALYSIS** Describe and correct the error in determining the value of x that makes the triangles congruent.

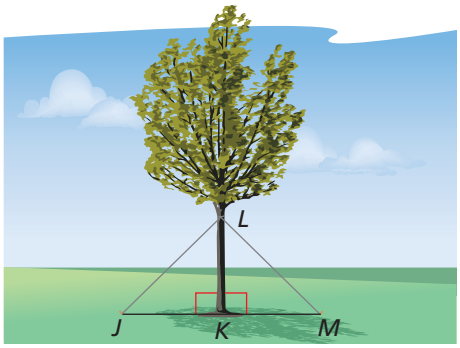


21. **MAKING AN ARGUMENT** Your friend claims that in order to use the SSS Congruence Theorem (Theorem 5.8) to prove that two triangles are congruent, both triangles must be equilateral triangles. Is your friend correct? Explain your reasoning.

22. **MODELING WITH MATHEMATICS** The distances between consecutive bases on a softball field are the same. The distance from home plate to second base is the same as the distance from first base to third base. The angles created at each base are 90°. Prove $\triangle HFS \cong \triangle FST \cong \triangle STH$. *(See Example 4.)*



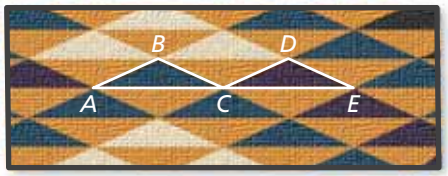
23. **REASONING** To support a tree, you attach wires from the trunk of the tree to stakes in the ground, as shown in the diagram.



a. What additional information do you need to use the HL Congruence Theorem (Theorem 5.9) to prove that $\triangle JKL \cong \triangle MKL$?

b. Suppose K is the midpoint of JM. Name a theorem you could use to prove that $\triangle JKL \cong \triangle MKL$. Explain your reasoning.

24. **REASONING** Use the photo of the Navajo rug, where $\overline{BC} \cong \overline{DE}$ and $\overline{AC} \cong \overline{CE}$.



a. What additional information do you need to use the SSS Congruence Theorem (Theorem 5.8) to prove that $\triangle ABC \cong \triangle CDE$?

b. What additional information do you need to use the HL Congruence Theorem (Theorem 5.9) to prove that $\triangle ABC \cong \triangle CDE$?

In Exercises 25–28, use the given coordinates to determine whether $\triangle ABC \cong \triangle DEF$.

25. $A(-2, -2), B(4, -2), C(4, 6), D(5, 7), E(5, 1), F(13, 1)$

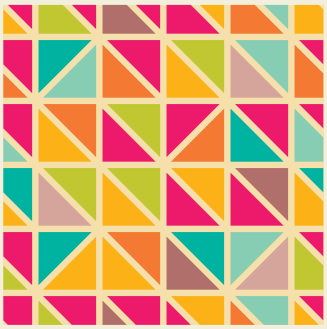
26. $A(-2, 1), B(3, -3), C(7, 5), D(3, 6), E(8, 2), F(10, 11)$

27. $A(0, 0), B(6, 5), C(9, 0), D(0, -1), E(6, -6), F(9, -1)$

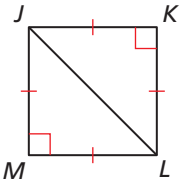
28. $A(-5, 7), B(-5, 2), C(0, 2), D(0, 6), E(0, 1), F(4, 1)$

29. **CRITICAL THINKING** You notice two triangles in the tile floor of a hotel lobby. You want to determine whether the triangles are congruent, but you only have a piece of string. Can you determine whether the triangles are congruent? Explain.

30. **HOW DO YOU SEE IT?** There are several theorems you can use to show that the triangles in the "square" pattern are congruent. Name two of them.



31. **MAKING AN ARGUMENT** Your cousin says that $\triangle JKL$ is congruent to $\triangle LMJ$ by the SSS Congruence Theorem (Thm. 5.8). Your friend says that $\triangle JKL$ is congruent to $\triangle LMJ$ by the HL Congruence Theorem (Thm. 5.9). Who is correct? Explain your reasoning.



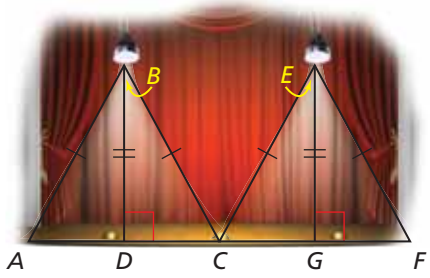
32. **THOUGHT PROVOKING** The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, do you think that two triangles are congruent if their corresponding sides are congruent? Justify your answer.

USING TOOLS **In Exercises 33 and 34, use the given information to sketch $\triangle LMN$ and $\triangle STU$. Mark the triangles with the given information.**

33. $\overline{LM} \perp \overline{MN}, \overline{ST} \perp \overline{TU}, \overline{LM} \cong \overline{NM} \cong \overline{UT} \cong \overline{ST}$

34. $\overline{LM} \perp \overline{MN}, \overline{ST} \perp \overline{TU}, \overline{LM} \cong \overline{ST}, \overline{LN} \cong \overline{SU}$

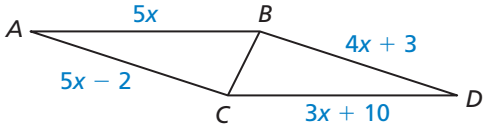
35. **CRITICAL THINKING** The diagram shows the light created by two spotlights. Both spotlights are the same distance from the stage.



a. Show that $\triangle ABD \cong \triangle CBD$. State which theorem or postulate you used and explain your reasoning.

b. Are all four right triangles shown in the diagram congruent? Explain your reasoning.

36. **MATHEMATICAL CONNECTIONS** Find all values of x that make the triangles congruent. Explain.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

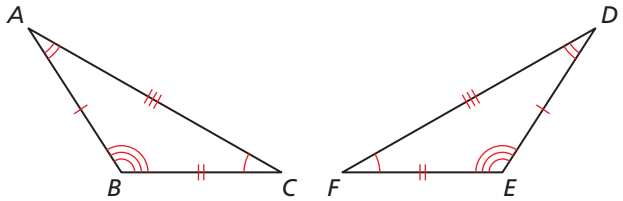
Use the congruent triangles. *(Section 5.2)*

37. Name the segment in $\triangle DEF$ that is congruent to $\overline{AC}$.

38. Name the segment in $\triangle ABC$ that is congruent to $\overline{EF}$.

39. Name the angle in $\triangle DEF$ that is congruent to $\angle B$.

40. Name the angle in $\triangle ABC$ that is congruent to $\angle F$.



5.6 Proving Triangle Congruence by ASA and AAS

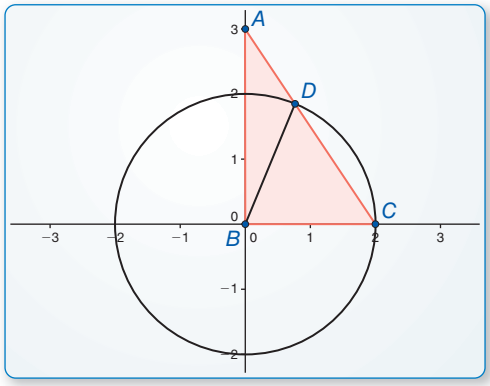
Essential Question What information is sufficient to determine whether two triangles are congruent?

EXPLORATION 1 Determining Whether SSA Is Sufficient

Work with a partner.

a. Use dynamic geometry software to construct $\triangle ABC$. Construct the triangle so that vertex B is at the origin, $\overline{AB}$ has a length of 3 units, and $\overline{BC}$ has a length of 2 units.

b. Construct a circle with a radius of 2 units centered at the origin. Locate point D where the circle intersects $\overline{AC}$. Draw $\overline{BD}$.



Sample

Points
$A(0, 3)$
$B(0, 0)$
$C(2, 0)$
$D(0.77, 1.85)$
Segments
$AB = 3$
$AC = 3.61$
$BC = 2$
$AD = 1.38$
Angle
$m\angle A = 33.69°$

c. $\triangle ABC$ and $\triangle ABD$ have two congruent sides and a nonincluded congruent angle. Name them.

d. Is $\triangle ABC \cong \triangle ABD$? Explain your reasoning.

e. Is SSA sufficient to determine whether two triangles are congruent? Explain your reasoning.

CONSTRUCTING VIABLE ARGUMENTS

To be proficient in math, you need to recognize and use counterexamples.

EXPLORATION 2 Determining Valid Congruence Theorems

Work with a partner. Use dynamic geometry software to determine which of the following are valid triangle congruence theorems. For those that are not valid, write a counterexample. Explain your reasoning.

Possible Congruence Theorem	Valid or not valid?
SSS	
SSA	
SAS	
AAS	
ASA	
AAA	

Communicate Your Answer

3. What information is sufficient to determine whether two triangles are congruent?

4. Is it possible to show that two triangles are congruent using more than one congruence theorem? If so, give an example.

5.6 Lesson

Core Vocabulary

Previous
congruent figures
rigid motion

What You Will Learn

- Use the ASA and AAS Congruence Theorems.

Using the ASA and AAS Congruence Theorems

Theorem

Theorem 5.10 Angle-Side-Angle (ASA) Congruence Theorem

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, and $\angle C \cong \angle F$, then $\triangle ABC \cong \triangle DEF$.

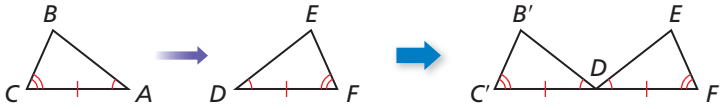
Proof p. 270

PROOF **Angle-Side-Angle (ASA) Congruence Theorem**

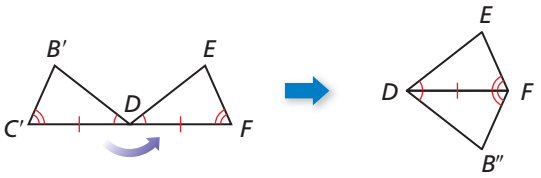
Given $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, $\angle C \cong \angle F$

Prove $\triangle ABC \cong \triangle DEF$

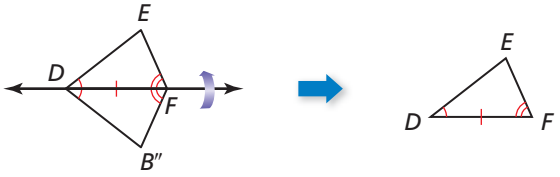
First, translate $\triangle ABC$ so that point A maps to point D, as shown below.



This translation maps $\triangle ABC$ to $\triangle DB'C'$. Next, rotate $\triangle DB'C'$ counterclockwise through $\angle C'DF$ so that the image of $\overrightarrow{DC'}$ coincides with $\overrightarrow{DF}$, as shown below.



Because $\overline{DC'} \cong \overline{DF}$, the rotation maps point C' to point F. So, this rotation maps $\triangle DB'C'$ to $\triangle DB''F$. Now, reflect $\triangle DB''F$ in the line through points D and F, as shown below.



Because points D and F lie on $\overleftrightarrow{DF}$, this reflection maps them onto themselves. Because a reflection preserves angle measure and $\angle B''DF \cong \angle EDF$, the reflection maps $\overrightarrow{DB''}$ to $\overrightarrow{DE}$. Similarly, because $\angle B''FD \cong \angle EFD$, the reflection maps $\overrightarrow{FB''}$ to $\overrightarrow{FE}$. The image of B'' lies on $\overrightarrow{DE}$ and $\overrightarrow{FE}$. Because $\overrightarrow{DE}$ and $\overrightarrow{FE}$ only have point E in common, the image of B'' must be E. So, this reflection maps $\triangle DB''F$ to $\triangle DEF$.

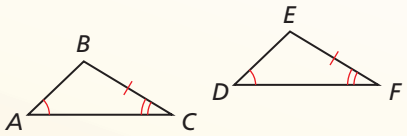
Because you can map $\triangle ABC$ to $\triangle DEF$ using a composition of rigid motions, $\triangle ABC \cong \triangle DEF$.

Theorem

Theorem 5.11 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

If $\angle A \cong \angle D$, $\angle C \cong \angle F$, and $\overline{BC} \cong \overline{EF}$, then $\triangle ABC \cong \triangle DEF$.

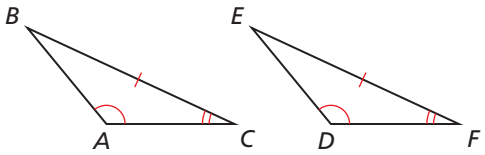


Proof p. 271

PROOF Angle-Angle-Side (AAS) Congruence Theorem

Given $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{BC} \cong \overline{EF}$

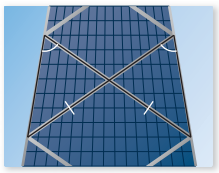
Prove $\triangle ABC \cong \triangle DEF$

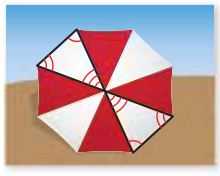


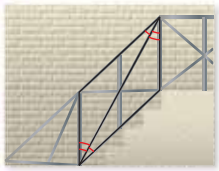
You are given $\angle A \cong \angle D$ and $\angle C \cong \angle F$. By the Third Angles Theorem (Theorem 5.4), $\angle B \cong \angle E$. You are given $\overline{BC} \cong \overline{EF}$. So, two pairs of angles and their included sides are congruent. By the ASA Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 1 Identifying Congruent Triangles

Can the triangles be proven congruent with the information given in the diagram? If so, state the theorem you would use.

a. 

b. 

c. 

COMMON ERROR

You need at least one pair of congruent corresponding sides to prove two triangles are congruent.

SOLUTION

a. The vertical angles are congruent, so two pairs of angles and a pair of non-included sides are congruent. The triangles are congruent by the AAS Congruence Theorem.

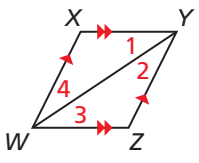
b. There is not enough information to prove the triangles are congruent, because no sides are known to be congruent.

c. Two pairs of angles and their included sides are congruent. The triangles are congruent by the ASA Congruence Theorem.

Monitoring Progress

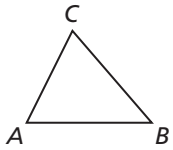
Help in English and Spanish at *BigIdeasMath.com*

1. Can the triangles be proven congruent with the information given in the diagram? If so, state the theorem you would use.



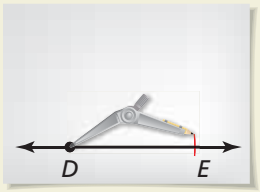
CONSTRUCTION Copying a Triangle Using ASA

Construct a triangle that is congruent to $\triangle ABC$ using the ASA Congruence Theorem. Use a compass and straightedge.



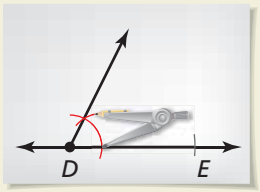
SOLUTION

Step 1



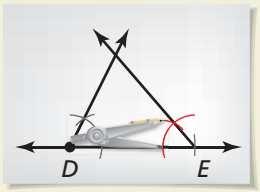
Construct a side
Construct $\overline{DE}$ so that it is congruent to $\overline{AB}$.

Step 2



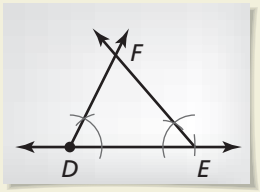
Construct an angle
Construct $\angle D$ with vertex D and side $\overrightarrow{DE}$ so that it is congruent to $\angle A$.

Step 3



Construct an angle
Construct $\angle E$ with vertex E and side $\overrightarrow{ED}$ so that it is congruent to $\angle B$.

Step 4



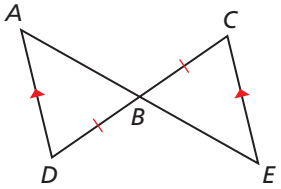
Label a point
Label the intersection of the sides of $\angle D$ and $\angle E$ that you constructed in Steps 2 and 3 as F. By the ASA Congruence Theorem, $\triangle ABC \cong \triangle DEF$.

EXAMPLE 2 Using the ASA Congruence Theorem

Write a proof.

Given $\overline{AD} \parallel \overline{EC}$, $\overline{BD} \cong \overline{BC}$

Prove $\triangle ABD \cong \triangle EBC$



SOLUTION

STATEMENTS	REASONS
1. $\overline{AD} \parallel \overline{EC}$	1. Given
A 2. $\angle D \cong \angle C$	2. Alternate Interior Angles Theorem (Thm. 3.2)
S 3. $\overline{BD} \cong \overline{BC}$	3. Given
A 4. $\angle ABD \cong \angle EBC$	4. Vertical Angles Congruence Theorem (Thm 2.6)
5. $\triangle ABD \cong \triangle EBC$	5. ASA Congruence Theorem

Monitoring Progress

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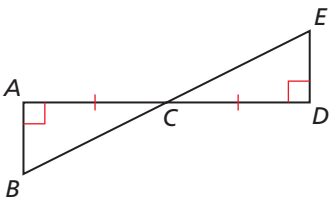
2. In the diagram, $\overline{AB} \perp \overline{AD}$, $\overline{DE} \perp \overline{AD}$, and $\overline{AC} \cong \overline{DC}$. Prove $\triangle ABC \cong \triangle DEC$.

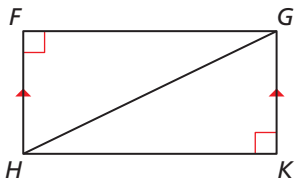


EXAMPLE 3 Using the AAS Congruence Theorem

Write a proof.

Given $\overline{HF} \parallel \overline{GK}$, $\angle F$ and $\angle K$ are right angles.

Prove $\triangle HFG \cong \triangle GKH$



SOLUTION

STATEMENTS	REASONS
1. $\overline{HF} \parallel \overline{GK}$	**1.** Given
A **2.** $\angle GHF \cong \angle HGK$	**2.** Alternate Interior Angles Theorem (Theorem 3.2)
3. $\angle F$ and $\angle K$ are right angles.	**3.** Given
A **4.** $\angle F \cong \angle K$	**4.** Right Angles Congruence Theorem (Theorem 2.3)
S **5.** $\overline{HG} \cong \overline{GH}$	**5.** Reflexive Property of Congruence (Theorem 2.1)
6. $\triangle HFG \cong \angle GKH$	**6.** AAS Congruence Theorem

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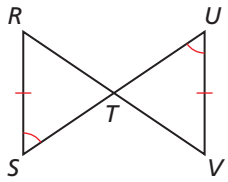
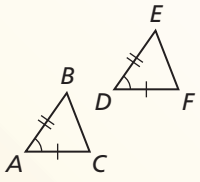
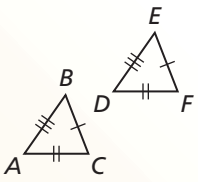
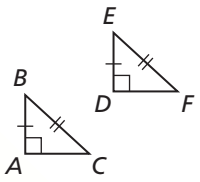
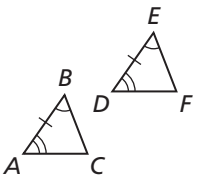
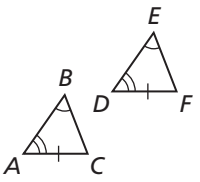
3. In the diagram, $\angle S \cong \angle U$ and $\overline{RS} \cong \overline{VU}$. Prove $\triangle RST \cong \triangle VUT$.



Concept Summary

Triangle Congruence Theorems

You have learned five methods for proving that triangles are congruent.

SAS	SSS	HL (right △ only)	ASA	AAS
				
Two sides and the included angle are congruent.	All three sides are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a non-included side are congruent.

In the Exercises, you will prove three additional theorems about the congruence of right triangles: **H**ypotenuse-**A**ngle, **L**eg-**L**eg, and **A**ngle-**L**eg.

5.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

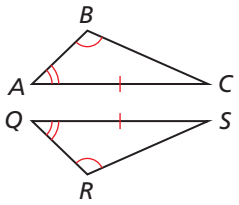
1. **WRITING** How are the AAS Congruence Theorem (Theorem 5.11) and the ASA Congruence Theorem (Theorem 5.10) similar? How are they different?

2. **WRITING** You know that a pair of triangles has two pairs of congruent corresponding angles. What other information do you need to show that the triangles are congruent?

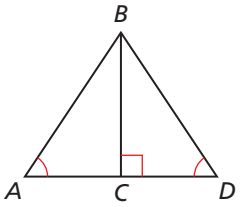
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, decide whether enough information is given to prove that the triangles are congruent. If so, state the theorem you would use. *(See Example 1.)*

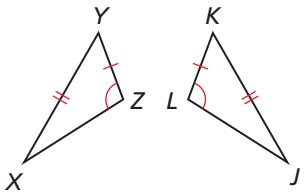
3. $\triangle ABC$, $\triangle QRS$



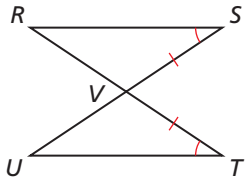
4. $\triangle ABC$, $\triangle DBC$



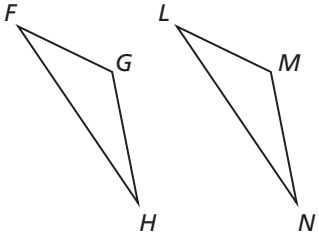
5. $\triangle XYZ$, $\triangle JKL$



6. $\triangle RSV$, $\triangle UTV$



In Exercises 7 and 8, state the third congruence statement that is needed to prove that $\triangle FGH \cong \triangle LMN$ using the given theorem.



7. **Given** $\overline{GH} \cong \overline{MN}$, $\angle G \cong \angle M$, ___ $\cong$ ____

   Use the AAS Congruence Theorem (Thm. 5.11).

8. **Given** $\overline{FG} \cong \overline{LM}$, $\angle G \cong \angle M$, ___ $\cong$ ____

   Use the ASA Congruence Theorem (Thm. 5.10).

In Exercises 9–12, decide whether you can use the given information to prove that $\triangle ABC \cong \triangle DEF$. Explain your reasoning.

9. $\angle A \cong \angle D$, $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DF}$

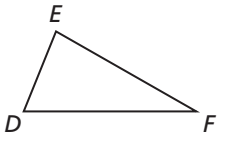
10. $\angle C \cong \angle F$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$

11. $\angle B \cong \angle E$,$\angle C \cong \angle F$, $\overline{AC} \cong \overline{DE}$

12. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\overline{BC} \cong \overline{EF}$

CONSTRUCTION **In Exercises 13 and 14, construct a triangle that is congruent to the given triangle using the ASA Congruence Theorem (Theorem 5.10). Use a compass and straightedge.**

13.



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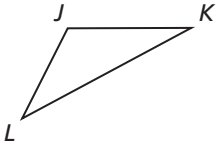
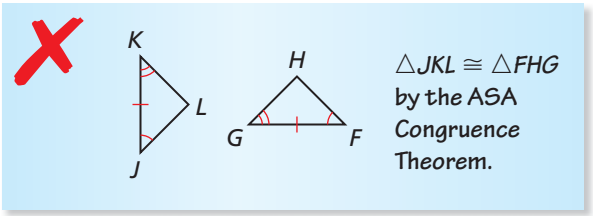


ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error.**

15.



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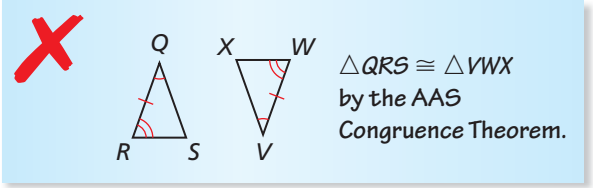
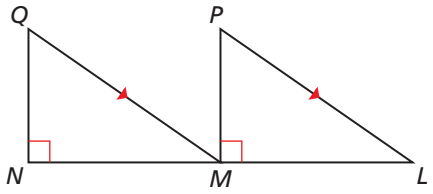


PROOF **In Exercises 17 and 18, prove that the triangles are congruent using the ASA Congruence Theorem (Theorem 5.10).** *(See Example 2.)*

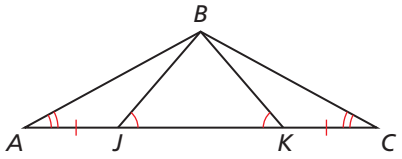
17. **Given** M is the midpoint of $\overline{NL}$.
$\overline{NL} \perp \overline{NQ}$, $\overline{NL} \perp \overline{MP}$, $\overline{QM} \parallel \overline{PL}$

Prove $\triangle NQM \cong \triangle MPL$



18. **Given** $\overline{AJ} \cong \overline{KC}$, $\angle BJK \cong \angle BKJ$, $\angle A \cong \angle C$

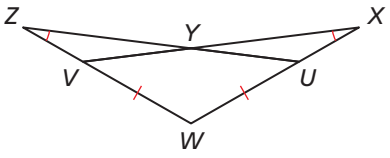
Prove $\triangle ABK \cong \triangle CBJ$



PROOF **In Exercises 19 and 20, prove that the triangles are congruent using the AAS Congruence Theorem (Theorem 5.11).** *(See Example 3.)*

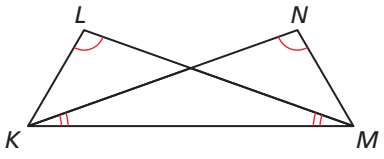
19. **Given** $\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$

Prove $\triangle XWV \cong \triangle ZWU$



20. **Given** $\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$

Prove $\triangle NMK \cong \triangle LKM$



PROOF **In Exercises 21–23, write a paragraph proof for the theorem about right triangles.**

21. **Hypotenuse-Angle (HA) Congruence Theorem** If an angle and the hypotenuse of a right triangle are congruent to an angle and the hypotenuse of a second right triangle, then the triangles are congruent.

22. **Leg-Leg (LL) Congruence Theorem** If the legs of a right triangle are congruent to the legs of a second right triangle, then the triangles are congruent.

23. **Angle-Leg (AL) Congruence Theorem** If an angle and a leg of a right triangle are congruent to an angle and a leg of a second right triangle, then the triangles are congruent.

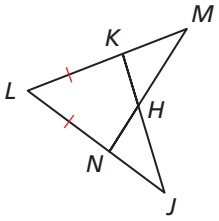
24. **REASONING** What additional information do you need to prove $\triangle JKL \cong \triangle MNL$ by the ASA Congruence Theorem (Theorem 5.10)?

Ⓐ $\overline{KM} \cong \overline{KJ}$

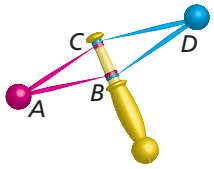
Ⓑ $\overline{KH} \cong \overline{NH}$

Ⓒ $\angle M \cong \angle J$

Ⓓ $\angle LKJ \cong \angle LNM$



25. **MATHEMATICAL CONNECTIONS** This toy contains $\triangle ABC$ and $\triangle DBC$. Can you conclude that $\triangle ABC \cong \triangle DBC$ from the given angle measures? Explain.



$m\angle ABC = (8x - 32)°$

$m\angle DBC = (4y - 24)°$

$m\angle BCA = (5x + 10)°$

$m\angle BCD = (3y + 2)°$

$m\angle CAB = (2x - 8)°$

$m\angle CDB = (y - 6)°$

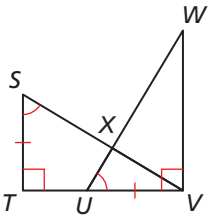
26. **REASONING** Which of the following congruence statements are true? Select all that apply.

Ⓐ $\overline{TU} \cong \overline{UV}$

Ⓑ $\triangle STV \cong \triangle XVW$

Ⓒ $\triangle TVS \cong \triangle VWU$

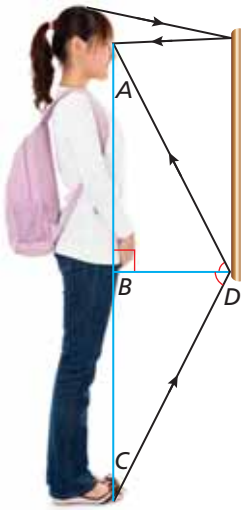
Ⓓ $\triangle VST \cong \triangle VUW$



27. **PROVING A THEOREM** Prove the Converse of the Base Angles Theorem (Theorem 5.7). (*Hint:* Draw an auxiliary line inside the triangle.)

28. **MAKING AN ARGUMENT** Your friend claims to be able to rewrite any proof that uses the AAS Congruence Theorem (Thm. 5.11) as a proof that uses the ASA Congruence Theorem (Thm. 5.10). Is this possible? Explain your reasoning.

29. MODELING WITH MATHEMATICS When a light ray from an object meets a mirror, it is reflected back to your eye. For example, in the diagram, a light ray from point C is reflected at point D and travels back to point A. The *law of reflection* states that the angle of incidence, $\angle CDB$, is congruent to the angle of reflection, $\angle ADB$.



a. Prove that $\triangle ABD$ is congruent to $\triangle CBD$.

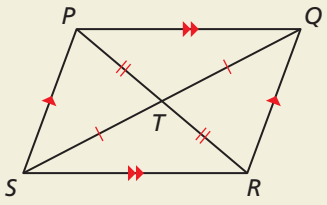
Given $\angle CDB \cong \angle ADB$, $\overline{DB} \perp \overline{AC}$

Prove $\triangle ABD \cong \triangle CBD$

b. Verify that $\triangle ACD$ is isosceles.

c. Does moving away from the mirror have any effect on the amount of his or her reflection a person sees? Explain.

30. HOW DO YOU SEE IT? Name as many pairs of congruent triangles as you can from the diagram. Explain how you know that each pair of triangles is congruent.



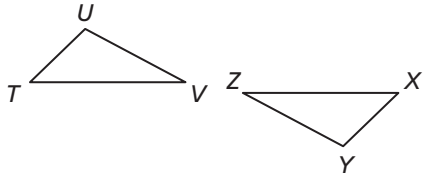
31. CONSTRUCTION Construct a triangle. Show that there is no AAA congruence rule by constructing a second triangle that has the same angle measures but is not congruent.

32. THOUGHT PROVOKING Graph theory is a branch of mathematics that studies vertices and the way they are connected. In graph theory, two polygons are *isomorphic* if there is a one-to-one mapping from one polygon's vertices to the other polygon's vertices that preserves adjacent vertices. In graph theory, are any two triangles isomorphic? Explain your reasoning.

33. MATHEMATICAL CONNECTIONS Six statements are given about $\triangle TUV$ and $\triangle XYZ$.

$\overline{TU} \cong \overline{XY}$ $\quad$ $\overline{UV} \cong \overline{YZ}$ $\quad$ $\overline{TV} \cong \overline{XZ}$

$\angle T \cong \angle X$ $\quad$ $\angle U \cong \angle Y$ $\quad$ $\angle V \cong \angle Z$



a. List all combinations of three given statements that would provide enough information to prove that $\triangle TUV$ is congruent to $\triangle XYZ$.

b. You choose three statements at random. What is the probability that the statements you choose provide enough information to prove that the triangles are congruent?

Maintaining Mathematical Proficiency

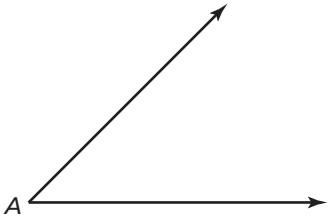
Reviewing what you learned in previous grades and lessons

Find the coordinates of the midpoint of the line segment with the given endpoints. *(Section 1.3)*

34. $C(1, 0)$ and $D(5, 4)$ $\quad$ **35.** $J(-2, 3)$ and $K(4, -1)$ $\quad$ **36.** $R(-5, -7)$ and $S(2, -4)$

Copy the angle using a compass and straightedge. *(Section 1.5)*

37.



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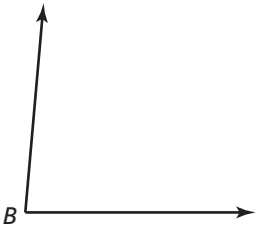
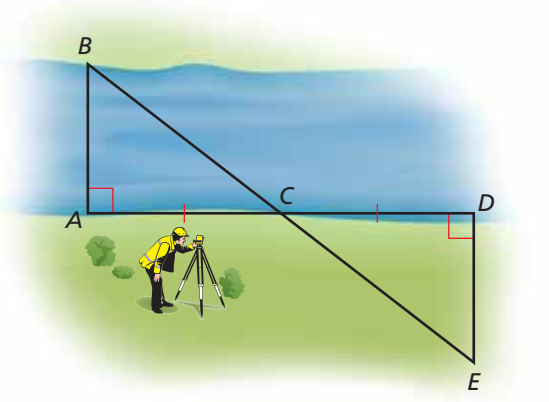


5.7 Using Congruent Triangles

Essential Question How can you use congruent triangles to make an indirect measurement?

EXPLORATION 1 Measuring the Width of a River

Work with a partner. The figure shows how a surveyor can measure the width of a river by making measurements on only one side of the river.



a. Study the figure. Then explain how the surveyor can find the width of the river.

b. Write a proof to verify that the method you described in part (a) is valid.

Given $\angle A$ is a right angle, $\angle D$ is a right angle, $\overline{AC} \cong \overline{CD}$

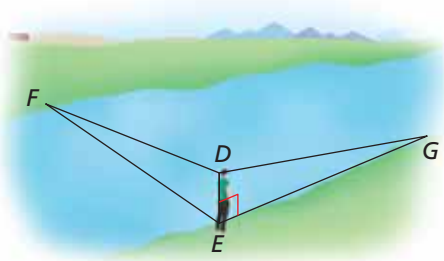
c. Exchange proofs with your partner and discuss the reasoning used.

CRITIQUING THE REASONING OF OTHERS

To be proficient in math, you need to listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

EXPLORATION 2 Measuring the Width of a River

Work with a partner. It was reported that one of Napoleon's officers estimated the width of a river as follows. The officer stood on the bank of the river and lowered the visor on his cap until the farthest thing visible was the edge of the bank on the other side. He then turned and noted the point on his side that was in line with the tip of his visor and his eye. The officer then paced the distance to this point and concluded that distance was the width of the river.



a. Study the figure. Then explain how the officer concluded that the width of the river is EG.

b. Write a proof to verify that the conclusion the officer made is correct.

Given $\angle DEG$ is a right angle, $\angle DEF$ is a right angle, $\angle EDG \cong \angle EDF$

c. Exchange proofs with your partner and discuss the reasoning used.

Communicate Your Answer

3. How can you use congruent triangles to make an indirect measurement?

4. Why do you think the types of measurements described in Explorations 1 and 2 are called *indirect* measurements?

5.7 Lesson

Core Vocabulary

Previous

congruent figures
corresponding parts
construction

What You Will Learn

- Use congruent triangles.
- Prove constructions.

Using Congruent Triangles

Congruent triangles have congruent corresponding parts. So, if you can prove that two triangles are congruent, then you know that their corresponding parts must be congruent as well.

EXAMPLE 1 Using Congruent Triangles

Explain how you can use the given information to prove that the hang glider parts are congruent.

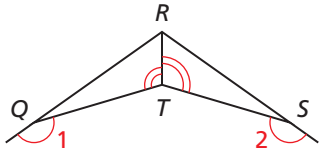
Given $\angle 1 \cong \angle 2$, $\angle RTQ \cong \angle RTS$

Prove $\overline{QT} \cong \overline{ST}$

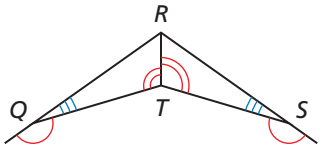
SOLUTION

If you can show that $\triangle QRT \cong \triangle SRT$, then you will know that $\overline{QT} \cong \overline{ST}$. First, copy the diagram and mark the given information. Then mark the information that you can deduce. In this case, $\angle RQT$ and $\angle RST$ are supplementary to congruent angles, so $\angle RQT \cong \angle RST$. Also, $\overline{RT} \cong \overline{RT}$ by the Reflexive Property of Congruence (Theorem 2.1).

Mark given information.



Mark deduced information.



Two angle pairs and a non-included side are congruent, so by the AAS Congruence Theorem (Theorem 5.11), $\triangle QRT \cong \triangle SRT$.

- Because corresponding parts of congruent triangles are congruent, $\overline{QT} \cong \overline{ST}$.

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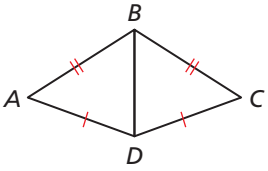
1. Explain how you can prove that $\angle A \cong \angle C$.



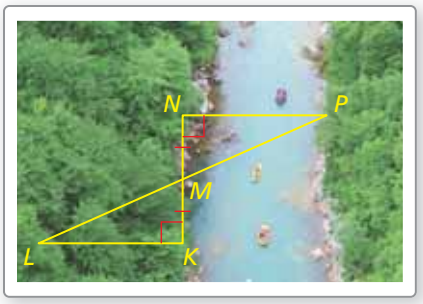
EXAMPLE 2 Using Congruent Triangles for Measurement

MAKING SENSE OF PROBLEMS

When you cannot easily measure a length directly, you can make conclusions about the length *indirectly*, usually by calculations based on known lengths.

Use the following method to find the distance across a river, from point N to point P.

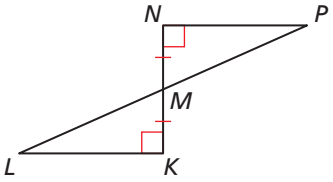
- Place a stake at K on the near side so that $\overline{NK} \perp \overline{NP}$.
- Find M, the midpoint of $\overline{NK}$.
- Locate the point L so that $\overline{NK} \perp \overline{KL}$ and L, P, and M are collinear.



Explain how this plan allows you to find the distance.

SOLUTION

Because $\overline{NK} \perp \overline{NP}$ and $\overline{NK} \perp \overline{KL}$, $\angle N$ and $\angle K$ are congruent right angles. Because M is the midpoint of $\overline{NK}$, $\overline{NM} \cong \overline{KM}$. The vertical angles $\angle KML$ and $\angle NMP$ are congruent. So, $\triangle MLK \cong \triangle MPN$ by the ASA Congruence Theorem (Theorem 5.10). Then because corresponding parts of congruent triangles are congruent, $\overline{KL} \cong \overline{NP}$. So, you can find the distance NP across the river by measuring $\overline{KL}$.

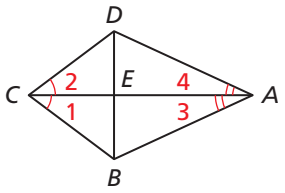


EXAMPLE 3 Planning a Proof Involving Pairs of Triangles

Use the given information to write a plan for proof.

Given $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove $\triangle BCE \cong \triangle DCE$



SOLUTION

In $\triangle BCE$ and $\triangle DCE$, you know that $\angle 1 \cong \angle 2$ and $\overline{CE} \cong \overline{CE}$. If you can show that $\overline{CB} \cong \overline{CD}$, then you can use the SAS Congruence Theorem (Theorem 5.5).

To prove that $\overline{CB} \cong \overline{CD}$, you can first prove that $\triangle CBA \cong \triangle CDA$. You are given $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. $\overline{CA} \cong \overline{CA}$ by the Reflexive Property of Congruence (Theorem 2.1). You can use the ASA Congruence Theorem (Theorem 5.10) to prove that $\triangle CBA \cong \triangle CDA$.

Plan for Proof Use the ASA Congruence Theorem (Theorem 5.10) to prove that $\triangle CBA \cong \triangle CDA$. Then state that $\overline{CB} \cong \overline{CD}$. Use the SAS Congruence Theorem (Theorem 5.5) to prove that $\triangle BCE \cong \triangle DCE$.

Monitoring Progress

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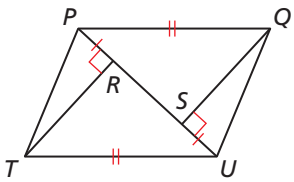
2. In Example 2, does it matter how far from point N you place a stake at point K? Explain.

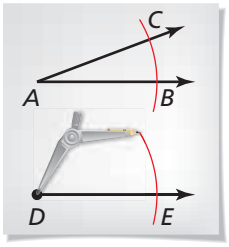
3. Write a plan to prove that $\triangle PTU \cong \triangle UQP$.



Proving Constructions

Recall that you can use a compass and a straightedge to copy an angle. The construction is shown below. You can use congruent triangles to prove that this construction is valid.

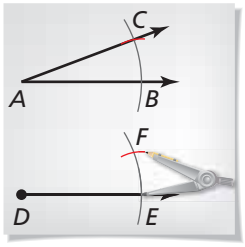
Step 1



Draw a segment and arcs

To copy $\angle A$, draw a segment with initial point D. Draw an arc with center A. Using the same radius, draw an arc with center D. Label points B, C, and E.

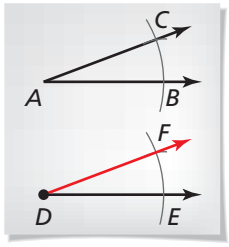
Step 2



Draw an arc

Draw an arc with radius BC and center E. Label the intersection F.

Step 3



Draw a ray

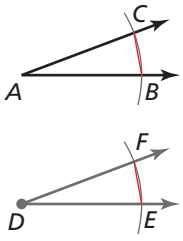
Draw $\overrightarrow{DF}$. In Example 4, you will prove that $\angle D \cong \angle A$.

EXAMPLE 4 Proving a Construction

Write a proof to verify that the construction for copying an angle is valid.

SOLUTION

Add $\overline{BC}$ and $\overline{EF}$ to the diagram. In the construction, one compass setting determines $\overline{AB}$, $\overline{DE}$, $\overline{AC}$, and $\overline{DF}$, and another compass setting determines $\overline{BC}$ and $\overline{EF}$. So, you can assume the following as given statements.



Given $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$

Prove $\angle D \cong \angle A$

Plan for Proof Show that $\triangle DEF \cong \triangle ABC$, so you can conclude that the corresponding parts $\angle D$ and $\angle A$ are congruent.

Plan in Action

STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$	1. Given
2. $\triangle DEF \cong \triangle ABC$	2. SSS Congruence Theorem (Theorem 5.8)
3. $\angle D \cong \angle A$	3. Corresponding parts of congruent triangles are congruent.

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4. Use the construction of an angle bisector on page 42. What segments can you assume are congruent?

5.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

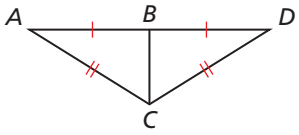
1. **COMPLETE THE SENTENCE** _____ parts of congruent triangles are congruent.

2. **WRITING** Describe a situation in which you might choose to use indirect measurement with congruent triangles to find a measure rather than measuring directly.

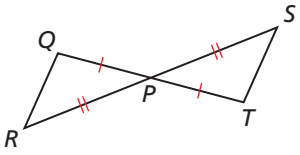
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, explain how to prove that the statement is true. *(See Example 1.)*

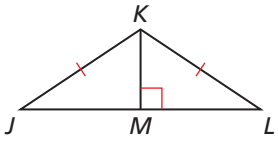
3. $\angle A \cong \angle D$



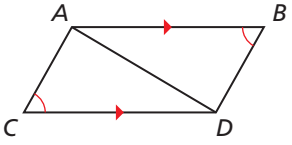
4. $\angle Q \cong \angle T$



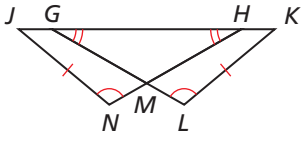
5. $\overline{JM} \cong \overline{LM}$



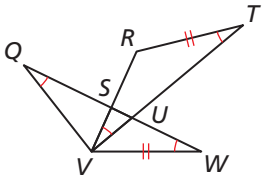
6. $\overline{AC} \cong \overline{DB}$



7. $\overline{GK} \cong \overline{HJ}$

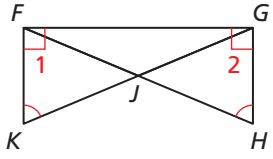


8. $\overline{QW} \cong \overline{VT}$

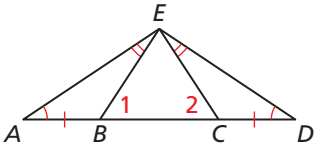


In Exercises 9–12, write a plan to prove that $\angle 1 \cong \angle 2$. *(See Example 3.)*

9.



10.



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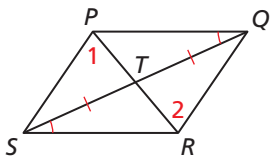
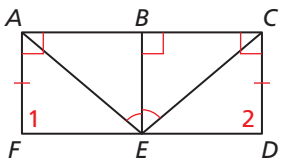
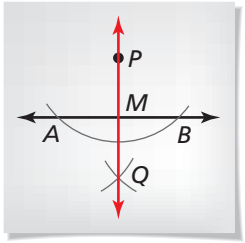


12.



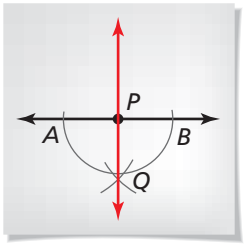
In Exercises 13 and 14, write a proof to verify that the construction is valid. *(See Example 4.)*

13. Line perpendicular to a line through a point not on the line



Plan for Proof Show that $\triangle APQ \cong \triangle BPQ$ by the SSS Congruence Theorem (Theorem 5.8). Then show that $\triangle APM \cong \triangle BPM$ using the SAS Congruence Theorem (Theorem 5.5). Use corresponding parts of congruent triangles to show that $\angle AMP$ and $\angle BMP$ are right angles.

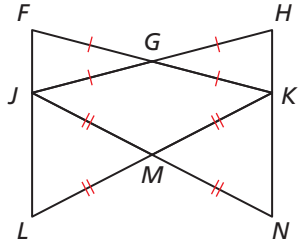
14. Line perpendicular to a line through a point on the line



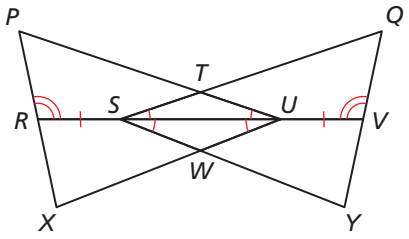
Plan for Proof Show that $\triangle APQ \cong \triangle BPQ$ by the SSS Congruence Theorem (Theorem 5.8). Use corresponding parts of congruent triangles to show that $\angle QPA$ and $\angle QPB$ are right angles.

In Exercises 15 and 16, use the information given in the diagram to write a proof.

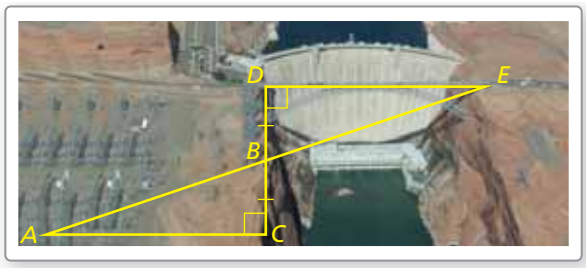
15. **Prove** $\overline{FL} \cong \overline{HN}$



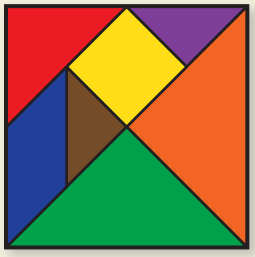
16. **Prove** $\triangle PUX \cong \triangle QSY$



17. **MODELING WITH MATHEMATICS** Explain how to find the distance across the canyon. *(See Example 2.)*



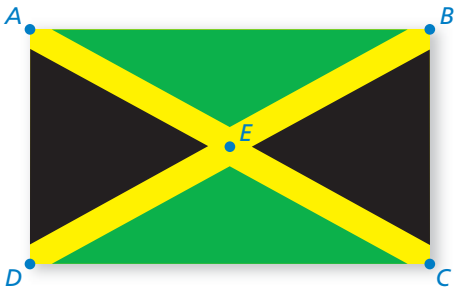
18. **HOW DO YOU SEE IT?** Use the tangram puzzle.



a. Which triangle(s) have an area that is twice the area of the purple triangle?

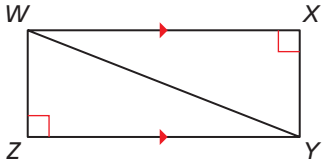
b. How many times greater is the area of the orange triangle than the area of the purple triangle?

19. **PROOF** Prove that the green triangles in the Jamaican flag are congruent if $\overline{AD} \parallel \overline{BC}$ and E is the midpoint of $\overline{AC}$.



20. **THOUGHT PROVOKING** The Bermuda Triangle is a region in the Atlantic Ocean in which many ships and planes have mysteriously disappeared. The vertices are Miami, San Juan, and Bermuda. Use the Internet or some other resource to find the side lengths, the perimeter, and the area of this triangle (in miles). Then create a congruent triangle on land using cities as vertices.

21. **MAKING AN ARGUMENT** Your friend claims that $\triangle WZY$ can be proven congruent to $\triangle YXW$ using the HL Congruence Theorem (Thm. 5.9). Is your friend correct? Explain your reasoning.

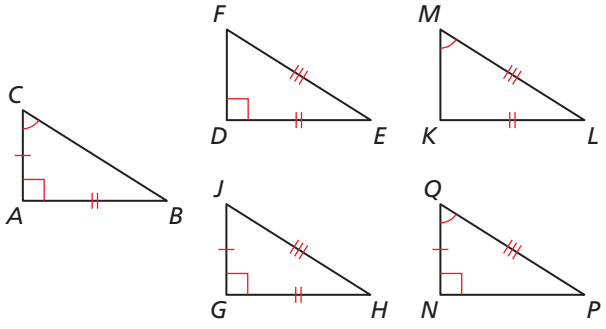


22. **CRITICAL THINKING** Determine whether each conditional statement is true or false. If the statement is false, rewrite it as a true statement using the converse, inverse, or contrapositive.

a. If two triangles have the same perimeter, then they are congruent.

b. If two triangles are congruent, then they have the same area.

23. **ATTENDING TO PRECISION** Which triangles are congruent to $\triangle ABC$? Select all that apply.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the perimeter of the polygon with the given vertices. *(Section 1.4)*

24. $A(-1, 1), B(4, 1), C(4, -2), D(-1, -2)$

25. $J(-5, 3), K(-2, 1), L(3, 4)$

5.8 Coordinate Proofs

Essential Question How can you use a coordinate plane to write a proof?

EXPLORATION 1 Writing a Coordinate Proof

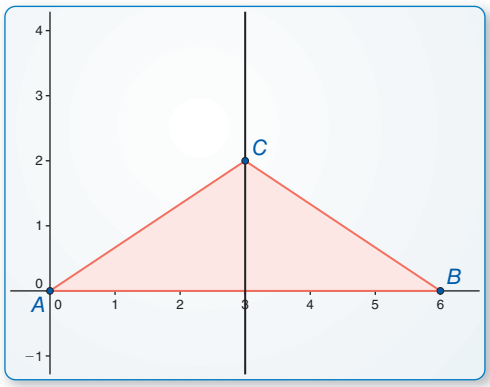
Work with a partner.

a. Use dynamic geometry software to draw $\overline{AB}$ with endpoints $A(0, 0)$ and $B(6, 0)$.

b. Draw the vertical line $x = 3$.

c. Draw $\triangle ABC$ so that C lies on the line $x = 3$.

d. Use your drawing to prove that $\triangle ABC$ is an isosceles triangle.



Sample

Points
$A(0, 0)$
$B(6, 0)$
$C(3, y)$
Segments
$AB = 6$
Line
$x = 3$

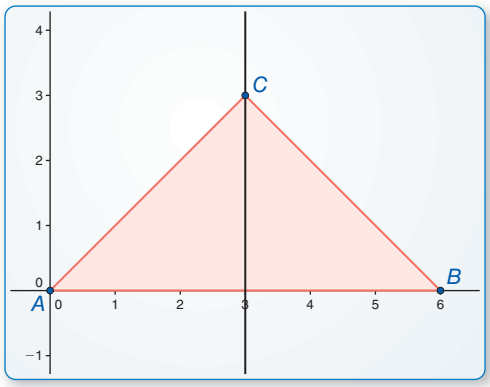
EXPLORATION 2 Writing a Coordinate Proof

Work with a partner.

a. Use dynamic geometry software to draw $\overline{AB}$ with endpoints $A(0, 0)$ and $B(6, 0)$.

b. Draw the vertical line $x = 3$.

c. Plot the point $C(3, 3)$ and draw $\triangle ABC$. Then use your drawing to prove that $\triangle ABC$ is an isosceles right triangle.



Sample

Points
$A(0, 0)$
$B(6, 0)$
$C(3, 3)$
Segments
$AB = 6$
$BC = 4.24$
$AC = 4.24$
Line
$x = 3$

CRITIQUING THE REASONING OF OTHERS

To be proficient in math, you need to understand and use stated assumptions, definitions, and previously established results.

d. Change the coordinates of C so that C lies below the x-axis and $\triangle ABC$ is an isosceles right triangle.

e. Write a coordinate proof to show that if C lies on the line $x = 3$ and $\triangle ABC$ is an isosceles right triangle, then C must be the point $(3, 3)$ or the point found in part (d).

Communicate Your Answer

3. How can you use a coordinate plane to write a proof?

4. Write a coordinate proof to prove that $\triangle ABC$ with vertices $A(0, 0)$, $B(6, 0)$, and $C(3, 3\sqrt{3})$ is an equilateral triangle.

5.8 Lesson

Core Vocabulary

coordinate proof, *p. 284*

What You Will Learn

- Place figures in a coordinate plane.
- Write coordinate proofs.

Placing Figures in a Coordinate Plane

A **coordinate proof** involves placing geometric figures in a coordinate plane. When you use variables to represent the coordinates of a figure in a coordinate proof, the results are true for all figures of that type.

EXAMPLE 1 Placing a Figure in a Coordinate Plane

Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

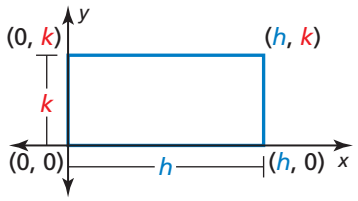
a. a rectangle

b. a scalene triangle

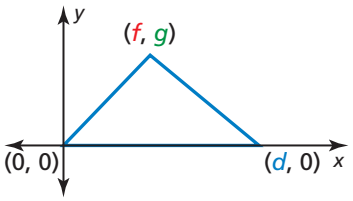
SOLUTION

It is easy to find lengths of horizontal and vertical segments and distances from (0, 0), so place one vertex at the origin and one or more sides on an axis.

a. Let h represent the length and k represent the width.



b. Notice that you need to use three different variables.



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1. Show another way to place the rectangle in Example 1 part (a) that is convenient for finding side lengths. Assign new coordinates.
2. A square has vertices (0, 0), $(m, 0)$, and $(0, m)$. Find the fourth vertex.

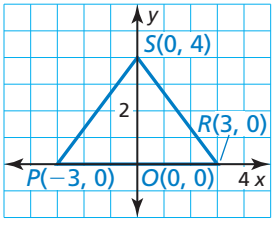
Once a figure is placed in a coordinate plane, you may be able to prove statements about the figure.

EXAMPLE 2 Writing a Plan for a Coordinate Proof

Write a plan to prove that $\overrightarrow{SO}$ bisects $\angle PSR$.

Given Coordinates of vertices of $\triangle POS$ and $\triangle ROS$

Prove $\overrightarrow{SO}$ bisects $\angle PSR$.



SOLUTION

Plan for Proof Use the Distance Formula to find the side lengths of $\triangle POS$ and $\triangle ROS$. Then use the SSS Congruence Theorem (Theorem 5.8) to show that $\triangle POS \cong \triangle ROS$. Finally, use the fact that corresponding parts of congruent triangles are congruent to conclude that $\angle PSO \cong \angle RSO$, which implies that $\overrightarrow{SO}$ bisects $\angle PSR$.

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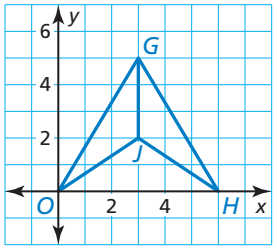
3. Write a plan for the proof.

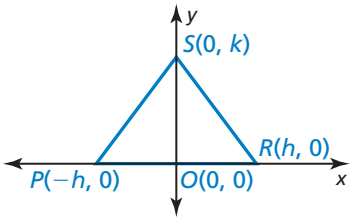
Given $\overrightarrow{GJ}$ bisects $\angle OGH$.

Prove $\triangle GJO \cong \triangle GJH$



The coordinate proof in Example 2 applies to a specific triangle. When you want to prove a statement about a more general set of figures, it is helpful to use variables as coordinates.

For instance, you can use variable coordinates to duplicate the proof in Example 2. Once this is done, you can conclude that $\overrightarrow{SO}$ bisects $\angle PSR$ for any triangle whose coordinates fit the given pattern.

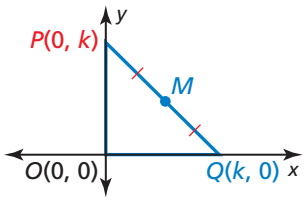


EXAMPLE 3 Applying Variable Coordinates

Place an isosceles right triangle in a coordinate plane. Then find the length of the hypotenuse and the coordinates of its midpoint M.

SOLUTION

Place $\triangle PQO$ with the right angle at the origin. Let the length of the legs be k. Then the vertices are located at $P(0, k)$, $Q(k, 0)$, and $O(0, 0)$.



Use the Distance Formula to find PQ, the length of the hypotenuse.

$$PQ = \sqrt{(k - 0)^2 + (0 - k)^2} = \sqrt{k^2 + (-k)^2} = \sqrt{k^2 + k^2} = \sqrt{2k^2} = k\sqrt{2}$$

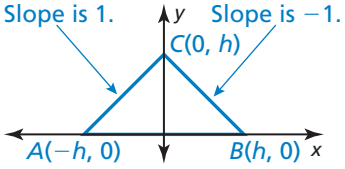
Use the Midpoint Formula to find the midpoint M of the hypotenuse.

$$M\left(\frac{0 + k}{2}, \frac{k + 0}{2}\right) = M\left(\frac{k}{2}, \frac{k}{2}\right)$$

▶ So, the length of the hypotenuse is $k\sqrt{2}$ and the midpoint of the hypotenuse is $\left(\frac{k}{2}, \frac{k}{2}\right)$.

FINDING AN ENTRY POINT

Another way to solve Example 3 is to place a triangle with point C at $(0, h)$ on the y-axis and hypotenuse $\overline{AB}$ on the x-axis. To make $\angle ACB$ a right angle, position A and B so that legs $\overline{CA}$ and $\overline{CB}$ have slopes of 1 and -1, respectively.



Length of hypotenuse $= 2h$

$$M\left(\frac{-h + h}{2}, \frac{0 + 0}{2}\right) = M(0, 0)$$

Monitoring Progress

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4. Graph the points $O(0, 0)$, $H(m, n)$, and $J(m, 0)$. Is $\triangle OHJ$ a right triangle? Find the side lengths and the coordinates of the midpoint of each side.

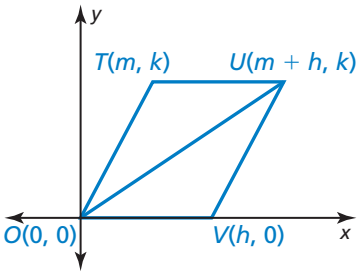
Writing Coordinate Proofs

EXAMPLE 4 Writing a Coordinate Proof

Write a coordinate proof.

Given Coordinates of vertices of quadrilateral $OTUV$

Prove $\triangle OTU \cong \triangle UVO$



SOLUTION

Segments $\overline{OV}$ and $\overline{UT}$ have the same length.

$$OV = |h - 0| = h$$

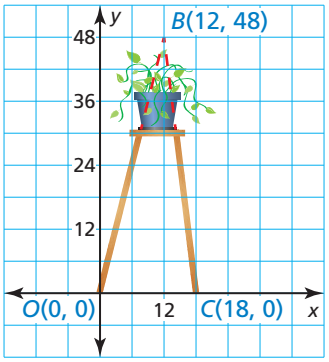
$$UT = |(m + h) - m| = h$$

Horizontal segments $\overline{UT}$ and $\overline{OV}$ each have a slope of 0, which implies that they are parallel. Segment $\overline{OU}$ intersects $\overline{UT}$ and $\overline{OV}$ to form congruent alternate interior angles, $\angle TUO$ and $\angle VOU$. By the Reflexive Property of Congruence (Theorem 2.1), $\overline{OU} \cong \overline{OU}$.

▶ So, you can apply the SAS Congruence Theorem (Theorem 5.5) to conclude that $\triangle OTU \cong \triangle UVO$.

EXAMPLE 5 Writing a Coordinate Proof

You buy a tall, three-legged plant stand. When you place a plant on the stand, the stand appears to be unstable under the weight of the plant. The diagram at the right shows a coordinate plane superimposed on one pair of the plant stand's legs. The legs are extended to form $\triangle OBC$. Prove that $\triangle OBC$ is a scalene triangle. Explain why the plant stand may be unstable.



SOLUTION

First, find the side lengths of $\triangle OBC$.

$$OB = \sqrt{(48 - 0)^2 + (12 - 0)^2} = \sqrt{2448} \approx 49.5$$

$$BC = \sqrt{(18 - 12)^2 + (0 - 48)^2} = \sqrt{2340} \approx 48.4$$

$$OC = |18 - 0| = 18$$

▶ Because $\triangle OBC$ has no congruent sides, $\triangle OBC$ is a scalene triangle by definition. The plant stand may be unstable because $\overline{OB}$ is longer than $\overline{BC}$, so the plant stand is leaning to the right.

Monitoring Progress

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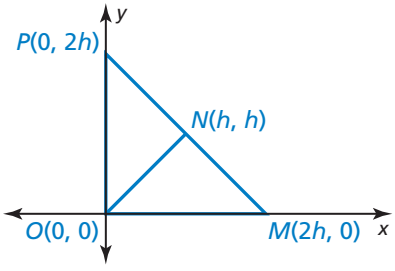
5. Write a coordinate proof.

Given Coordinates of vertices of $\triangle NPO$ and $\triangle NMO$

Prove $\triangle NPO \cong \triangle NMO$



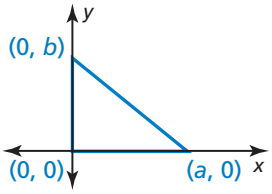
5.8 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** How is a *coordinate proof* different from other types of proofs you have studied? How is it the same?

2. **WRITING** Explain why it is convenient to place a right triangle on the grid as shown when writing a coordinate proof.



Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex. Explain the advantages of your placement. *(See Example 1.)*

3. a right triangle with leg lengths of 3 units and 2 units

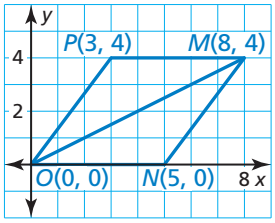
4. a square with a side length of 3 units

5. an isosceles right triangle with leg length p

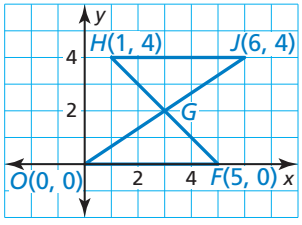
6. a scalene triangle with one side length of $2m$

In Exercises 7 and 8, write a plan for the proof. *(See Example 2.)*

7. **Given** Coordinates of vertices of $\triangle OPM$ and $\triangle ONM$
   Prove $\triangle OPM$ and $\triangle ONM$ are isosceles triangles.



8. **Given** G is the midpoint of $\overline{HF}$.
   Prove $\triangle GHJ \cong \triangle GFO$



In Exercises 9–12, place the figure in a coordinate plane and find the indicated length.

9. a right triangle with leg lengths of 7 and 9 units; Find the length of the hypotenuse.

10. an isosceles triangle with a base length of 60 units and a height of 50 units; Find the length of one of the legs.

11. a rectangle with a length of 5 units and a width of 4 units; Find the length of the diagonal.

12. a square with side length n; Find the length of the diagonal.

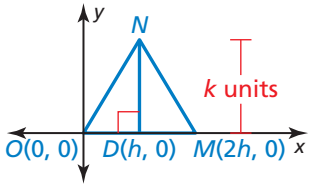
In Exercises 13 and 14, graph the triangle with the given vertices. Find the length and the slope of each side of the triangle. Then find the coordinates of the midpoint of each side. Is the triangle a right triangle? isosceles? Explain. (Assume all variables are positive and $m \neq n$.) *(See Example 3.)*

13. $A(0, 0)$, $B(h, h)$, $C(2h, 0)$

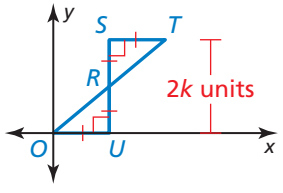
14. $D(0, n)$, $E(m, n)$, $F(m, 0)$

In Exercises 15 and 16, find the coordinates of any unlabeled vertices. Then find the indicated length(s).

15. Find ON and MN.

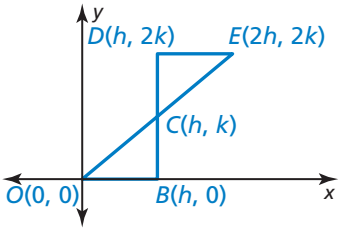


16. Find OT.

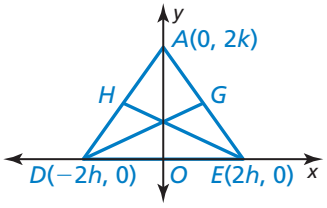


PROOF In Exercises 17 and 18, write a coordinate proof. *(See Example 4.)*

17. Given Coordinates of vertices of $\triangle DEC$ and $\triangle BOC$
Prove $\triangle DEC \cong \triangle BOC$



18. Given Coordinates of $\triangle DEA$, H is the midpoint of $\overline{DA}$, G is the midpoint of $\overline{EA}$.
Prove $\overline{DG} \cong \overline{EH}$



19. MODELING WITH MATHEMATICS You and your cousin are camping in the woods. You hike to a point that is 500 meters east and 1200 meters north of the campsite. Your cousin hikes to a point that is 1000 meters east of the campsite. Use a coordinate proof to prove that the triangle formed by your position, your cousin's position, and the campsite is isosceles. *(See Example 5.)*

20. MAKING AN ARGUMENT Two friends see a drawing of quadrilateral $PQRS$ with vertices $P(0, 2)$, $Q(3, -4)$, $R(1, -5)$, and $S(-2, 1)$. One friend says the quadrilateral is a parallelogram but not a rectangle. The other friend says the quadrilateral is a rectangle. Which friend is correct? Use a coordinate proof to support your answer.

21. MATHEMATICAL CONNECTIONS Write an algebraic expression for the coordinates of each endpoint of a line segment whose midpoint is the origin.

22. REASONING The vertices of a parallelogram are $(w, 0)$, $(0, v)$, $(-w, 0)$, and $(0, -v)$. What is the midpoint of the side in Quadrant III?

Ⓐ $\left(\frac{w}{2}, \frac{v}{2}\right)$ Ⓑ $\left(-\frac{w}{2}, -\frac{v}{2}\right)$

Ⓒ $\left(-\frac{w}{2}, \frac{v}{2}\right)$ Ⓓ $\left(\frac{w}{2}, -\frac{v}{2}\right)$

23. REASONING A rectangle with a length of $3h$ and a width of k has a vertex at $(-h, k)$. Which point cannot be a vertex of the rectangle?

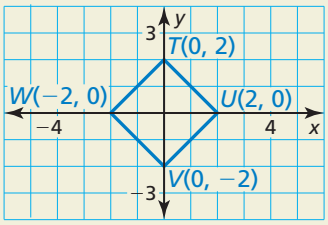
Ⓐ (h, k) Ⓑ $(-h, 0)$

Ⓒ $(2h, 0)$ Ⓓ $(2h, k)$

24. THOUGHT PROVOKING Choose one of the theorems you have encountered up to this point that you think would be easier to prove with a coordinate proof than with another type of proof. Explain your reasoning. Then write a coordinate proof.

25. CRITICAL THINKING The coordinates of a triangle are $(5d, -5d)$, $(0, -5d)$, and $(5d, 0)$. How should the coordinates be changed to make a coordinate proof easier to complete?

26. HOW DO YOU SEE IT? Without performing any calculations, how do you know that the diagonals of square $TUVW$ are perpendicular to each other? How can you use a similar diagram to show that the diagonals of any square are perpendicular to each other?



27. PROOF Write a coordinate proof for each statement.

a. The midpoint of the hypotenuse of a right triangle is the same distance from each vertex of the triangle.

b. Any two congruent right isosceles triangles can be combined to form a single isosceles triangle.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

$\overrightarrow{YW}$ bisects $\angle XYZ$ such that $m\angle XYW = (3x - 7)°$ and $m\angle WYZ = (2x + 1)°$. *(Section 1.5)*

28. Find the value of x.

29. Find $m\angle XYZ$.

5.5–5.8 What Did You Learn?

Core Vocabulary

legs (of a right triangle), *p. 264*
hypotenuse (of a right triangle), *p. 264*
coordinate proof, *p. 284*

Core Concepts

Theorem 5.8 Side-Side-Side (SSS) Congruence Theorem, *p. 262*
Theorem 5.9 Hypotenuse-Leg (HL) Congruence Theorem, *p. 264*
Theorem 5.10 Angle-Side-Angle (ASA) Congruence Theorem, *p. 270*
Theorem 5.11 Angle-Angle-Side (AAS) Congruence Theorem, *p. 271*
Using Congruent Triangles, *p. 278*
Proving Constructions, *p. 280*
Placing Figures in a Coordinate Plane, *p. 284*
Writing Coordinate Proofs, *p. 286*

Mathematical Practices

1. Write a simpler problem that is similar to Exercise 22 on page 267. Describe how to use the simpler problem to gain insight into the solution of the more complicated problem in Exercise 22.
2. Make a conjecture about the meaning of your solutions to Exercises 21–23 on page 275.
3. Identify at least two external resources that you could use to help you solve Exercise 20 on page 282.

Performance Task

Creating the Logo

Congruent triangles are often used to create company logos. Why are they used and what are the properties that make them attractive? Following the required constraints, create your new logo and justify how your shape contains the required properties.

To explore the answers to these questions and more, go to ***BigIdeasMath.com***.

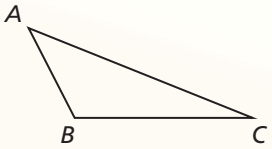
5 Chapter Review

Dynamic Solutions available at *BigIdeasMath.com*

5.1 Angles of Triangles *(pp. 231–238)*

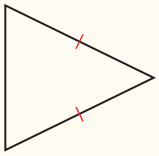
Classify the triangle by its sides and by measuring its angles.

The triangle does not have any congruent sides, so it is scalene. The measure of $\angle B$ is 117°, so the triangle is obtuse.



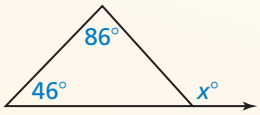
▶ The triangle is an obtuse scalene triangle.

1. Classify the triangle at the right by its sides and by measuring its angles.

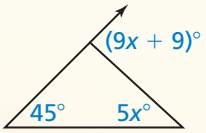


Find the measure of the exterior angle.

2.

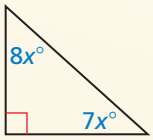


3.

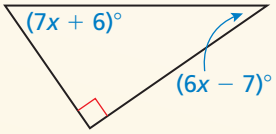


Find the measure of each acute angle.

4.



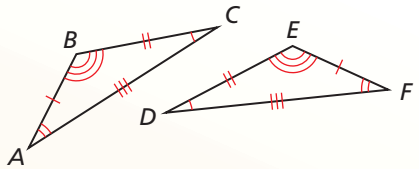
5.



5.2 Congruent Polygons *(pp. 239–244)*

Write a congruence statement for the triangles. Identify all pairs of congruent corresponding parts.

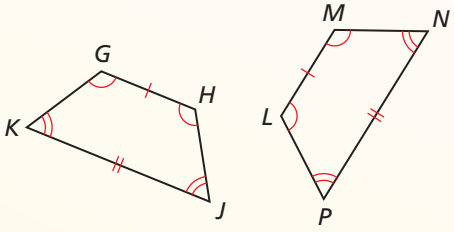
The diagram indicates that $\triangle ABC \cong \triangle FED$.



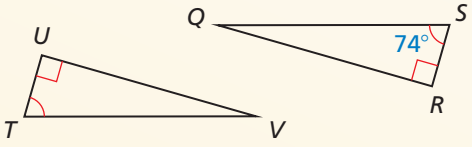
Corresponding angles $\angle A \cong \angle F, \angle B \cong \angle E, \angle C \cong \angle D$

Corresponding sides $\overline{AB} \cong \overline{FE}, \overline{BC} \cong \overline{ED}, \overline{AC} \cong \overline{FD}$

6. In the diagram, $GHJK \cong LMNP$. Identify all pairs of congruent corresponding parts. Then write another congruence statement for the quadrilaterals.



7. Find $m\angle V$.

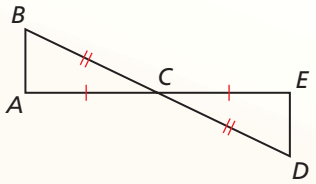


5.3 Proving Triangle Congruence by SAS *(pp. 245–250)*

Write a proof.

Given $\overline{AC} \cong \overline{EC}, \overline{BC} \cong \overline{DC}$

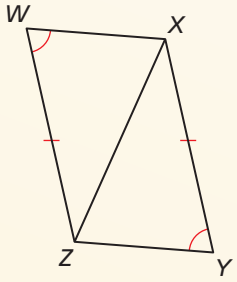
Prove $\triangle ABC \cong \triangle EDC$



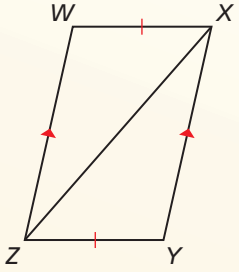
STATEMENTS	REASONS
1. $\overline{AC} \cong \overline{EC}$	**1.** Given
2. $\overline{BC} \cong \overline{DC}$	**2.** Given
3. $\angle ACB \cong \angle ECD$	**3.** Vertical Angles Congruence Theorem (Theorem 2.6)
4. $\triangle ABC \cong \triangle EDC$	**4.** SAS Congruence Theorem (Theorem 5.5)

Decide whether enough information is given to prove that $\triangle WXZ \cong \triangle YZX$ using the SAS Congruence Theorem (Theorem 5.5). If so, write a proof. If not, explain why.

8.



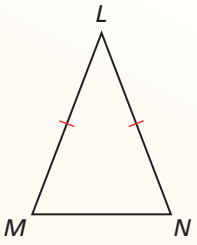
9.



5.4 Equilateral and Isosceles Triangles *(pp. 251–258)*

In $\triangle LMN$, $\overline{LM} \cong \overline{LN}$. Name two congruent angles.

▶ $\overline{LM} \cong \overline{LN}$, so by the Base Angles Theorem (Theorem 5.6), $\angle M \cong \angle N$.



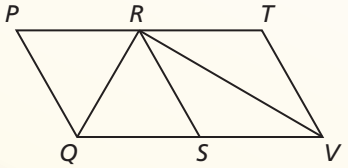
Copy and complete the statement.

10. If $\overline{QP} \cong \overline{QR}$, then $\angle\_\_ \cong \angle\_\_$.

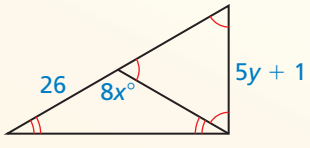
11. If $\angle TRV \cong \angle TVR$, then $\_\_\_ \cong \_\_\_$.

12. If $\overline{RQ} \cong \overline{RS}$, then $\angle\_\_ \cong \angle\_\_$.

13. If $\angle SRV \cong \angle SVR$, then $\_\_\_ \cong \_\_\_$.



14. Find the values of x and y in the diagram.

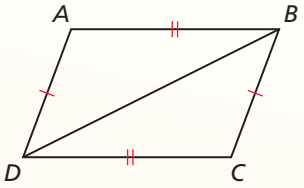


5.5 Proving Triangle Congruence by SSS *(pp. 261–268)*

Write a proof.

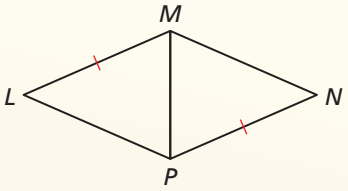
Given $\overline{AD} \cong \overline{CB}, \overline{AB} \cong \overline{CD}$

Prove $\triangle ABD \cong \triangle CDB$

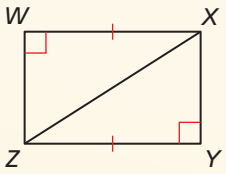


STATEMENTS	REASONS
1. $\overline{AD} \cong \overline{CB}$	**1.** Given
2. $\overline{AB} \cong \overline{CD}$	**2.** Given
3. $\overline{BD} \cong \overline{DB}$	**3.** Reflexive Property of Congruence (Theorem 2.1)
4. $\triangle ABD \cong \triangle CDB$	**4.** SSS Congruence Theorem (Theorem 5.8)

15. Decide whether enough information is given to prove that $\triangle LMP \cong \triangle NPM$ using the SSS Congruence Theorem (Thm. 5.8). If so, write a proof. If not, explain why.



16. Decide whether enough information is given to prove that $\triangle WXZ \cong \triangle YZX$ using the HL Congruence Theorem (Thm. 5.9). If so, write a proof. If not, explain why.

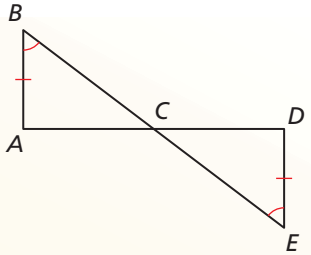


5.6 Proving Triangle Congruence by ASA and AAS *(pp. 269–276)*

Write a proof.

Given $\overline{AB} \cong \overline{DE}, \angle ABC \cong \angle DEC$

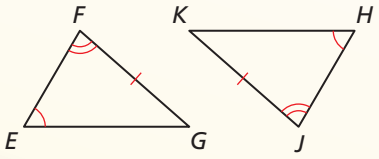
Prove $\triangle ABC \cong \triangle DEC$



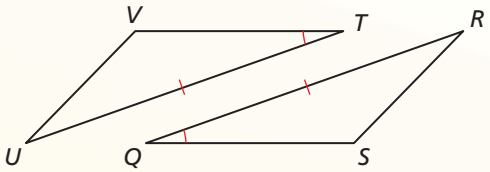
STATEMENTS	REASONS
1. $\overline{AB} \cong \overline{DE}$	**1.** Given
2. $\angle ABC \cong \angle DEC$	**2.** Given
3. $\angle ACB \cong \angle DCE$	**3.** Vertical Angles Congruence Theorem (Thm. 2.6)
4. $\triangle ABC \cong \triangle DEC$	**4.** AAS Congruence Theorem (Thm. 5.11)

Decide whether enough information is given to prove that the triangles are congruent using the AAS Congruence Theorem (Thm. 5.11). If so, write a proof. If not, explain why.

17. $\triangle EFG$, $\triangle HJK$

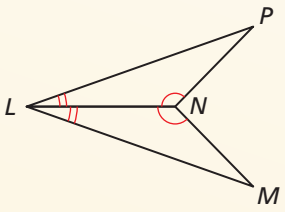


18. $\triangle TUV$, $\triangle QRS$

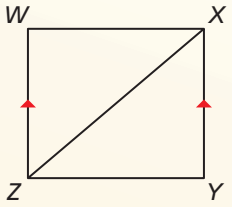


Decide whether enough information is given to prove that the triangles are congruent using the ASA Congruence Theorem (Thm. 5.10). If so, write a proof. If not, explain why.

19. $\triangle LPN$, $\triangle LMN$



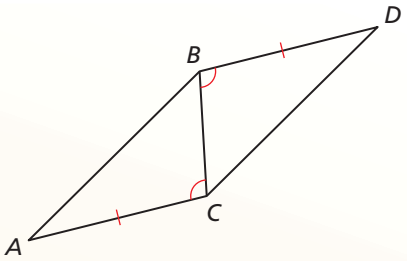
20. $\triangle WXZ$, $\triangle YZX$



5.7 Using Congruent Triangles *(pp. 277–282)*

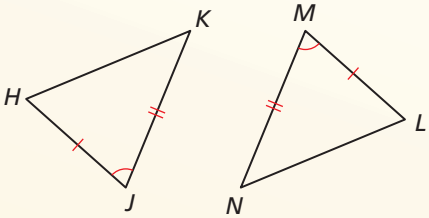
Explain how you can prove that $\angle A \cong \angle D$.

If you can show that $\triangle ABC \cong \triangle DCB$, then you will know that $\angle A \cong \angle D$. You are given $\overline{AC} \cong \overline{DB}$ and $\angle ACB \cong \angle DBC$. You know that $\overline{BC} \cong \overline{CB}$ by the Reflexive Property of Congruence (Thm. 2.1). Two pairs of sides and their included angles are congruent, so by the SAS Congruence Theorem (Thm. 5.5), $\triangle ABC \cong \triangle DCB$.

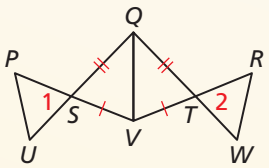


▶ Because corresponding parts of congruent triangles are congruent, $\angle A \cong \angle D$.

21. Explain how to prove that $\angle K \cong \angle N$.



22. Write a plan to prove that $\angle 1 \cong \angle 2$.

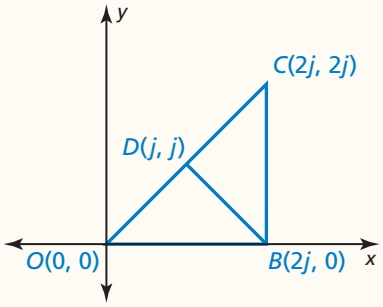


5.8 Coordinate Proofs *(pp. 283–288)*

Write a coordinate proof.

Given Coordinates of vertices of $\triangle ODB$ and $\triangle BDC$

Prove $\triangle ODB \cong \triangle BDC$



Segments $\overline{OD}$ and $\overline{BD}$ have the same length.

$$OD = \sqrt{(j-0)^2 + (j-0)^2} = \sqrt{j^2 + j^2} = \sqrt{2j^2} = j\sqrt{2}$$

$$BD = \sqrt{(j-2j)^2 + (j-0)^2} = \sqrt{(-j)^2 + j^2} = \sqrt{2j^2} = j\sqrt{2}$$

Segments $\overline{DB}$ and $\overline{DC}$ have the same length.

$$DB = BD = j\sqrt{2}$$

$$DC = \sqrt{(2j-j)^2 + (2j-j)^2} = \sqrt{j^2 + j^2} = \sqrt{2j^2} = j\sqrt{2}$$

Segments $\overline{OB}$ and $\overline{BC}$ have the same length.

$$OB = |2j - 0| = 2j$$

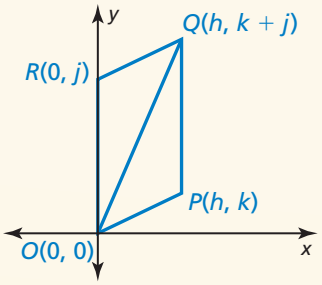
$$BC = |2j - 0| = 2j$$

▶ So, you can apply the SSS Congruence Theorem (Theorem 5.8) to conclude that $\triangle ODB \cong \triangle BDC$.

23. Write a coordinate proof.

Given Coordinates of vertices of quadrilateral $OPQR$

Prove $\triangle OPQ \cong \triangle QRO$



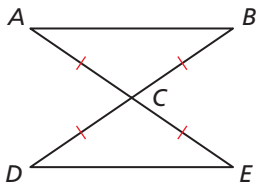
24. Place an isosceles triangle in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

25. A rectangle has vertices $(0, 0)$, $(2k, 0)$, and $(0, k)$. Find the fourth vertex.

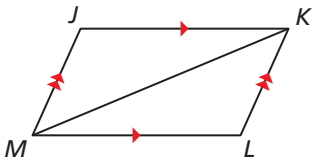
5 Chapter Test

Write a proof.

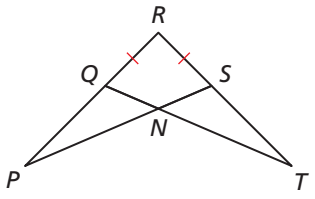
1. **Given** $\overline{CA} \cong \overline{CB} \cong \overline{CD} \cong \overline{CE}$
Prove $\triangle ABC \cong \triangle EDC$



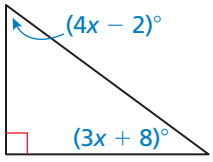
2. **Given** $\overline{JK} \parallel \overline{ML}, \overline{MJ} \parallel \overline{KL}$
Prove $\triangle MJK \cong \triangle KLM$



3. **Given** $\overline{QR} \cong \overline{RS}, \angle P \cong \angle T$
Prove $\triangle SRP \cong \triangle QRT$



4. Find the measure of each acute angle in the figure at the right.

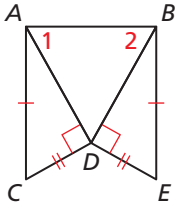


5. Is it possible to draw an equilateral triangle that is not equiangular? If so, provide an example. If not, explain why.

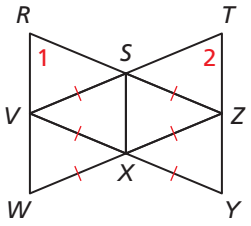
6. Can you use the Third Angles Theorem (Theorem 5.4) to prove that two triangles are congruent? Explain your reasoning.

Write a plan to prove that $\angle 1 \cong \angle 2$.

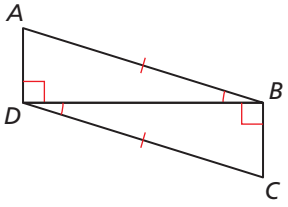
7.

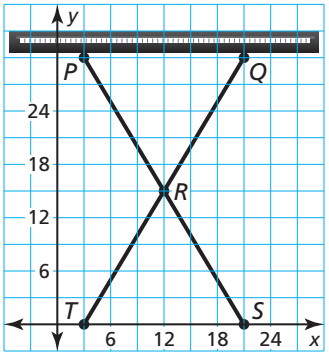


8.



9. Is there more than one theorem that could be used to prove that $\triangle ABD \cong \triangle CDB$? If so, list all possible theorems.





10. Write a coordinate proof to show that the triangles created by the keyboard stand are congruent.

11. The picture shows the Pyramid of Cestius, which is located in Rome, Italy. The measure of the base for the triangle shown is 100 Roman feet. The measures of the other two sides of the triangle are both 144 Roman feet.

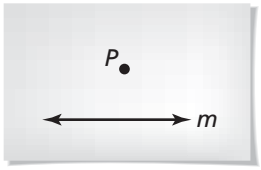
a. Classify the triangle shown by its sides.

b. The measure of $\angle 3$ is 40°. What are the measures of $\angle 1$ and $\angle 2$? Explain your reasoning.

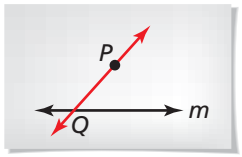
5 Cumulative Assessment

1. Your friend claims that the Exterior Angle Theorem (Theorem 5.2) can be used to prove the Triangle Sum Theorem (Theorem 5.1). Is your friend correct? Explain your reasoning.

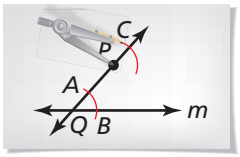
2. Use the steps in the construction to explain how you know that the line through point P is parallel to line m.



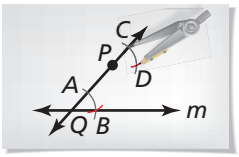
Step 1



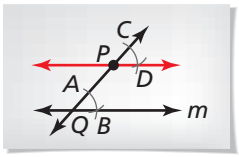
Step 2



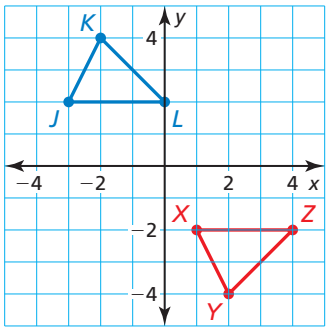
Step 3



Step 4



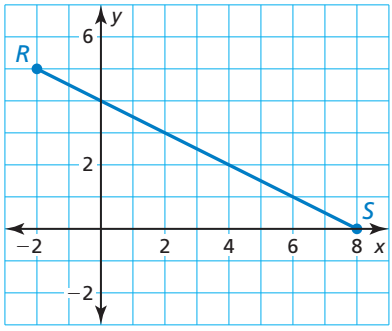
3. The coordinate plane shows $\triangle JKL$ and $\triangle XYZ$.



a. Write a composition of transformations that maps $\triangle JKL$ to $\triangle XYZ$.

b. Is the composition a congruence transformation? If so, identify all congruent corresponding parts.

4. The directed line segment RS is shown. Point Q is located along $\overline{RS}$ so that the ratio of RQ to QS is 2 to 3. What are the coordinates of point Q?

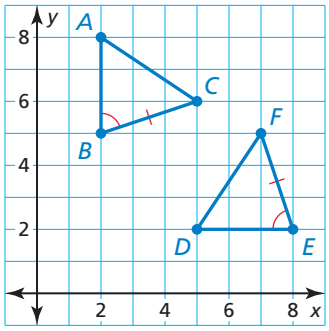


Ⓐ $Q(1.2, 3)$ Ⓑ $Q(4, 2)$ Ⓒ $Q(2, 3)$ Ⓓ $Q(-6, 7)$

5. The coordinate plane shows $\triangle ABC$ and $\triangle DEF$.

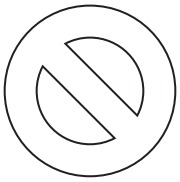
a. Prove $\triangle ABC \cong \triangle DEF$ using the given information.

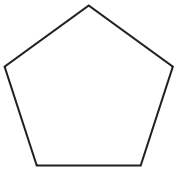
b. Describe the composition of rigid motions that maps $\triangle ABC$ to $\triangle DEF$.



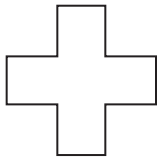
6. The vertices of a quadrilateral are $W(0, 0)$, $X(-1, 3)$, $Y(2, 7)$, and $Z(4, 2)$. Your friend claims that point W will not change after dilating quadrilateral $WXYZ$ by a scale factor of 2. Is your friend correct? Explain your reasoning.

7. Which figure(s) have rotational symmetry? Select all that apply.

Ⓐ 

Ⓑ 

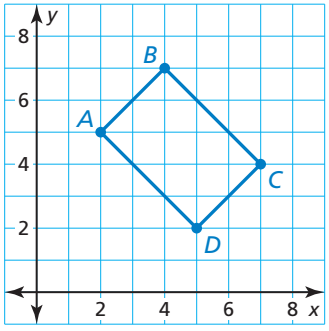
Ⓒ 

Ⓓ 

8. Write a coordinate proof.

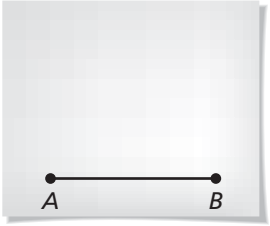
Given Coordinates of vertices of quadrilateral $ABCD$

Prove Quadrilateral $ABCD$ is a rectangle.

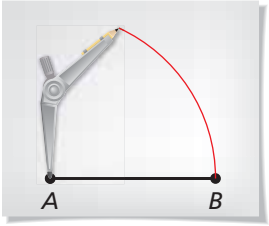


9. Write a proof to verify that the construction of the equilateral triangle shown below is valid.

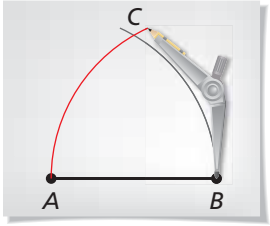
Step 1



Step 2



Step 3



Step 4